Cosmology and Particle Physics

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Lecture 4

Before the hot epoch

With Big Bang nucleosynthesis theory and observations

we are confident of the theory of the early Universe

at temperatures up to $T \simeq 1$ MeV, age $t \simeq 1$ second

With the LHC, we hope to be able to go

up to temperatures $T \sim 100$ GeV, age $t \sim 10^{-10}$ second

Are we going to have a handle on even earlier epoch?

Key: cosmological perturbations

Our Universe is not exactly homogeneous.

Inhomogeneities: ⊙ density perturbations and associated gravitational potentials (3d scalar), observed;

⊙ gravitational waves (3d tensor), not observed (yet?).

Today: inhomogeneities strong and non-linear

In the past: amplitudes small,

$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5}$$

Linear analysis appropriate.

How are they measured?

- Cosmic microwave background: photographic picture of the Universe at age 380 000 yrs, $T = 3000$ K
  - Temperature anisotropy
  - Polarization
- Deep surveys of galaxies and quasars, cover good part of entire visible Universe
- Gravitational lensing, etc.

We have already learned a number of fundamental things

Extrapolation back in time with known laws of physics and known elementary particles and fields $\Longrightarrow$ hot Universe, starts from Big Bang singularity (infinite temperature, infinite expansion rate)

We know that this is not the whole story!

Properties of perturbations in conventional ("hot") Universe.

Reminder:

Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = dt^2 - a^2(t) d\vec{x}^{\,2}$$

$a(t) \propto t^{1/2}$ at radiation domination stage (before $T \simeq 1$ eV, $t \simeq 60$ thousand years)
$a(t) \propto t^{2/3}$ at matter domination stage (until recently).

Cosmological horizon at time t (assuming that nothing preceeded hot epoch): distance that light travels from Big Bang moment,

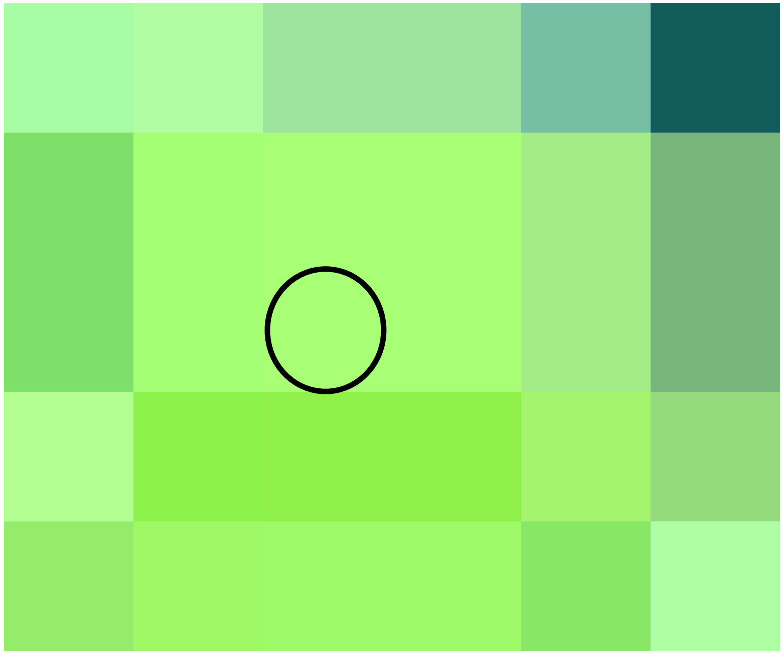
$$l_{H,t} \sim H^{-1}(t) \sim t$$

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

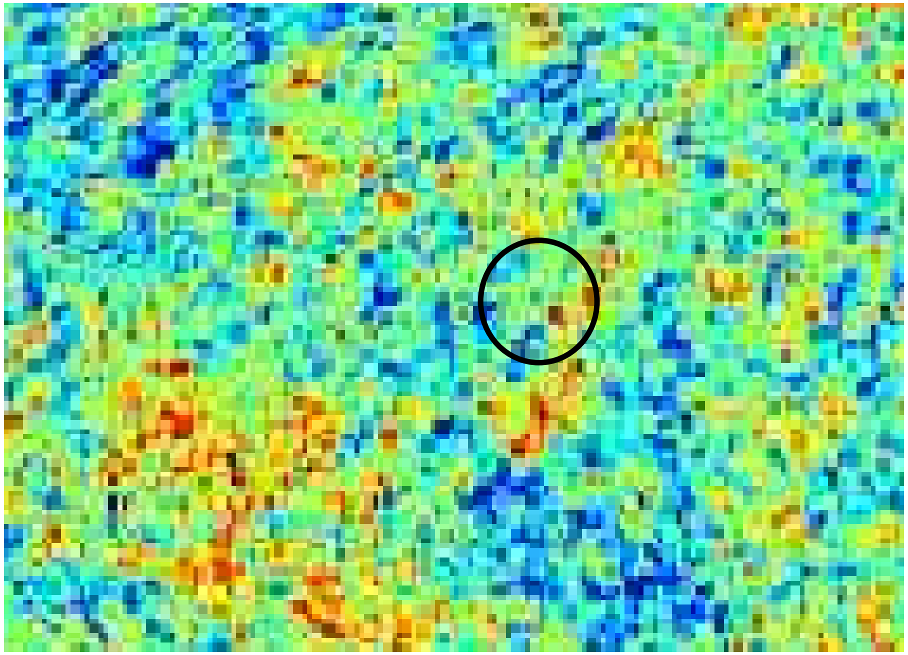
$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_{H,t} \propto t$$

Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.



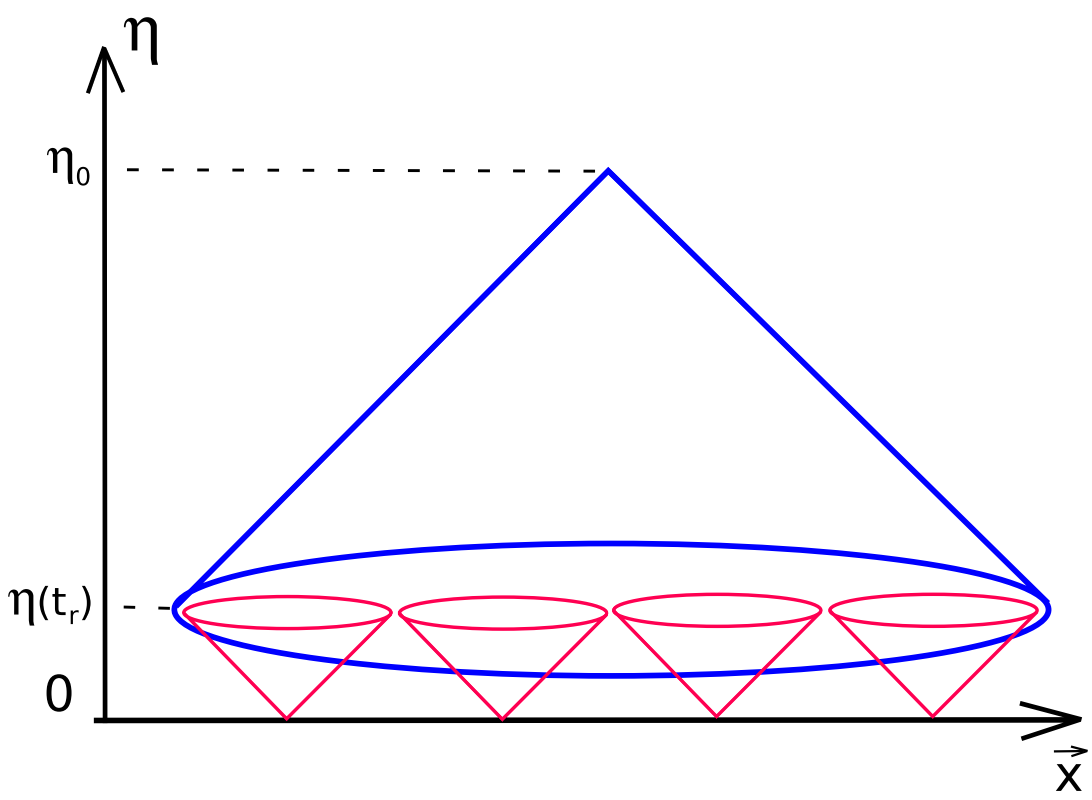
superhorizon mode



subhorizon mode

Causal structure of space-time in hot Big Bang theory (no inflation or anything else before the hot epoch)

$$\eta = \int \frac{dt}{a(t)} \,, \quad \text{conformal time}$$



Major issue: origin of perturbations

Causality $\Longrightarrow$ perturbations can be generated only when they are subhorizon.

Off-hand possibilities:

- Perturbations were never superhorizon, they were generated at the hot cosmological epoch by some causal mechanism.

  E.g., seeded by topological defects (cosmic strings, etc.)

  The only possibility, if expansion started from hot Big Bang.

No longer an option!

- Hot epoch was preceeded by some other epoch. Perturbations were generated then.

Perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase. Why?

Subhorion regime (late times): acoustic oscillations

$$\frac{\delta\rho}{\rho}(\vec{k},t) = A(\vec{k})\mathrm{e}^{i\vec{k}\vec{x}}\cos\left(\int_0^t v_s\,\frac{k}{a(t)}\,dt + \psi\right), \qquad \psi = \text{ arbitrary phase}$$

NB: Physicsl distance $dl = adx \Longleftrightarrow$ physical momentum k/a, gets redshifted.

Sound velocity $v_s \approx 1/\sqrt{3}$.

Solutions to wave equation in superhorizon regime in expanding Universe

$$\frac{\delta\rho}{\rho} = \text{const} \quad \text{and} \quad \frac{\delta\rho}{\rho} = \frac{\text{const}}{t^{3/2}}$$

Assume that modes were superhorizon. Consistency of the picture: the Universe was not very inhomogeneous at early times, the initial condition is (up to amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \implies \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\implies$ phase of oscillations well defined.

$$\frac{\delta\rho}{\rho}(\vec{k},t) = A(\vec{k})\mathrm{e}^{i\vec{k}\vec{x}}\cos\left(\int_0^t v_s \frac{k}{a(t)}\, dt\right),$$

no arbitrary phase

Perturbations come to the time of photon last scattering (= recombination) at different phases, depending on wave vector:

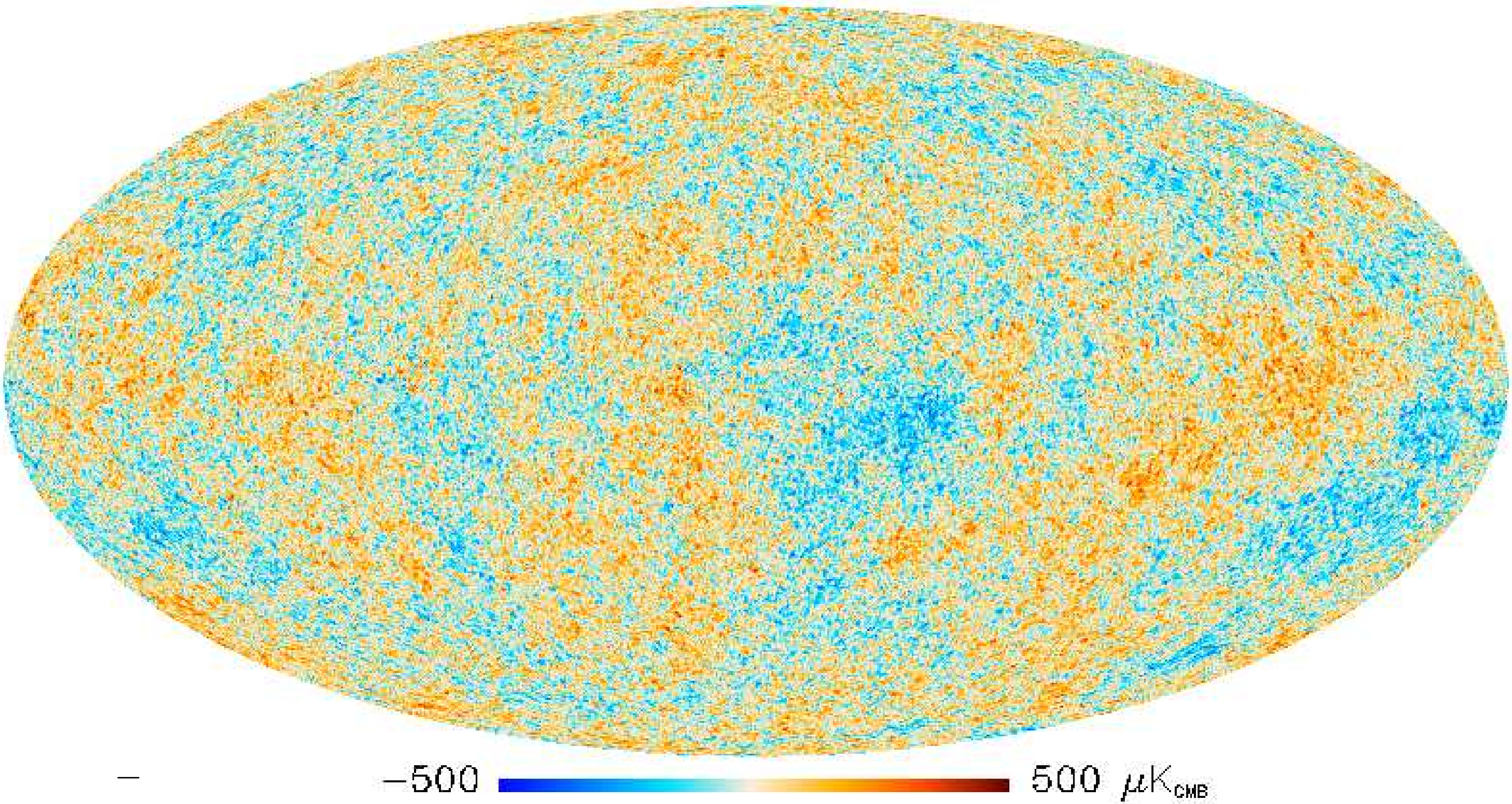
$$\delta(t_r) \equiv \frac{\delta\rho}{\rho}(t_r) \propto \cos\left(k\int_0^{t_r} dt\ \frac{v_s}{a(t)}\right) = \cos(kr_s)$$

r_s: sound horizon at recombination, $a_0 r_s = 150$ Mpc.

Waves with $k = \pi n/r_s$ have large $|\delta\rho|$, while waves with $k = (\pi n + 1/2)/r_s$ have $|\delta\rho| = 0$ in baryon-photon component.

This translates into oscillations in CMB angular spectrum

$$T = 2.726^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck

Fourier decomposition of temperatue fluctuations:

$$\frac{\delta T}{T}(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)$$

a_{lm}: independent Gaussian random variables, $\langle a_{lm} a^*_{l'm'} \rangle \propto \delta_{ll'} \delta_{mm'}$

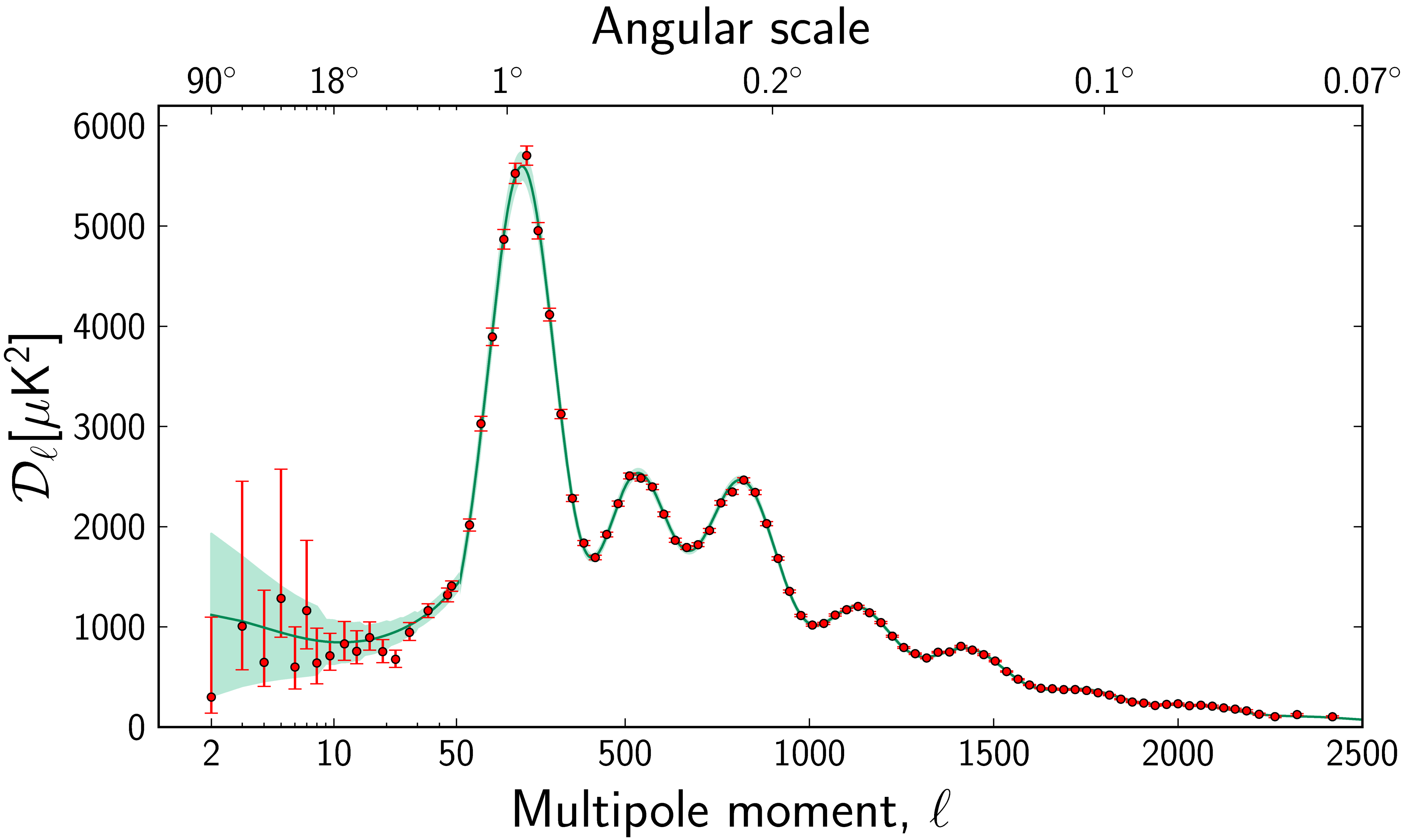
$\langle a^*_{lm} a_{lm} \rangle = C_l$ are measured; usually shown $D_l = \frac{l(l+1)}{2\pi} C_l$

larger $l \Longleftrightarrow$ smaller angular scales, shorter wavelengths

NB: One Universe, one realization of an ensemble $\Longrightarrow$ cosmic variance $\Delta C_l / C_l \simeq 1/\sqrt{2l}$

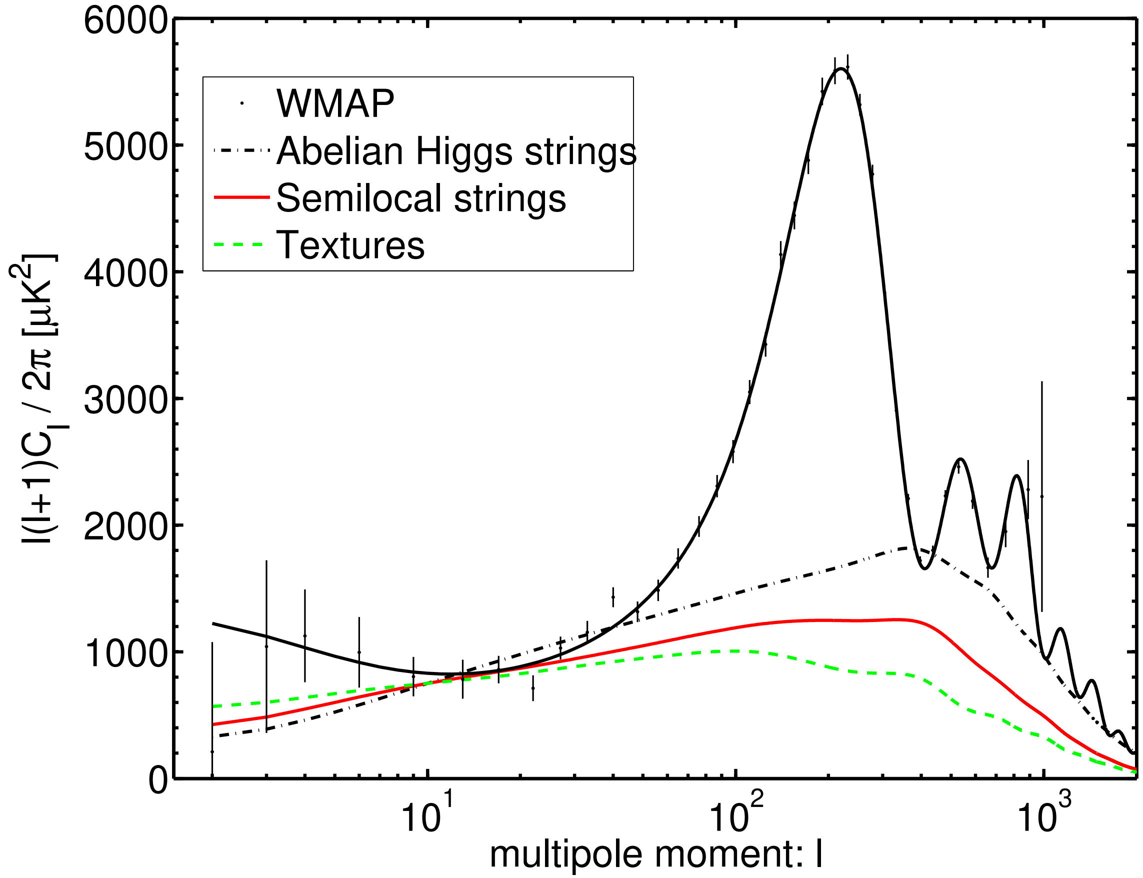
- Physics:
  - Primordial perturbations
  - Development of sound waves in cosmic plasma from early hot stage to recombination
    $\Longrightarrow$ composition of cosmic plasma
  - Propagation of photons after recombination
    $\Longrightarrow$ expansion history of the Universe

CMB angular spectrum



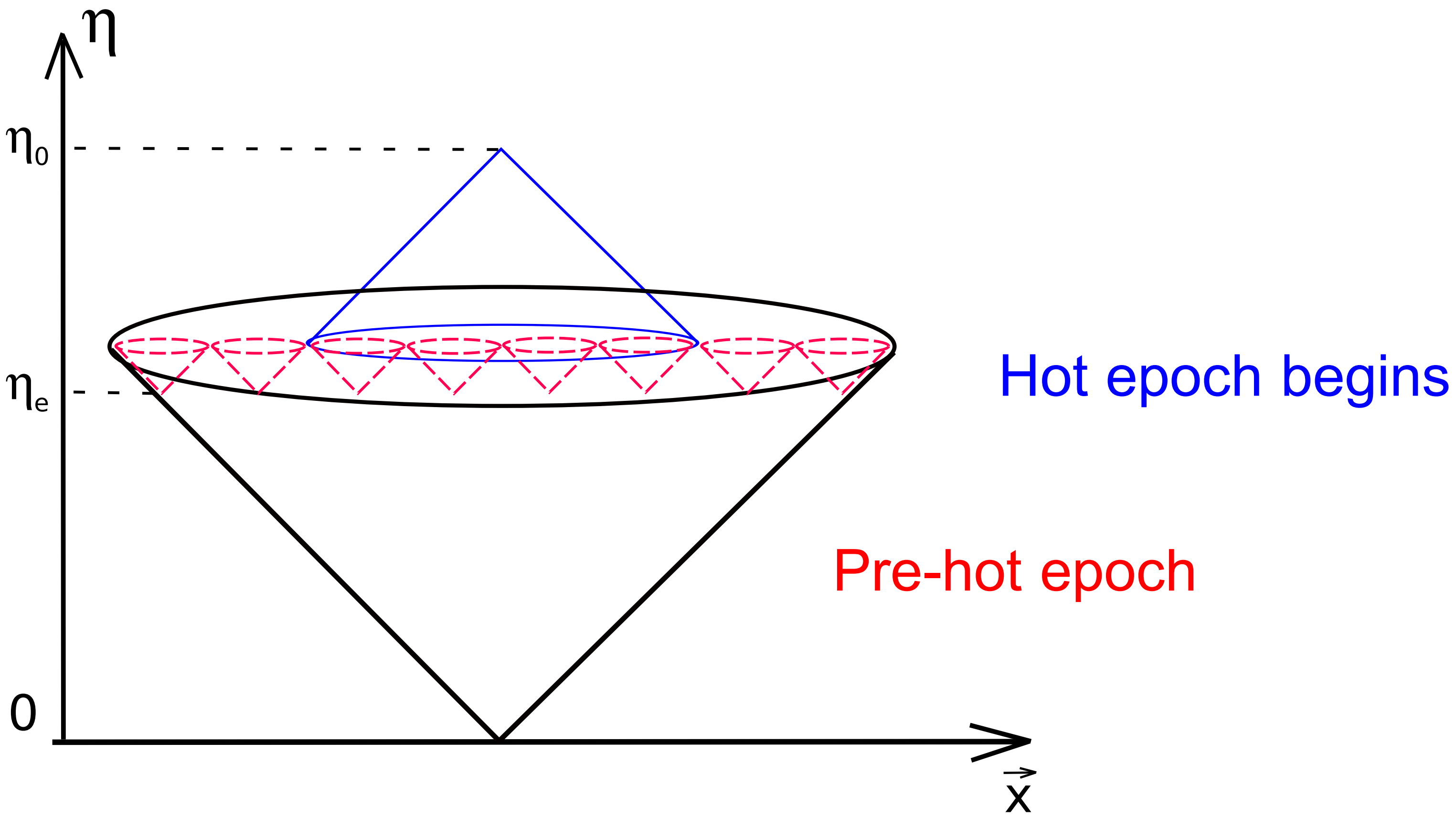
Furthermore, there are perturbations which were superhorizon at the time of photon last scattering (low multipoles, $l \lesssim 50$)

These properties would not be present if perturbations were generated at hot epoch in causal manner: phase ψ would be random function of k, no oscillations in CMB angular spectrum.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

That epoch must have been long (in conformal time) and unusual: perturbations were subhorizon early at that epoch, our visible part of the Universe was in a causally connected region.



Excellent guess: inflation

Starobinsky'79; Guth'81; Linde'82; Albrecht and Steinhardt'82

Exponential expansion with almost constant Hubble rate,

$$a(t) = \mathrm{e}^{\int H dt}, \qquad H \approx \text{const}$$

- Initially Planck-size region expands to entire visible Universe in $t \sim 100\, H^{-1}$ $\Longrightarrow$ for $t \gg 100\, H^{-1}$ the Universe is VERY large
- Perturbations subhorizon early at inflation:

$$\lambda(t) = 2\pi \frac{a(t)}{k} \ll H^{-1}$$

since $a(t) \propto \mathrm{e}^{Ht}$ and $H \approx$ const;
wavelengths gets redshifted, the Hubble parameter stays constant

Alternatives to inflation:

Contraction — Bounce — Expansion,

Start up from static state (“Genesis”)

Difficult, but not impossible.

Other suggestive observational facts about density perturbations

(valid within certain error bars!)

- Perturbations in overall density, not in composition (jargon: "adiabatic")

$$\frac{\text{baryon density}}{\text{entropy density}} = \frac{\text{dark matter density}}{\text{entropy density}} = \text{const in space}$$

Consistent with generation of baryon asymmetry and dark matter at hot stage.

Perturbation in chemical composition (jargon: "isocurvature" or "entropy") $\Longrightarrow$ wrong prediction for CMB angular spectrum $\Longleftrightarrow$ strong constraints from Planck.

NB: even weak variation of composition over space would mean exotic mechanism of baryon asymmetry and/or dark matter generation.

- Primordial perturbations are Gaussian.

Gaussian random field $\delta(\mathbf{k})$: correlators obey Wick's theorem,

$$
\begin{aligned}
\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle &= 0 \\
\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle &= \langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle \cdot \langle \delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle \\
&+ \text{ permutations of momenta}
\end{aligned}
$$

- $\langle \delta(\mathbf{k})\delta^*(\mathbf{k}')\rangle$ means averaging over ensemble of Universes. Realization in our Universe is intrinsically unpredictable.
- strong hint on the origin: enhanced vacuum fluctuations of free quantum field
  Free quantum field

$$\phi(\mathbf{x},t) = \int d^3k e^{-i\mathbf{kx}} \left(f_{\mathbf{k}}^{(+)}(t) a_{\mathbf{k}}^{\dagger} + e^{i\mathbf{kx}} f_{\mathbf{k}}^{(-)}(t) a_{\mathbf{k}} \right)$$

In vacuo $f_{\mathbf{k}}^{(\pm)}(t) = \mathrm{e}^{\pm i\omega_k t}$

Enhanced perturbations: large $f_{\mathbf{k}}^{(\pm)}$. But in any case, Wick's theorem valid

- Inflation does the job very well: vacuum fluctuations of all light fields get enhanced greatly due to fast expansion of the Universe.

  Including the field that dominates energy density (inflaton) $\Longrightarrow$ perturbations in energy density.

  Mukhanov, Chibisov'81; Hawking'82; Starobinsky'82; Guth, Pi'82; Bardeen et.al.'83

- Enhancement of vacuum fluctuations is less automatic in alternative scenarios

- Non-Gaussianity: big issue
  - Very small in the simplest inflationary theories
  - Sizeable in more contrived inflationary models and in alternatives to inflation. Often begins with bispectrum (3-point function; vanishes for Gaussian field)

$$\langle \delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\rangle = \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\; G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$$

    Shape of $G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$ different in different models $\Longrightarrow$ potential discriminator.

  - In some models bispectrum vanishes, e.g., due to some symmetries. But trispectrum (connected 4-point function) may be measurable.

Non-Gaussianity has not been detected yet
strong constraints from Planck

Primordial power spectrum is nearly flat

Homogeneity and anisotropy of Gaussian random field:

$$\langle \frac{\delta\rho}{\rho}(\vec{k}) \frac{\delta\rho}{\rho}(\vec{k}') \rangle = \frac{1}{4\pi k^3} \mathscr{P}(k) \delta(\vec{k} + \vec{k}')$$

$\mathscr{P}(k) =$ power spectrum, gives fluctuation in logarithmic interval of momenta,

$$\langle \left(\frac{\delta\rho}{\rho}(\vec{x}) \right)^2 \rangle = \int_0^\infty \frac{dk}{k} \mathscr{P}(k)$$

Flat spectrum: $\mathscr{P}$ is independent of k

Harrison' 70; Zeldovich' 72, Peebles,Yu' 70

Parametrization

$$\mathscr{P}(k) = A \left(\frac{k}{k_*} \right)^{n_s - 1}$$

$A =$ amplitude, $(n_s - 1) =$ tilt, $k_* =$ fiducial momentum (matter of convention). Flat spectrum $\Longleftrightarrow n_s = 1$.
Experiment: $n_s = 0.96 \pm 0.01$ (WMAP, Planck, ...)

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

Relevant symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

- Alternative: conformal symmetry

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.
$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97
Concrete models: V.R.' 09;
Creminelli, Nicolis, Trincherini' 10.

Exploratory stage: toy models + general arguments so far.

- Tensor modes = primordial gravitational waves

Sizeable amplitude, (almost) flat power spectrum predicted by simplest (and hence most plausible) inflationary models
but not alternatives to inflation

Smoking gun for inflation

Tensor perturbations = gravity waves

Metric perturbations

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$h_{ij} = h_{ij}(\vec{x}, t)$, $h^i_i = \partial_i h^i_j = 0$, spin 2.

Gravity waves: effects on CMB

- Temperature anisotropy (in addition to effect of scalar perturbations)

V.R., Sazhin, Veryaskin' 1982; Fabbri, Pollock' 83

WMAP, Planck

NB: gravity wave amplitudes are time-independent when superhorizon and decay as $h_{ij} \propto a^{-1}(t)$ in subhorizon regime.

Strongest contribution to δT at large angles

$$\Delta\theta \gtrsim 2^o, \quad l \lesssim 50, \quad \text{Present wavelengths } \sim 1 \text{ Gpc}$$

- Polarization

Basko, Polnarev' 1980; Polnarev' 1985; Sazhin, Benitez' 1995

especially B-mode

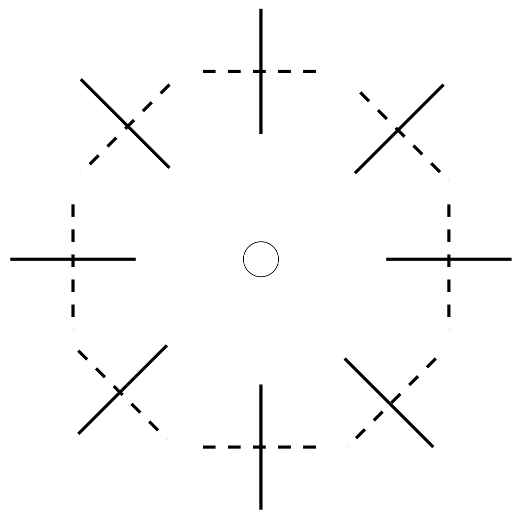
Kamionkowski, Kosowski, Stebbins' 1997; Seljak, Zaldarriaga' 1997

Weak signal, degree of polarization $P(l) \propto l$ at $l \lesssim 50$ and decays with l at $l > 50$.

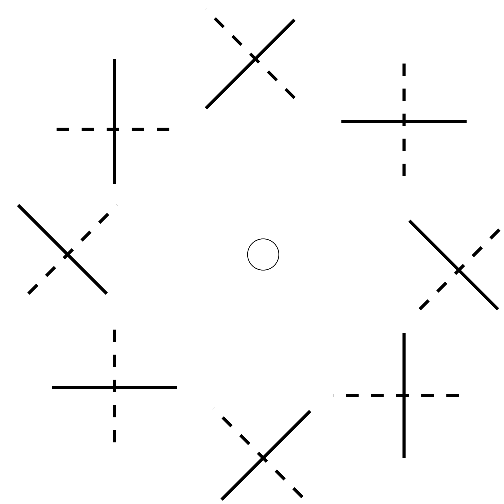
Amplitude at $r = 0.2$:

$$P(l \sim 30) \sim 3 \cdot 10^{-8} \Longrightarrow P \cdot T \sim 0.1 \ \mu\text{K}$$

Linear polariation: E- and B-modes



E-mode, parity even



B-mode, parity odd

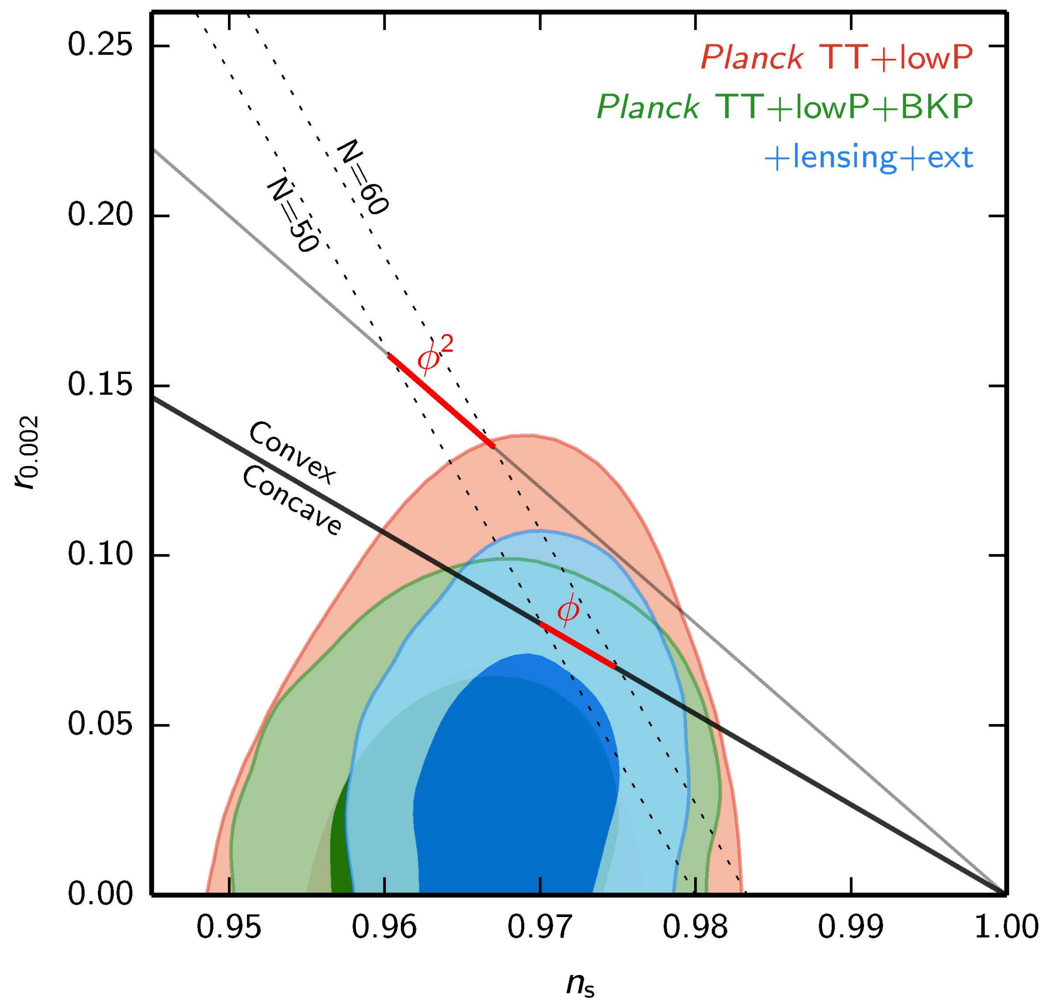
From both scalar
and tensor perturbations

From tensors only

(+ lensing by structures
at relatively small angular scales)

Planck and everybody else

Scalar spectral index vs. power of tensors



To summarize:

- Available data on cosmological perturbations (notably, CMB anisotropies) give confidence that the hot stage of the cosmological evolution was preceeded by some other epoch, at which these perturbations were generated.

- Inflation is consistent with all data. But there are competitors: the data may rather be viewed as pointing towards early conformal epoch of the cosmological evolution.

  More options:
  Matter bounce,
  Negative exponential potential,
  Lifshitz scalar, ...

- Only very basic things are known for the time being.

Good chance for future

- Detection of B-mode of CMB polarization generated by primordial gravity waves $\Longrightarrow$ simple inflation
  - Together with scalar and tensor tilts $\Longrightarrow$ properties of inflaton
- Non-trivial correlation properties of density perturbations (non-Gaussianity) $\Longrightarrow$ contrived inflation, or something entirely different.
  - Shape of non-Gaussianity $\Longrightarrow$ choice between various alternatives
- Statistical anisotropy $\Longrightarrow$ anisotropic pre-hot epoch.
  - Shape of statistical anisotropy $\Longrightarrow$ specific anisotropic model
- Admixture of entropy (isocurvature) perturbations $\Longrightarrow$ generaion of dark matter and/or baryon asymmetry before the hot epoch

At the eve of new physics

LHC $\Longleftrightarrow$ Planck,
dedicated CMB polarization experiments,
data and theoretical understanding
of structure formation ...

Good chance to learn
what preceeded the hot Big Bang epoch

Barring the possibility that Nature is dull