Radiative recoil corrections to the hyperfine splitting of light muonic atoms

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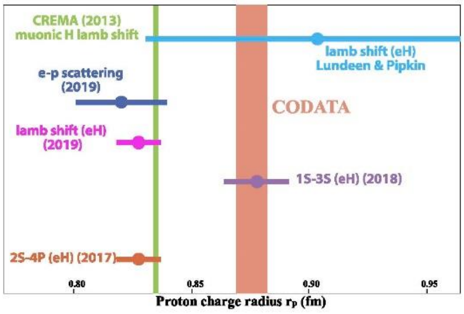
Dorokhov A.E. (JINR), Martynenko F.A. (Samara University)



«Proton Radius Puzzle»

-a disagreement between the value of the proton charge radius r_p obtained from experiments involving muonic hydrogen and those based on electron-proton systems.

(eh): $r_p = 0.8775(51)\ fm$ — *CODATA*

(μh): $r_p = 0.84087(39)\ fm$ — *A. Antognini et al.[CREMA], Science **339**, 417 (2013)*

Recent Researches

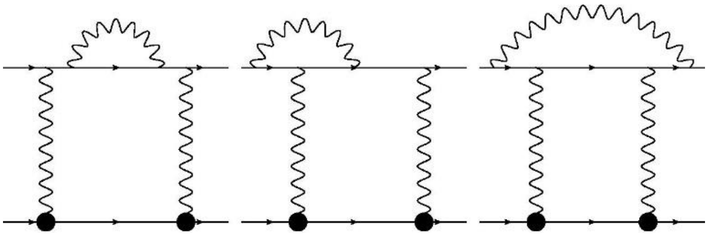
$r_p = 0.8335(95)\ fm$ — *A. Beyer, et al., Science **358**, 79–85 (2017)*

$r_p = 0.877(13)\ fm$ — *H. Fleurbaey et al. Phys. Rev. Lett. **120**, 183001 (2018)*

$r_p = 0.831(7)_{stat}(12)_{sys}\ fm$ — *W. Xiong et al., Nature **575**, 147(2019)*

$r_p = 0.833(10)\ fm$ — *N. Bezginov et al., Science **365**, 1007-1012(2019)*

Radiative recoil corrections to the hyperfine structure of S-states of muonic deuterium



1. Radiative recoil corrections to the muon line. 1- muon self-energy, 2- vertex, 3- spanning photon contributions

The muon-deuteron interaction amplitude can be presented in the form:

$$M_{direct} = \frac{-i(Z\alpha)^2}{\pi^2}\int d^4k[\bar{u}(q_1)L_{\mu\nu}u(p_1)]D_{\mu\omega}(k)D_{\nu\lambda}(k)\times$$
$$[\varepsilon^*_\rho(q_2)\Gamma_{\omega,\rho\beta}(q_2,p_2+k)D_{2,\beta\tau}(p_2+k)\Gamma_{\lambda,\tau\alpha}(p_2+k,p_2)\varepsilon_\alpha(p_2)],$$
$$M_{crossed} = \frac{-i(Z\alpha)^2}{\pi^2}\int d^4k[\bar{u}(q_1)L_{\mu\nu}u(p_1)]D_{\mu\lambda}(k)D_{\nu\omega}(k)\times$$
$$[\varepsilon^*_\rho(q_2)\Gamma_{\omega,\rho\beta}(q_2,p_2-k)D_{2,\beta\tau}(p_2-k)\Gamma_{\lambda,\tau\alpha}(p_2-k,p_2)\varepsilon_\alpha(p_2)],$$

where $p_{1,2}, q_{1,2}$ are 4-momenta of muon and deuteron in initial and final states, k is photon 4-momentum, $\varepsilon_\alpha(p_2), \varepsilon_\rho(q_2)$ are deuteron polarization vectors of initial and final states, $D_{\mu\omega}, D_{\nu\lambda}$ are photon propagators, $D_{2,\beta\tau}(p_2+k)$ is a deuteron propagator, $\Gamma_{\omega,\rho\beta}(q_2,p_2+k), \Gamma_{\lambda,\tau\alpha}(p_2+k,p_2)$ are deuteron-photon vertex operators. They are parametrized by three form factors:

$$\Gamma_{\omega,\rho\beta}(q_2,p_2+k) = \frac{(2p_2+k)_\omega}{2m_2}g_{\rho\beta}F_1(k^2) - \frac{(2p_2+k)_\omega}{2m_2}\frac{k_\rho k_\beta}{2m_2^2}F_2(k^2)$$
$$-(g_{\rho\gamma}g_{\beta\omega} - g_{\rho\omega}g_{\beta\gamma})\frac{k_\gamma}{2m_2}F_3(k^2),\ p_{1,2}\approx q_{1,2}$$

Form factors $F_{1,2,3}$ are determined by charge (G_E), magnetic (G_M) and quadrupole (G_Q) deuteron form factors ($\eta = \frac{k^2}{4m_2^2}$):

$$G_E = F_1 + \frac{2}{3}\eta[F_1 + (1-\eta)F_2 - F_3], G_M = F_3, G_Q = F_1 + (1-\eta)F_2 - F_3$$

For electromagnetic form factors we use phenomenological parametrization based on electron-deuteron elastic scattering data.

Lepton tensor with taking into account a radiative photon $L_{\mu\nu}$ is determined for each diagram separately. For example, for self-energy diagram:

$$L^{SE}_{\mu\nu} = \frac{\alpha}{4\pi}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu\frac{\hat{p}_1-\hat{k}+m_1}{(p_1-k)^2+m_1^2}\gamma_\xi\frac{\hat{p}_1-\hat{k}-\hat{q}+m_1}{(p_1-k-q)^2+m_1^2}\gamma_\eta\frac{\hat{p}_1-\hat{k}+m_1}{(p_1-k)^2+m_1^2}\gamma_\nu D_{\xi\eta}(q)$$

For radiative photon we use the Fried-Yennie gauge because it allows us to get the infrared finite result for each diagram separately. Packages FeynCalc and FeynPar in Wolfram Mathematica for $L_{\mu\nu}$ construction are used. They allow to make transformation using Feynman parametrization method. The general structure of $L_{\mu\nu}$ for three types of diagrams is:

$$L^\Sigma_{\mu\nu} = -\frac{3\alpha}{4\pi}\int_0^1(1-x)dx\frac{\gamma^\mu(\hat{p}_1-\hat{k})\gamma^\nu}{m_1^2 - x(m_1^2+k_2)+2p_1kx}$$
$$L^\Lambda_{\mu\nu} = -\frac{\alpha}{4\pi}\int_0^1 dx\int_0^1 dz\frac{\gamma^\mu(\hat{p}_1-\hat{k}+m_1)}{(p_1-k)^2-m_1^2}\left[F_\nu^{(1)}+\frac{F_\nu^{(2)}}{\Delta}+\frac{F_\nu^{(3)}}{\Delta^2}\right]$$
$$L^\Omega_{\mu\nu} = -\frac{2\alpha}{4\pi}\int_0^1 x^2(1-x)dx\int_0^1(1-z)dz\left[\frac{F_{\mu\nu}^{(1)}}{\Delta}+\frac{F_{\mu\nu}^{(2)}}{\Delta^2}+\frac{F_{\mu\nu}^{(3)}}{\Delta^3}\right]$$
$$\Delta = x^2m_1^2 - xz(1-xz)k^2 + 2kp_1xz(1-x)$$

Functions $F_\nu^{(i)}$ and $F_{\mu\nu}^{(i)}$ have large structure. They are presented in [1,2].

Our approach to evaluating the muon-proton interaction amplitude is based on inserting of projection operators on the bound states with definite total angular momenta $F = \frac{1}{2},\frac{3}{2}$. We construct them from wave functions of muon and deuteron:

$$\Pi_{\frac{1}{2}} = [u(p_1)\varepsilon_\alpha(p_2)]_{\frac{1}{2}} = \frac{i}{\sqrt{3}}\gamma_5(\gamma_\alpha - v_\alpha)\Psi(P),$$
$$\Pi_{\frac{3}{2}} = [u(p_1)\varepsilon_\alpha(p_2)]_{\frac{3}{2}} = \Phi_\alpha(P),$$
$$\sum\bar{\Psi}(P)\Psi(P) = \frac{1}{4}(1+\hat{v}),\ \sum\overline{\Phi_\rho}(P)\Phi_\alpha(P) = -\frac{1}{8}(1+\hat{v})\left[g_{\rho\alpha}-\frac{1}{3}\gamma_\rho\gamma_\alpha-\frac{2}{3}v_\rho v_\alpha+\frac{1}{3}(v_\rho\gamma_\alpha - v_\alpha\gamma_\rho)\right]$$

Using projecting operators allows us to present numerator of amplitude as a trace:

$$T_{SE}^{F=\frac{1}{2}} = \frac{1}{48}Tr\{(1+\hat{v})(\gamma_\rho - v_\rho)\gamma_5(1+\hat{v})\gamma_\mu(\hat{p}_1-\hat{k})\gamma_\nu(1+\hat{v})\gamma_5(\gamma_\alpha - v_\alpha)\}$$
$$T_{SE}^{F=\frac{3}{2}} = \frac{1}{32}Tr\left\{(1+\hat{v})\left[g_{\rho\alpha}-\frac{1}{3}\gamma_\rho\gamma_\alpha-\frac{2}{3}v_\rho v_\alpha+\frac{1}{3}(v_\rho\gamma_\alpha - v_\alpha\gamma_\rho)\right](1+\hat{v})\gamma_\mu(\hat{p}_1-\hat{k})\gamma_\nu(1+\hat{v})\right\}$$

We utilize package FORM for trace calculating and Lorentz indexes collapsing. The result of these calculations for muon self-energy diagram is:

$$T_{SE}^{hfs} = F_3^2(-3k_0^4 + 3k^2k_0^2 - 3xk_0^4 + 3xk^2k_0^2 - 3xk^2k_0^4 + 3xk^4k_0^2)$$
$$+ F_1F_3(-12k_0^4 + 12k^4 - 12xk_0^4 + 24xk^2k_0^2 - 12xk^4$$
$$- 12xk^2k_0^4 + 12xk^6).$$

That result is presented taking into account that we keep only first order of nuclear finite size corrections $\frac{m_1}{m_2}$ ($\left(\frac{m_1}{m_2}\right)^2 = 0$). The results for two other diagrams have the same but more complex form.

After that we transform muon-deuteron interaction amplitude in Euclidean space and obtain contribution to the HFS in integral form (where we introduce the dimensionless variable $k \to m_1k$):

$$\Delta E_{SE}^{hfs} = -\frac{3\mu^3\alpha(Z\alpha)^5}{\pi^3n^3m_1^2}\int_0^\infty\frac{kdk}{k^4}\int_0^\pi Sin\phi^2d\phi\frac{1}{k^2+4\left(\frac{m_1}{m_2}\right)^{-2}Cos\phi^2}\times$$
$$\int_0^1(1-x)dx\frac{1}{(1-x+xk^2)^2+4x^2k^2Cos\phi^2}(T_{SE}^{hfs}),$$

Where in T_{SE}^{hfs} we also go to Euclidean space and introduce dimensionless variable. Interaction operators for two other diagrams have the same but more complex form. It is possible to integrate analytically over ϕ in the case of all three diagrams. The integrals for self-energy diagram have the form:

$$I_{1,2,3} = \int_0^\pi Sin\phi^2\frac{1}{k^2+4\left(\frac{m_1}{m_2}\right)^{-2}Cos\phi^2}\frac{1}{(1-x+xk^2)^2+4x^2k^2Cos\phi^2}\times(1,y^2,y^4)$$

$$I_1 = \frac{\mu_1\pi(-\sqrt{4+k^2\mu_1^2} - (-1+k^2)\sqrt{4+k^2\mu_1^2}x + k\mu_1\sqrt{1+x(-2+x+k^4x+2k^2(1+x))})}{4k(1+(-1+k^2)x)(-1+x+k^2(-1+\mu_1)x)(1+(-1+k^2(1+\mu_1))x)},$$

$$I_2 = -\frac{\mu_1^2\pi\left(-1+(-1+2k^2+k^4(-1+\mu_1^2)-k^3\mu_1\sqrt{4+k^2\mu_1^2})x^2+\sqrt{1+2(-1+k^2)x+(1+k^2)^2x^2}+(-1+k^2)x\left(-2+\sqrt{1+2(-1+k^2)x+(1+k^2)^2x^2}\right)\right)}{16k^2x^2(-1-2(-1+k^2)x+(-1+2k^2+k^4(-1+\mu_1^2))x^2)},$$

$$I_3 = \frac{\mu_1^2\pi\left(-2+2k^2\mu_1^2+k^4\mu_1^4-k^3\mu_1^3\sqrt{4+k^2\mu_1^2}+\frac{(1+(-1+k^2)x)^3\left(-1+x-k^2x+\frac{2k^4x^4}{(1+(-1+k^2)x)^3}-\frac{2k^2x^2}{1+(-1+k^2)x}+\sqrt{1+2(-1+k^2)x+(1+k^2)^2x^2}\right)}{k^4x^4}\right)}{64(-1-2(-1+k^2)x+(-1+2k^2+k^4(-1+\mu_1^2))x^2)}.$$

Integration over x and k after that is performed numerically using Monte-Carlo method in Wolfram Mathematica.

1. Radiative recoil diagrams contribution to the hyperfine splitting of S-states of muonic deuterium

Diagram	Radiative nonrecoil correction	Radiative nonrecoil + recoil correction
Self-energy	0.0014 meV	0.0014, meV
Vertex	-0.0042 meV	-0.0038, meV
Jellyfish	-0.0011 meV	-0.0018, meV

Obtained contribution improves previous results and must be taken into account for comparison with more precise experimental data.

[1] R.N. Faustov, A.P. Martynenko, G.A. Martynenko, V.V. Sorokin, Phys. Lett. B **733** 354-358 (2014)

[2] R.N. Faustov, A.P. Martynenko, F.A. Martynenko, V.V. Sorokin, Phys. Lett. B**775** 79-83 (2017)

[3] R.N. Faustov, A.P. Martynenko, F.A. Martynenko, V.V. Sorokin, Phys. Part. Nucl. **48** no.5, 819-821 (2017)