

MC for fermion pair bremsstrahlung in decays

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- ▶ With the increasing precision of measurements more detailed theoretical calculations are needed for interpretation of results in the language of physics parameters such as masses or couplings of Z and W bosons.
- ▶ My goal is to improve PHOTOS¹ algorithm for simulation of the additional pair emissions in decays of Z boson.
- ▶ Single lepton emission kernel can be used for other than Z to l^+l^- . I attempt to do it for $\tau \rightarrow \mu\nu\bar{\nu}l^+l^-$. But it should be useful for l^+l^- pair emission in any decay.
- ▶ As in the past for PHOTOS bremsstrahlung it may generalize to decay of any particle and into any decay channel.

¹N. Davidson et al., [<http://photospp.web.cern.ch/photospp/>];
in current study a version of PHOTOS dated 17.11.2017 is used.

- ▶ PHOTOS algorithm is of the after-burner type.
- ▶ For the previously generated event, with a certain probability, a decay vertex can be replaced with the one featuring additional photons. A similar solution for additional lepton pairs is installed.
- ▶ PHOTOS uses the exact phase space parametrizations thus work is to calculate and implement matrix elements.
- ▶ Such matrix elements could be exact ones or approximate ones, main requirement is that they should be applicable universally.

Extra pair emission

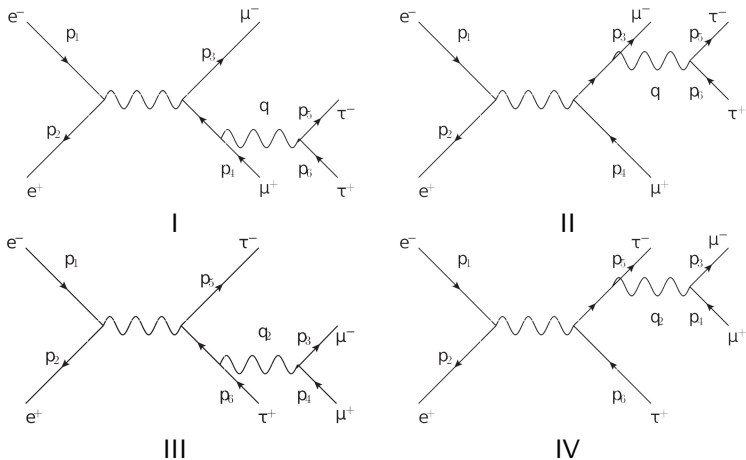


Figure 1: Feynman graphs corresponding to real pair emissions for an example process.

$$\begin{aligned}
 M_1 + M_2 + M_3 + M_4 &= \frac{-e^4}{(p_1 + p_2)^2 q^2} [\bar{v}(p_2) \gamma_\mu u(p_1)] \times & (1) \\
 &\times \left[\bar{u}(p_3) \left(\gamma^\alpha \frac{\not{p}_3 + \not{q} + m_\mu}{(p_3 + q)^2 - m_\mu^2} \gamma^\mu - \gamma^\mu \frac{\not{p}_4 + \not{q} - m_\mu}{(p_4 + q)^2 - m_\mu^2} \gamma^\alpha \right) v(p_4) \right] [\bar{u}(p_5) \gamma_\alpha v(p_6)] - \\
 &- \frac{e^4}{(p_1 + p_2)^2 q_2^2} [\bar{v}(p_2) \gamma_\mu u(p_1)] \times \\
 &\times \left[\bar{u}(p_5) \left(\gamma^\alpha \frac{\not{p}_5 + \not{q}_2 + m_\tau}{(p_5 + q_2)^2 - m_\tau^2} \gamma^\mu - \gamma^\mu \frac{\not{p}_6 + \not{q}_2 - m_\tau}{(p_6 + q_2)^2 - m_\tau^2} \gamma^\alpha \right) v(p_6) \right] [\bar{u}(p_3) \gamma_\alpha v(p_4)]
 \end{aligned}$$

Complete amplitude

$$|M_1 + M_2 + M_3 + M_4|^2 = \frac{e^8}{(p_1 + p_2)^4} \times \quad (2)$$

$$\begin{aligned} & \left\{ \frac{1}{q^4} [\bar{v}(p_2) \gamma_\mu u(p_1)] \left[\bar{u}(p_3) \left(\frac{2p_3^\alpha \gamma^\mu + \gamma^\alpha \not{q} \gamma^\mu}{(p_3 + q)^2 - m_\mu^2} - \frac{2p_4^\alpha \gamma^\mu + \gamma^\mu \not{q} \gamma^\alpha}{(p_4 + q)^2 - m_\mu^2} \right) v(p_4) \right] [\bar{u}(p_5) \gamma_\alpha v(p_6)] \times \right. \\ & \times [\bar{v}(p_6) \gamma_\beta u(p_5)] \left[\bar{v}(p_4) \left(\frac{2p_3^\beta \gamma^\nu + \gamma^\nu \not{q} \gamma^\beta}{(p_3 + q)^2 - m_\mu^2} - \frac{2p_4^\beta \gamma^\nu + \gamma^\beta \not{q} \gamma^\nu}{(p_4 + q)^2 - m_\mu^2} \right) u(p_3) \right] [\bar{u}(p_1) \gamma_\nu v(p_2)] + \\ & + \frac{1}{q^2} [\bar{v}(p_2) \gamma_\mu u(p_1)] \left[\bar{u}(p_5) \left(\frac{2p_5^\alpha \gamma^\mu + \gamma^\alpha \not{q}_2 \gamma^\mu}{(p_5 + q_2)^2 - m_\tau^2} - \frac{2p_6^\alpha \gamma^\mu + \gamma^\mu \not{q}_2 \gamma^\alpha}{(p_6 + q_2)^2 - m_\tau^2} \right) v(p_6) \right] [\bar{u}(p_3) \gamma_\alpha v(p_4)] \times \\ & \times [\bar{v}(p_4) \gamma_\beta u(p_3)] \left[\bar{v}(p_6) \left(\frac{2p_5^\beta \gamma^\nu + \gamma^\nu \not{q}_2 \gamma^\beta}{(p_5 + q_2)^2 - m_\tau^2} - \frac{2p_6^\beta \gamma^\nu + \gamma^\beta \not{q}_2 \gamma^\nu}{(p_6 + q_2)^2 - m_\tau^2} \right) u(p_5) \right] [\bar{u}(p_1) \gamma_\nu v(p_2)] + \\ & \frac{1}{q^2 q_2^2} [\bar{v}(p_2) \gamma_\mu u(p_1)] \left[\bar{u}(p_3) \left(\frac{2p_3^\alpha \gamma^\mu + \gamma^\alpha \not{q} \gamma^\mu}{(p_3 + q)^2 - m_\mu^2} - \frac{2p_4^\alpha \gamma^\mu + \gamma^\mu \not{q} \gamma^\alpha}{(p_4 + q)^2 - m_\mu^2} \right) v(p_4) \right] [\bar{u}(p_5) \gamma_\alpha v(p_6)] \times \\ & \times \left[\bar{v}(p_6) \left(\frac{2p_5^\beta \gamma^\nu + \gamma^\nu \not{q}_2 \gamma^\beta}{(p_5 + q_2)^2 - m_\tau^2} - \frac{2p_6^\beta \gamma^\nu + \gamma^\beta \not{q}_2 \gamma^\nu}{(p_6 + q_2)^2 - m_\tau^2} \right) u(p_5) \right] [\bar{v}(p_4) \gamma_\beta u(p_3)] [\bar{u}(p_1) \gamma_\nu v(p_2)] + \\ & \frac{1}{q^2 q_2^2} [\bar{v}(p_2) \gamma_\mu u(p_1)] \left[\bar{u}(p_5) \left(\frac{2p_5^\alpha \gamma^\mu + \gamma^\alpha \not{q}_2 \gamma^\mu}{(p_5 + q_2)^2 - m_\tau^2} - \frac{2p_6^\alpha \gamma^\mu + \gamma^\mu \not{q}_2 \gamma^\alpha}{(p_6 + q_2)^2 - m_\tau^2} \right) v(p_6) \right] [\bar{u}(p_3) \gamma_\alpha v(p_4)] \times \\ & \times \left[\bar{v}(p_4) \left(\frac{2p_3^\beta \gamma^\nu + \gamma^\nu \not{q} \gamma^\beta}{(p_3 + q)^2 - m_\mu^2} - \frac{2p_4^\beta \gamma^\nu + \gamma^\beta \not{q} \gamma^\nu}{(p_4 + q)^2 - m_\mu^2} \right) u(p_3) \right] [\bar{v}(p_6) \gamma_\beta u(p_5)] [\bar{u}(p_1) \gamma_\nu v(p_2)] \left. \right\} \end{aligned}$$

Spin summation

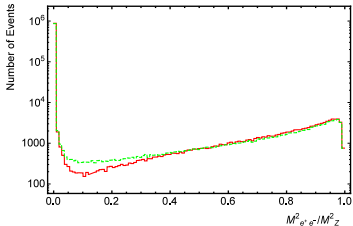
Lepton pair interference is switched off here and appear to be numerically unimportant (formally this is gauge dependent)

$$\begin{aligned}
 \sum_{\text{spins}} |M_1 + M_2|^2 &= \frac{\alpha^4 (4\pi)^4}{(\rho_1 + \rho_2)^4} \frac{1}{q^4} \left\{ \text{Tr} [(\not{\rho}_1 + m_e) \gamma_\mu (\not{\rho}_2 - m_e) \gamma_\nu] \text{Tr} [(\not{\rho}_3 + m_\mu) \gamma^\mu (\not{\rho}_4 - m_\mu) \gamma^\nu] \times \quad (3) \right. \\
 &\times \text{Tr} [(\not{\rho}_5 + m_\tau) \gamma_\alpha (\not{\rho}_6 - m_\tau) \gamma_\beta] \left(\frac{2\rho_3^\alpha}{(\rho_3 + q)^2 - m_\mu^2} - \frac{2\rho_4^\alpha}{(\rho_4 + q)^2 - m_\mu^2} \right) \left(\frac{2\rho_3^\beta}{(\rho_3 + q)^2 - m_\mu^2} - \frac{2\rho_4^\beta}{(\rho_4 + q)^2 - m_\mu^2} \right) + \\
 &+ \text{Tr} [(\not{\rho}_1 + m_e) \gamma_\mu (\not{\rho}_2 - m_e) \gamma_\nu] \text{Tr} [(\not{\rho}_5 + m_\tau) \gamma_\alpha (\not{\rho}_6 - m_\tau) \gamma_\beta] \times \\
 &\times \text{Tr} \left[(\not{\rho}_3 + m_\mu) \left(\frac{\gamma^\alpha \not{q} \gamma^\mu}{(\rho_3 + q)^2 - m_\mu^2} - \frac{\gamma^\mu \not{q} \gamma^\alpha}{(\rho_4 + q)^2 - m_\mu^2} \right) (\not{\rho}_4 - m_\mu) \left(\frac{\gamma^\nu \not{q} \gamma^\beta}{(\rho_3 + q)^2 - m_\mu^2} - \frac{\gamma^\beta \not{q} \gamma^\nu}{(\rho_4 + q)^2 - m_\mu^2} \right) \right] + \\
 &+ \text{Tr} [(\not{\rho}_1 + m_e) \gamma_\mu (\not{\rho}_2 - m_e) \gamma_\nu] \text{Tr} \left[(\not{\rho}_5 + m_\tau) \left(\frac{2\not{\rho}_3}{(\rho_3 + q)^2 - m_\mu^2} - \frac{2\not{\rho}_4}{(\rho_4 + q)^2 - m_\mu^2} \right) (\not{\rho}_6 - m_\tau) \gamma_\beta \right] \times \\
 &\times \text{Tr} \left[(\not{\rho}_3 + m_\mu) \gamma^\mu (\not{\rho}_4 - m_\mu) \left(\frac{\gamma^\nu \not{q} \gamma^\beta}{(\rho_3 + q)^2 - m_\mu^2} - \frac{\gamma^\beta \not{q} \gamma^\nu}{(\rho_4 + q)^2 - m_\mu^2} \right) \right] + \\
 &+ \text{Tr} [(\not{\rho}_1 + m_e) \gamma_\mu (\not{\rho}_2 - m_e) \gamma_\nu] \text{Tr} \left[(\not{\rho}_5 + m_\tau) \gamma_\alpha (\not{\rho}_6 - m_\tau) \left(\frac{2\not{\rho}_3}{(\rho_3 + q)^2 - m_\mu^2} - \frac{2\not{\rho}_4}{(\rho_4 + q)^2 - m_\mu^2} \right) \right] \times \\
 &\times \text{Tr} \left[(\not{\rho}_3 + m_\mu) \left(\frac{\gamma^\alpha \not{q} \gamma^\mu}{(\rho_3 + q)^2 - m_\mu^2} - \frac{\gamma^\mu \not{q} \gamma^\alpha}{(\rho_4 + q)^2 - m_\mu^2} \right) (\not{\rho}_4 - m_\mu) \gamma^\nu \right] \left. \right\}
 \end{aligned}$$

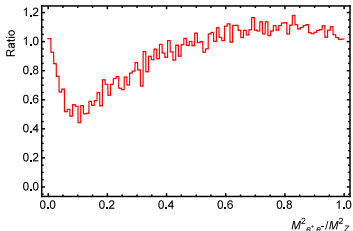
- ▶ KORALW¹ Monte Carlo has been used to generate $e^+e^- \rightarrow 4f$ processes and has provided source of benchmarks for tests². For that purpose it was necessary to run the program for the Center of Mass Energy equal to Z boson mass and Z width was set to a very small value, effectively switching off emission of pair from the initial state.
- ▶ PYTHIA has been used to generate equal number of $e^+e^- \rightarrow Z \rightarrow e^+e^-$ and $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$ events as input for PHOTOS, then PHOTOS has been used to generate $e^+e^- \mu^+\mu^-$ final states. Normalization for the sample size was fixed to assure 1M of four-fermion events.

¹S. Jadach, W. Placzek, M. Skrzypek, B. Ward, and Z. Was, *Comput.Phys.Commun.* 119 (1999) 272–311, hep-ph/9906277.

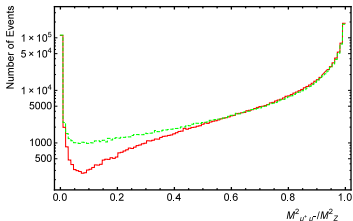
²S. Antropov, A. Arbuzov, R. Sadykov and Z. Was, *Acta Phys. Polon. B* 48, 1469 (2017) doi:10.5506/APhysPolB.48.1469 [arXiv:1706.05571 [hep-ph]].



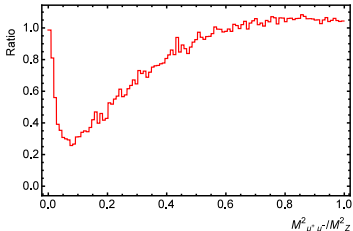
a) Spectrum of electron pair mass squared.



b) Ratio of PHOTOS generated spectrum to the one generated by KORALW.



a) Spectrum of muon pair mass squared.



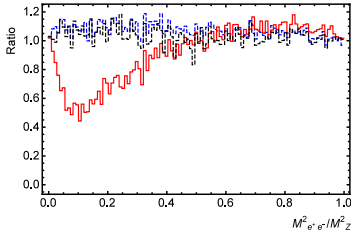
b) Ratio of PHOTOS generated spectrum to the one generated by KORALW.

Figure 2: Lepton pair invariant mass spectra in the channel $Z \rightarrow \mu^+ \mu^- e^+ e^-$.

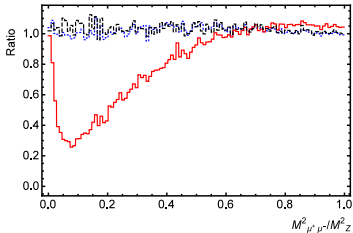
Results generated by published PHOTOS (solid red line) are obtained from samples of equal number of $Z \rightarrow e^+ e^-$ and $Z \rightarrow \mu^+ \mu^-$ decays. They are compared with results from KORALW (dashed green line) where four fermion final state matrix elements are used.

Improvement. Hard pair emission kernel

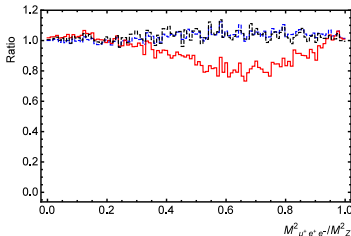
$$\begin{aligned}
 & \sum_{\text{spins}} |M_1 + M_2|^2 - \sum_{\text{spins}} |M_1 + M_2|_{\text{soft}}^2 = \frac{\alpha^4 (4\pi)^4}{(\rho_1 + \rho_2)^4} \frac{1}{q^4} \text{Tr}[(\not{p}_1 + m_e)\gamma_\mu(\not{p}_2 - m_e)\gamma_\nu] \times \quad (4) \\
 & \times \left\{ \frac{16}{(2(\rho_3 q) + q^2)^2} \left[4g^{\mu\nu} m_\mu^2 \frac{q^2}{2} \left(\frac{q^2}{2} + m_\tau^2 \right) + \text{Tr}[\not{p}_3 \gamma^\mu \not{p}_4 \gamma^\nu] \frac{q^2}{2} \cdot p_3 q + \text{Tr}[\not{p}_4 \gamma^\mu (\not{p}_5 - \not{p}_6) \gamma^\nu] \frac{q^2}{2} \cdot p_3 (p_5 - p_6) - \right. \right. \\
 & \left. \left. - \text{Tr}[\not{p}_4 \gamma^\mu \not{p}_5 \gamma^\nu] \left(2(p_3 p_6)^2 + m_\mu^2 \frac{q^2}{2} \right) - \text{Tr}[\not{p}_4 \gamma^\mu \not{p}_6 \gamma^\nu] \left(2(p_3 p_5)^2 + m_\mu^2 \frac{q^2}{2} \right) \right] + \right. \\
 & \left. + \frac{16}{(2(\rho_4 q) + q^2)^2} \left[4g^{\mu\nu} m_\mu^2 \frac{q^2}{2} \left(\frac{q^2}{2} + m_\tau^2 \right) + \text{Tr}[\not{p}_3 \gamma^\mu \not{p}_4 \gamma^\nu] \frac{q^2}{2} \cdot p_4 q + \text{Tr}[\not{p}_3 \gamma^\mu (\not{p}_5 - \not{p}_6) \gamma^\nu] \frac{q^2}{2} \cdot p_4 (p_5 - p_6) - \right. \right. \\
 & \left. \left. - \text{Tr}[\not{p}_3 \gamma^\mu \not{p}_5 \gamma^\nu] \left(2(p_4 p_6)^2 + m_\mu^2 \frac{q^2}{2} \right) - \text{Tr}[\not{p}_3 \gamma^\mu \not{p}_6 \gamma^\nu] \left(2(p_4 p_5)^2 + m_\mu^2 \frac{q^2}{2} \right) \right] + \right. \\
 & \left. + \frac{16}{(2(\rho_3 q) + q^2)(2(\rho_4 q) + q^2)} \left[-2 \text{Tr}[\not{p}_5 \gamma^\mu \not{p}_6 \gamma^\nu] (m_\mu^2 m_\tau^2 + p_3 p_6 \cdot p_4 p_5 + p_3 p_5 \cdot p_4 p_6 - p_3 p_4 \cdot p_5 p_6) - \right. \right. \\
 & \left. \left. - (\text{Tr}[\not{p}_5 \gamma^\mu \not{p}_5 \gamma^\nu] + \text{Tr}[\not{p}_6 \gamma^\mu \not{p}_6 \gamma^\nu]) \left(m_\mu^2 \frac{q^2}{2} + m_\tau^2 \frac{q_2^2}{2} \right) + \text{Tr}[\not{q}_2 \gamma^\mu \not{q}_1 \gamma^\nu] \left(p_3 p_4 \frac{q^2}{2} \right) - \right. \right. \\
 & \left. \left. - \text{Tr}[\not{p}_3 \gamma^\mu \not{p}_3 \gamma^\nu] p_4 q \frac{q^2}{2} - \text{Tr}[\not{p}_4 \gamma^\mu \not{p}_4 \gamma^\nu] p_3 q \frac{q^2}{2} + \right. \right. \\
 & \left. \left. + 2 \text{Tr}[(\not{p}_3 + \not{p}_4 + \not{p}_6) \gamma^\mu \not{p}_6 \gamma^\nu] (p_3 p_5 \cdot p_4 p_5) + 2 \text{Tr}[(\not{p}_3 + \not{p}_4 + \not{p}_5) \gamma^\mu \not{p}_5 \gamma^\nu] (p_3 p_6 \cdot p_4 p_6) + \right. \right. \\
 & \left. \left. + 4g^{\mu\nu} \left(2m_\tau^2 (p_3 p_5 \cdot p_4 p_5 + p_3 p_6 \cdot p_4 p_6) - 2p_5 p_6 (p_3 p_6 \cdot p_4 p_5 + p_3 p_5 \cdot p_4 p_6) - 2m_\tau^2 \frac{q^2}{2} \left(\frac{q_2^2}{2} + p_3 p_4 \right) \right) \right] + \right. \\
 & \left. + 16m_\tau^2 \left(\frac{1}{(2(\rho_3 q) + q^2)} + \frac{1}{(2(\rho_4 q) + q^2)} \right) \times \right. \\
 & \left. \times \left(\frac{\text{Tr}[\not{p}_3 \gamma^\mu \not{q}_1 \gamma^\nu] p_4 q}{2\rho_4 q + q^2} + \frac{\text{Tr}[\not{p}_4 \gamma^\mu \not{q}_1 \gamma^\nu] p_3 q}{2\rho_3 q + q^2} - \frac{\text{Tr}[\not{p}_3 \gamma^\mu \not{p}_4 \gamma^\nu] \frac{q^2}{2}}{2(\rho_3 q) + q^2} - \frac{\text{Tr}[\not{p}_3 \gamma^\mu \not{p}_4 \gamma^\nu] \frac{q^2}{2}}{2(\rho_4 q) + q^2} \right) \right\}
 \end{aligned}$$



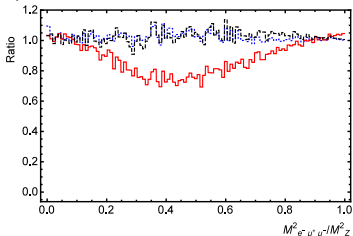
a) e^+e^- .



b) $\mu^+\mu^-$.



c) $\mu^+e^+e^-$.



d) $e^-\mu^+\mu^-$.

Figure 3: Normalized to M_Z^2 ratio of PHOTOS generated spectra in the channel $Z \rightarrow \mu^+\mu^-e^+e^-$ to the one generated by KORALW. Results generated by PHOTOS are obtained from samples of equal number of $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$ decays. Solid red line represents data that correspond to matrix element of published PHOTOS version. Blue and black dashed lines represent data that correspond to my matrix element (4) and further simplified formula (4).

- ▶ Complete matrix element (4) $e^+e^- \rightarrow Z \rightarrow l^+l^- + (L^+L^-)$ is installed and works perfectly.
- ▶ Simplified matrix element (4) $e^+e^- \rightarrow Z \rightarrow l^+l^- + (L^+L^-)$ is installed and works as good as complete one.
- ▶ Matrix element kernel, as of Fig. 3, works better than matrix element of published version of PHOTOS. Difference is up to factor of 3.5 for some parts of the spectra.
- ▶ Kernel, as of Fig. 3, has a potential of being used universally.

Way to make a comparison with tau decays generated by TAUOLA

- ▶ TAUOLA¹ is a MC Generator dedicated to generating tau-lepton decays.
- ▶ Phase space generation is precise and similar to one by PHOTOS.
- ▶ Matrix element for $\tau \rightarrow \mu\nu\bar{\nu}$ is complete down to $O(\alpha)$.
- ▶ Now I want to compare TAUOLA-PHOTOS outputs for $\tau \rightarrow \mu\nu\bar{\nu}$.

¹M. Chruszcz, T. Przedzinski, Z. Was and J. Zaremba, Comput. Phys. Commun. 232, 220 (2018) doi:10.1016/j.cpc.2018.05.017 [arXiv:1609.04617 [hep-ph]].

Extra pair emission for tau decay

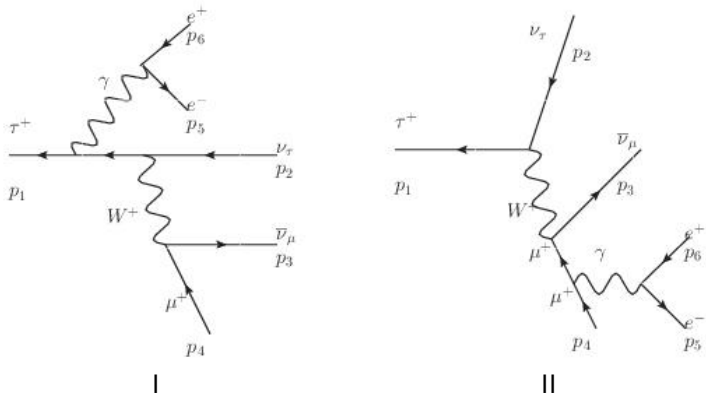


Figure 4: Feynman graphs corresponding to real pair emissions for $\tau^+ \rightarrow \mu^+ \nu \bar{\nu} e^+ e^-$ decay process.

Amplitude for tau decay with e^+e^- pair emission

$$\begin{aligned}
 \sum_{\text{spins}} |M_1 + M_2|^2 &= \frac{e^4}{\left((p_1 - p_2)^2 - M_W^2\right)^2} \frac{e^4}{64 \cdot \sin^4 \theta_W} \frac{e^4}{q^4} \text{Tr} [(\not{p}_5 + m_e) \gamma_\alpha (\not{p}_6 - m_e) \gamma_\beta] \times \quad (5) \\
 &\times \left\{ \text{Tr} [(\not{p}_1 - m_\tau) \gamma^\mu (1 - \gamma^5) (\not{p}_2 - m_\nu) (1 + \gamma^5) \gamma_\nu] \times \right. \\
 &\times \text{Tr} \left[(\not{p}_3 + m_{\bar{\nu}}) \gamma_\mu (1 - \gamma^5) \frac{\not{q} \gamma^\alpha + 2p_4^\alpha}{(q + p_4)^2 - m_\mu^2} (\not{p}_4 - m_\mu) \frac{\gamma^\beta \not{q} + 2p_4^\beta}{(q + p_4)^2 - m_\mu^2} (1 + \gamma^5) \gamma^\nu \right] + \\
 &+ \text{Tr} \left[(\not{p}_1 - m_\tau) \gamma^\mu (1 - \gamma^5) (\not{p}_2 - m_\nu) (1 + \gamma^5) \gamma_\nu \frac{\not{q} \gamma^\beta + 2p_1^\beta}{(p_1 - q)^2 - m_\tau^2} \right] \times \\
 &\times \text{Tr} \left[(\not{p}_3 + m_{\bar{\nu}}) \gamma_\mu (1 - \gamma^5) \frac{\not{q} \gamma^\alpha + 2p_4^\alpha}{(q + p_4)^2 - m_\mu^2} (\not{p}_4 - m_\mu) (1 + \gamma^5) \gamma^\nu \right] + \\
 &+ \text{Tr} \left[(\not{p}_1 - m_\tau) \frac{\gamma^\alpha \not{q} + 2p_1^\alpha}{(p_1 - q)^2 - m_\tau^2} \gamma^\mu (1 - \gamma^5) (\not{p}_2 - m_\nu) (1 + \gamma^5) \gamma_\nu \right] \times \\
 &\times \text{Tr} \left[(\not{p}_3 + m_{\bar{\nu}}) \gamma_\mu (1 - \gamma^5) (\not{p}_4 - m_\mu) \frac{\gamma^\beta \not{q} + 2p_4^\beta}{(q + p_4)^2 - m_\mu^2} (1 + \gamma^5) \gamma^\nu \right] + \\
 &+ \text{Tr} \left[(\not{p}_1 - m_\tau) \frac{\gamma^\alpha \not{q} + 2p_1^\alpha}{(p_1 - q)^2 - m_\tau^2} \gamma^\mu (1 - \gamma^5) (\not{p}_2 - m_\nu) (1 + \gamma^5) \gamma_\nu \frac{\gamma^\alpha \not{q} + 2p_1^\alpha}{(p_1 - q)^2 - m_\tau^2} \right] \times \\
 &\left. \times \text{Tr} [(\not{p}_3 + m_{\bar{\nu}}) \gamma_\mu (1 - \gamma^5) (\not{p}_4 - m_\mu) (1 + \gamma^5) \gamma^\nu] \right\}
 \end{aligned}$$

Amplitude for tau decay with soft pair emission

$$\begin{aligned} \sum_{\text{spins}} |M_1 + M_2|_{\text{soft}}^2 &= \frac{e^4}{\left((p_1 - p_2)^2 - M_W^2\right)^2 64 \cdot \sin^4 \theta_W} \times \quad (6) \\ &\times \text{Tr} \left[(\not{p}_1 - m_\tau) \gamma^\mu (1 - \gamma^5) (\not{p}_2 - m_\nu) (1 + \gamma^5) \gamma_\nu \right] \times \\ &\times \text{Tr} \left[(\not{p}_3 + m_{\bar{\nu}}) \gamma_\mu (1 - \gamma^5) (\not{p}_4 - m_\mu) (1 + \gamma^5) \gamma^\nu \right] \times \\ &\times \frac{e^4}{q^4} \text{Tr} \left[(\not{p}_5 + m_e) \gamma_\alpha (\not{p}_6 - m_e) \gamma_\beta \right] \times \\ &\times \left(\frac{p_4}{qp_4 + \frac{q^2}{2}} - \frac{p_1}{qp_1 - \frac{q^2}{2}} \right)_\alpha \left(\frac{p_4}{qp_4 + \frac{q^2}{2}} - \frac{p_1}{qp_1 - \frac{q^2}{2}} \right)_\beta \end{aligned}$$

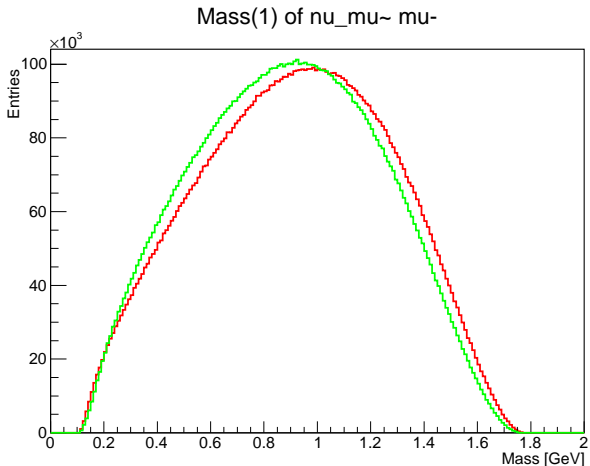


Figure 5: Invariant mass of $\mu^- \nu_\mu$ ¹. Red line corresponds to the data from $\tau^- \rightarrow \mu^- \nu \bar{\nu}$ channel, while green line corresponds to the data from $\tau^- \rightarrow \mu^- \nu \bar{\nu} e^+ e^-$ channel.

¹All TAUOLA modifications are provided by dr inz. Jakub Zaremba, data are prepared by him as well.

- ▶ Full agreement between PHOTOS and KORALW is reached in the channel $Z \rightarrow 4f$ both for exact matrix element installed in PHOTOS and for the one by formula (4).
- ▶ This agreement is reached without terms in matrix element that correspond to interference between lepton pairs.
- ▶ Construction of emission kernel for individual lepton is rather straightforward.
- ▶ First TAUOLA tests are done and look promising. Preparation of tests for $\tau^- \rightarrow \pi^- \nu_\tau e^+ e^-$ is rather straightforward¹.
- ▶ Indication that kernel to be installed into PHOTOS may be universal.

¹A. Guevara, G. Lopez Castro and P. Roig, Phys. Rev. D 88, no. 3, 033007 (2013)
doi:10.1103/PhysRevD.88.033007 [arXiv:1306.1732 [hep-ph]].

I hope that my work may be useful for Belle II and τ -charm phenomenology soon.

Thank you for your attention!

WeakSingleBoson:ffbar2gmZ = on

23:onMode = off

23:onIfAny = 11

23:mMin = 10.0

23:mMax = 200.0

HadronLevel:Hadronize = off

SpaceShower:QEDshowerByL = off

SpaceShower:QEDshowerByQ = off

PartonLevel:ISR = off

PartonLevel:FSR = off

Beams:idA = 2212

Beams:idB = 2212

Beams:eCM = 14000.0

$pp \rightarrow Z \rightarrow e^+e^-(e^+e^-, \mu^+\mu^-)$

WeakSingleBoson:ffbar2gmZ = on

23:onMode = off

23:onIfAny = 13

23:mMin = 10.0

23:mMax = 200.0

HadronLevel:Hadronize = off

SpaceShower:QEDshowerByL = off

SpaceShower:QEDshowerByQ = off

PartonLevel:ISR = off

PartonLevel:FSR = off

Beams:idA = 2212

Beams:idB = 2212

Beams:eCM = 14000.0

$pp \rightarrow Z \rightarrow \mu^+\mu^-(e^+e^-, \mu^+\mu^-)$

Initialization parameters for PYTHIA.