

MUON ANOMALOUS MAGNETIC MOMENT. THEORY

Ivan Logashenko

BINP

Moscow
International
School of
Physics 2022
(MISP 2022)

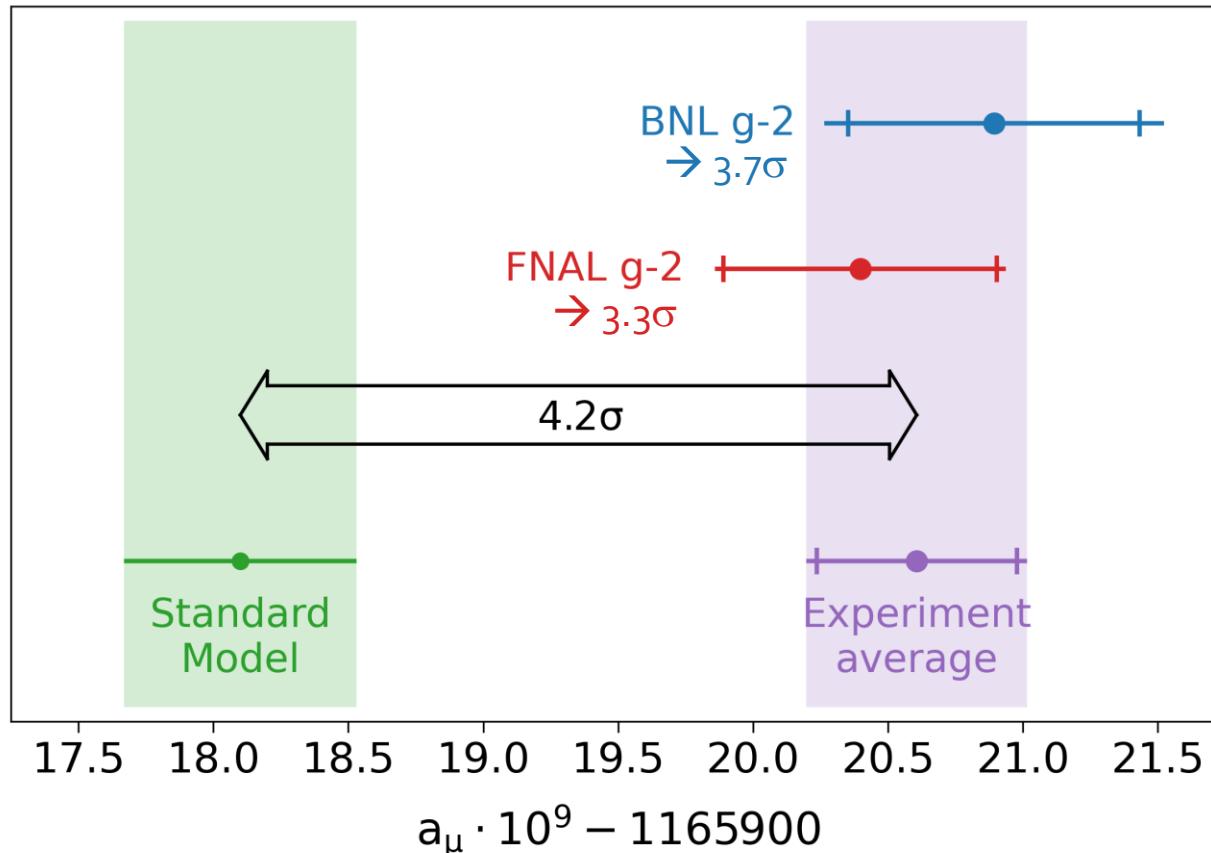
Disclaimer

В лекции использованы
материалы школы
«International Physics School on
Muon Dipole Moments and
Hadronic Effects»
и материалы совещания
«ShwingerFest2022: muon g-2»
(June 14-17, 2022)
и слайды из презентаций
коллег по коллаборации Muon
g-2



Напоминание из прошлой лекции

$$a_\mu(\text{SM}) = 0.00116591810(43) \rightarrow 368 \text{ ppb}$$



$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = 0.000000000251(59) \rightarrow 4.2\sigma$$

On a
theoretical
side...

Интересно не само значение аномального момента мюона, а его
отличие от предсказания Стандартной модели

$$\Delta a_\mu(\text{New Physics}) = a_\mu(\text{exp}) - a_\mu(\text{SM})$$

Вычисление a_μ в Стандартной модели:

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{Had}} + a_\mu^{\text{Weak}}$$

Расчет $a_\mu(\text{SM})$ не менее важен, чем измерение $a_\mu(\text{exp})!$

From F.Jegerlehner book

The closer you look the more there is to see

“It seems to be a strange enterprise to attempt write a physics book about a single number. It was not my idea to do so, but why not. In mathematics, maybe, one would write a book about π . Certainly, **the muon's anomalous magnetic moment is a very special number and today reflects almost the full spectrum of effects incorporated in today's Standard Model (SM) of fundamental interactions, including the electromagnetic, the weak and the strong forces....**”

693 pages book on muon (g-2)!

Springer Tracts in Modern Physics 274

Friedrich Jegerlehner

The Anomalous Magnetic Moment of the Muon

Second Edition

 Springer

Muon g-2 Theory Initiative

A consortium of > 100 theorists formed in advance of the new FNAL results to compile all the theoretical inputs and provide recommendations

- Organized 6 workshops between 2017-2020
- **White Paper posted 10 June 2020** (132 authors, 82 institutions, 21 countries)
[T. Aoyama et al, [arXiv:2006.04822](https://arxiv.org/abs/2006.04822), *Phys. Repts.* 887 (2020) 1-166.]
- (Virtual) plenary workshop in summer 2021 hosted by KEK (Japan)

The anomalous magnetic moment of the muon in the Standard Model

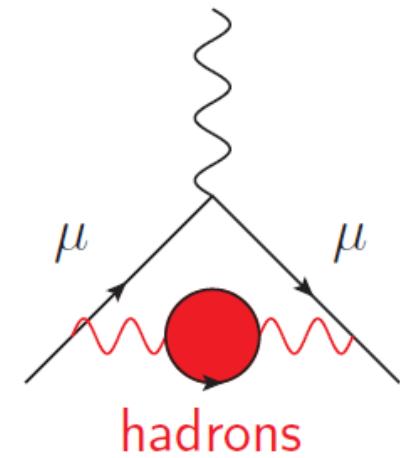
T. Aoyama^{1,2,3}, N. Asmussen⁴, M. Benayoun⁵, J. Bijnens⁶, T. Blum^{7,8}, M. Bruno⁹, I. Caprini¹⁰, C. M. Carloni Calame¹¹, M. Cè^{9,12,13}, G. Colangelo^{†14}, F. Curciarello^{15,16}, H. Czyż¹⁷, I. Danilkin¹², M. Davier^{†18}, C. T. H. Davies¹⁹, M. Della Morte²⁰, S. I. Eidelman^{†21,22}, A. X. El-Khadra^{†23,24}, A. Gérardin²⁵, D. Giusti^{26,27}, M. Golterman²⁸, Steven Gottlieb²⁹, V. Gülpers³⁰, F. Hagelstein¹⁴, M. Hayakawa^{31,2}, G. Herdoíza³², D. W. Hertzog³³, A. Hoecker³⁴, M. Hoferichter^{†14,35}, B.-L. Hoid³⁶, R. J. Hudspith^{12,13}, F. Ignatov²¹, T. Izubuchi^{37,8}, F. Jegerlehner³⁸, L. Jin^{7,8}, A. Keshavarzi³⁹, T. Kinoshita^{40,41}, B. Kubis³⁶, A. Kupich²¹, A. Kupsc^{42,43}, L. Laub¹⁴, C. Lehner^{†26,37}, L. Lellouch²⁵, I. Logashenko²¹, B. Malaescu⁵, K. Maltman^{44,45}, M. K. Marinković^{46,47}, P. Masjuan^{48,49}, A. S. Meyer³⁷, H. B. Meyer^{12,13}, T. Mibe^{†1}, K. Miura^{12,13,3}, S. E. Müller⁵⁰, M. Nio^{2,51}, D. Nomura^{52,53}, A. Nyffeler^{†12}, V. Pascalutsa¹², M. Passera⁵⁴, E. Perez del Rio⁵⁵, S. Peris^{48,49}, A. Portelli³⁰, M. Procura⁵⁶, C. F. Redmer¹², B. L. Roberts^{†57}, P. Sánchez-Puertas⁴⁹, S. Serednyakov²¹, B. Shwartz²¹, S. Simula²⁷, D. Stöckinger⁵⁸, H. Stöckinger-Kim⁵⁸, P. Stoffer⁵⁹, T. Teubner^{†60}, R. Van de Water²⁴, M. Vanderhaeghen^{12,13}, G. Venanzoni⁶¹, G. von Hippel¹², H. Wittig^{12,13}, Z. Zhang¹⁸, M. N. Achasov²¹, A. Bashir⁶², N. Cardoso⁴⁷, B. Chakraborty⁶³, E.-H. Chao¹², J. Charles²⁵, A. Crivellin^{64,65}, O. Deineka¹², A. Denig^{12,13}, C. DeTar⁶⁶, C. A. Dominguez⁶⁷, A. E. Dorokhov⁶⁸, V. P. Druzhinin²¹, G. Eichmann^{69,47}, M. Fael⁷⁰, C. S. Fischer⁷¹, E. Gámiz⁷², Z. Gelzer²³, J. R. Green⁹, S. Guellati-Khelifa⁷³, D. Hatton¹⁹, N. Hermansson-Truedsson¹⁴, S. Holz³⁶, B. Hörz⁷⁴, M. Knecht²⁵, J. Koponen¹, A. S. Kronfeld²⁴, J. Laiho⁷⁵, S. Leupold⁴², P. B. Mackenzie²⁴, W. J. Marciano³⁷, C. McNeile⁷⁶, D. Mohler^{12,13}, J. Monnard¹⁴, E. T. Neil⁷⁷, A. V. Nesterenko⁶⁸, K. Ott nad¹², V. Pauk¹², A. E. Radzhabov⁷⁸, E. de Rafael²⁵, K. Raya⁷⁹, A. Risch¹², A. Rodríguez-Sánchez⁶, P. Roig⁸⁰, T. San José^{12,13}, E. P. Solodov²¹, R. Sugar⁸¹, K. Yu. Todyshev²¹, A. Vainshtein⁸², A. Vaquero Avilés-Casco⁶⁶, E. Weil⁷¹, J. Wilhelm¹², R. Williams⁷¹, A. S. Zhevlov⁷⁸

a_μ в Стандартной модели

	$a_\mu [10^{-11}]$	$\Delta a_\mu [10^{-11}]$
experiment	116 592 061.	41.
QED $\mathcal{O}(\alpha)$	116 140 973.321	0.023
QED $\mathcal{O}(\alpha^2)$	413 217.626	0.007
QED $\mathcal{O}(\alpha^3)$	30 141.902	0.000
QED $\mathcal{O}(\alpha^4)$	381.004	0.017
QED $\mathcal{O}(\alpha^5)$	5.078	0.006
QED total	116 584 718.931	0.030
electroweak	153.6	1.0
had. VP (LO)	6931.	40.
had. VP (NLO)	-98.3	0.7
had. LbL	92.	19.
total	116 591 810.	43.

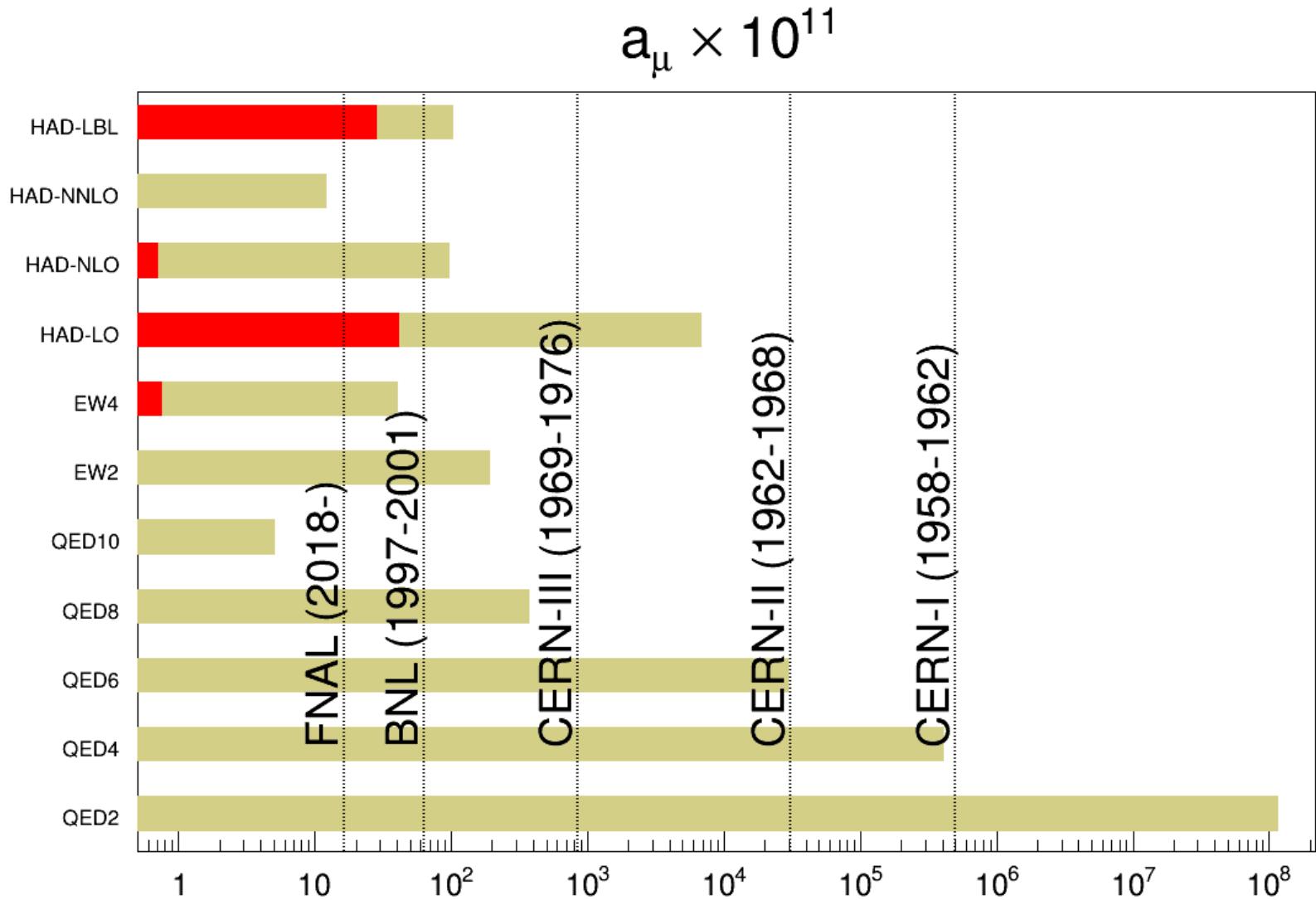
BNL E821 2006
+ Fermilab 2021

Aoyama et al. 2020



Точность измерения и вычисления

a_μ



QED contribution

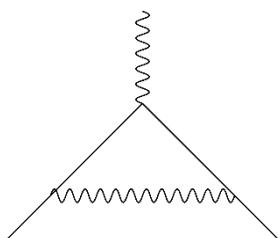
$$a_\mu^{QED} = C_1 \left(\frac{\alpha}{\pi} \right) + C_2 \left(\frac{\alpha}{\pi} \right)^2 + C_3 \left(\frac{\alpha}{\pi} \right)^3 + C_4 \left(\frac{\alpha}{\pi} \right)^4 + C_5 \left(\frac{\alpha}{\pi} \right)^5 + \dots$$

$$a_\mu^{QED} = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau)$$

универсальный
 $A_1(\mu) = A_1(e)$

отличаются для e, μ, τ
лептоны в петле отличаются от внешних лептонов

$$A_i = A_i^{(2)} \left(\frac{\alpha}{\pi} \right) + A_i^{(4)} \left(\frac{\alpha}{\pi} \right)^2 + A_i^{(6)} \left(\frac{\alpha}{\pi} \right)^3 + A_i^{(8)} \left(\frac{\alpha}{\pi} \right)^4 + A_i^{(10)} \left(\frac{\alpha}{\pi} \right)^5 + \dots$$



$$C_1 = A_1^{(2)} = \frac{1}{2}$$

QED contribution

$$a_\mu^{QED} = C_1 \left(\frac{\alpha}{\pi} \right) + C_2 \left(\frac{\alpha}{\pi} \right)^2 + C_3 \left(\frac{\alpha}{\pi} \right)^3 + C_4 \left(\frac{\alpha}{\pi} \right)^4 + C_5 \left(\frac{\alpha}{\pi} \right)^5 + \dots$$

Порядок	C_i^e	C_i^μ	$C_i^\mu \cdot (\alpha/\pi)^i, \times 10^{11}$
1	0.5	0.5	116 140 973.2420(260)
2	-0.328 478 444 00	0.765 857 423(16)	413 217.6270(90)
3	1.181 234 017	24.050 509 82(28)	30 141.9022(4)
4	-1.9113(18)	130.8734(60)	380.9900(170)
5	9.16(58)	751.92(93)	5.0845(63)

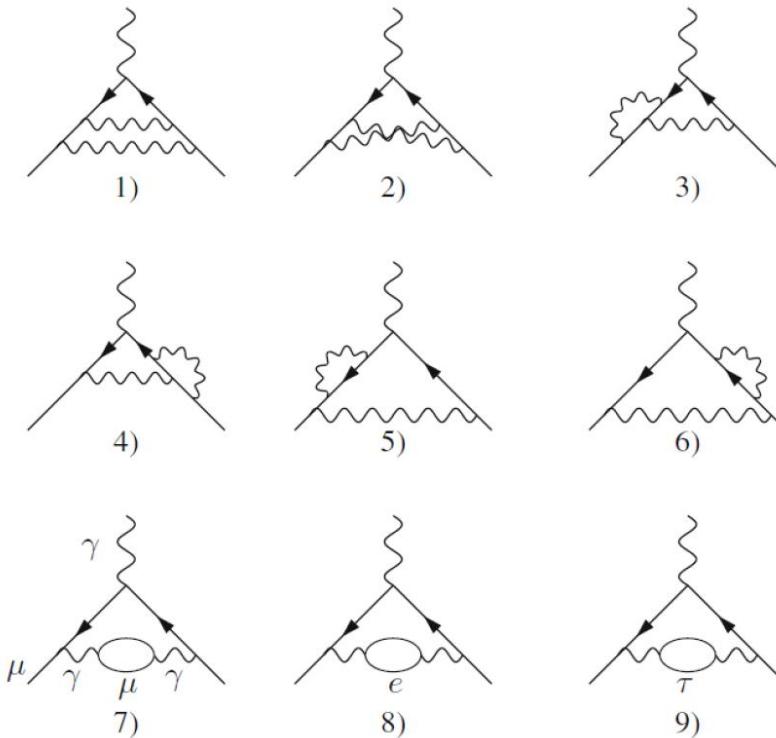
Таблица 1 — Вклады различного порядка теории возмущений в a_μ^{QED} .

$$a_\mu^{QED} = 116 584 718.859 (.026)(.009)(.017)(.006) [.034] \times 10^{-11},$$

0.29 ppb

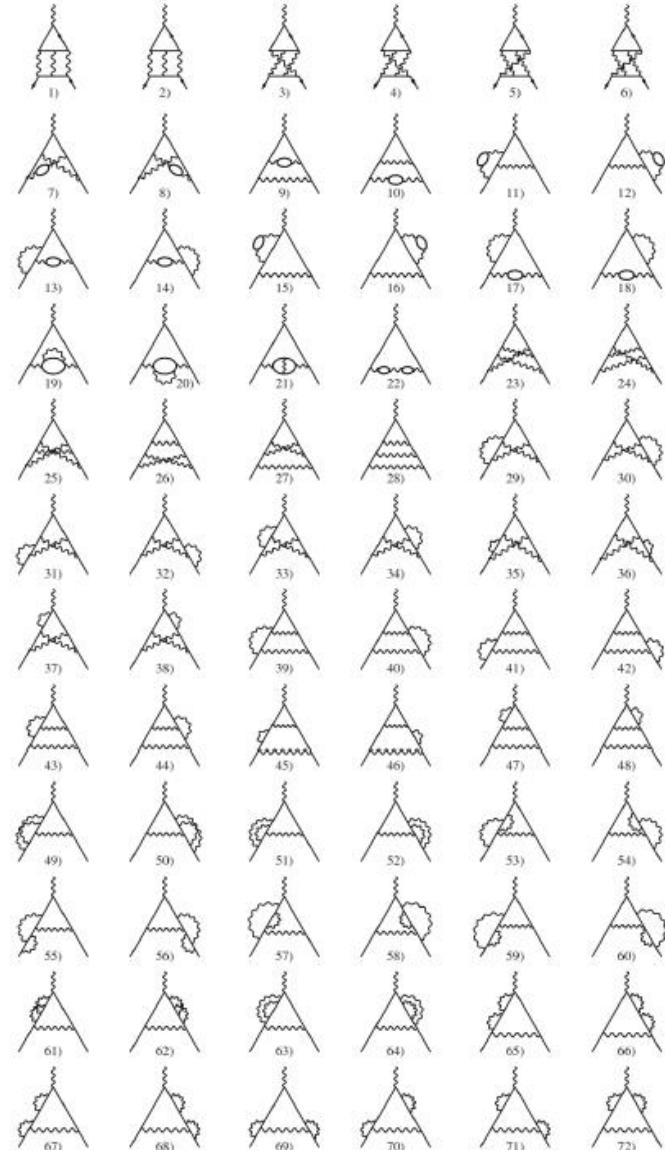
QED contribution two-loop

$$\begin{aligned} A_1^{(4)} &= -0.328\ 478\ 965\ 579\ 193\ 78\dots \\ A_2^{(4)}(m_\mu/m_e) &= 1.094\ 258\ 3111(84) \\ A_2^{(4)}(m_\mu/m_\tau) &= 0.000\ 078\ 064(25) \\ C_2 &= 0.765857410(27). \end{aligned}$$



QED contribution three-loop

$$\begin{aligned} A_1^{(6)} &= 1.181\,241\,456\,587\dots \\ A_2^{(6)}(m_\mu/m_e) &= 22.868\,380\,02\,(20) \\ A_2^{(6)}(m_\mu/m_\tau) &= 0.000\,360\,51\,(21) \\ A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) &= 0.000\,527\,66\,(17) \\ C_3 &= 24.050\,509\,64\,(43). \end{aligned}$$



Logarithmic enhancement

the logarithmic enhancement $\ln(m_\mu/m_e) \approx 5.3$

note: It does not exist for the lightest lepton, electron.

Two sources of the logarithm

1. Charge renormalization of the vacuum-polarization(VP) function

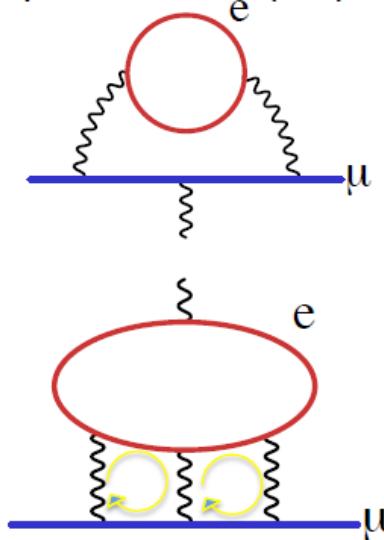
2nd-order VP arises

$$\frac{2}{3} \ln(m_\mu/m_e) - \frac{5}{9} \sim 3$$

"Renormalization Group" estimate

2. Light-by-light scattering diagram

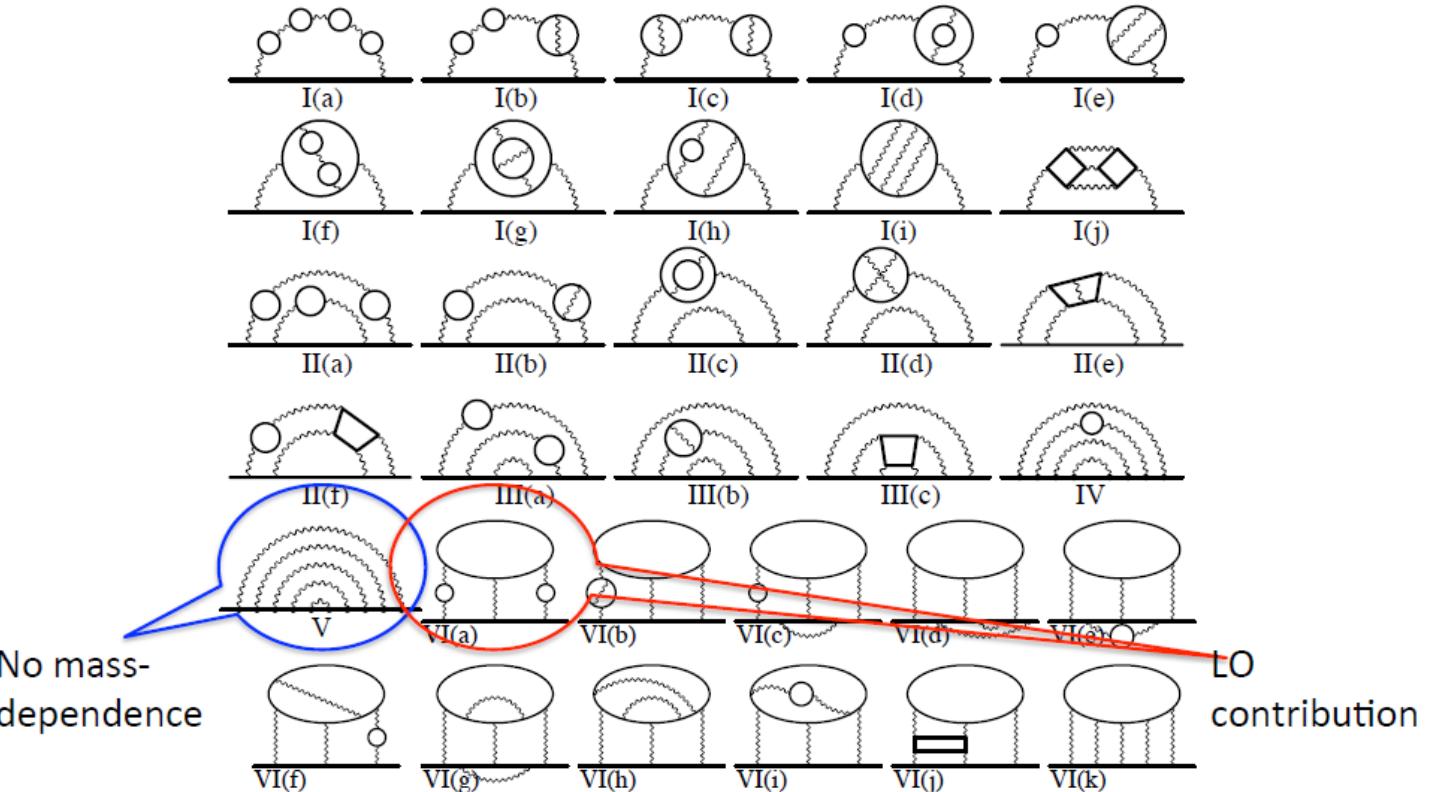
$$\frac{2}{3} \pi^2 \ln(m_\mu/m_e) \sim 35$$



Coulomb photon loops provide the factor π^2

QED contribution five-loop

12,672 Feynman vertex diagrams contribute to the 10th order .
They are classified into 32 gauge-invariant subsets over 6 sets.



QED contribution four- and five- loop

$$A_2^{(8)}(m_\mu/m_e) = 132.6852 \text{ (60)}$$

$$A_2^{(8)}(m_\mu/m_\tau) = 0.042\ 34 \text{ (12)}$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\ 72 \text{ (4)}$$

$$A_2^{(10)}(m_\mu/m_e) = 742.18 \text{ (87)}$$

$$A_2^{(10)}(m_\mu/m_\tau) = -0.068 \text{ (5)}$$

$$A_3^{(10)}(m_\mu/m_e, m_\mu/m_\tau) = 2.011 \text{ (10)}$$

QED contributions to
the muon g-2 is now
firmly established.

Rough estimate of the 12th-order contribution:

6th-order light-by-light x three 2nd-order vp x 10 ways

$$\sim 20 \times 3^3 \times 10 (\alpha/\pi)^6 \sim 5,000 (\alpha/\pi)^6 \sim 0.08 \times 10^{-11}$$

Recall the aimed goal of the on-going experiments $\sim 12 \times 10^{-11}$

Electroweak contribution

Electroweak (EW) contribution:

$$a_\mu(\text{EW}) = \underbrace{19.48 \times 10^{-10}}_{\text{1-loop}} + \underbrace{(-4.12(10) \times 10^{-10})}_{\text{2-loop}} + \underbrace{\mathcal{O}(10^{-12})}_{\text{3-loop leading log}} \\ = 15.36(10) \times 10^{-10}, \quad (\text{Number taken from PDG 2020})$$

where the uncertainty mainly comes from quark loops.

- 1-loop result published by many groups ([Bardeen-Gastmans-Lautrup](#), [Altarelli-Cabibbo-Maiani](#), [Jackiw-Weinberg](#), [Bars-Yoshimura](#), [Fujikawa-Lee-Sanda](#)) in 1972, and now a textbook exercise ([Peskin & Schroeder's textbook](#), [Problems 6.3 \(Higgs\)](#) and [21.1 \(W, Z\)](#))
- 2-loop contribution (~ 1700 diagrams in the 't Hooft-Feynman gauge) enhanced by $\ln(m_Z/m_\mu)$ and also by a factor of $\mathcal{O}(10)$,

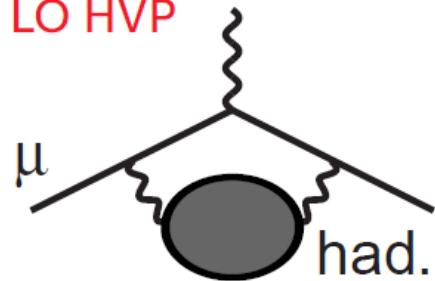
$$a_\mu(\text{EW, 2-loop}) \simeq -10 \left(\frac{\alpha}{\pi} \right) a_\mu(\text{EW, 1-loop}) \left(\ln \frac{m_Z}{m_\mu} + 1 \right),$$

where the factor of 10 appears since many “order one” diagrams accidentally add up. ([Czarnecki-Krause-Marciano](#))

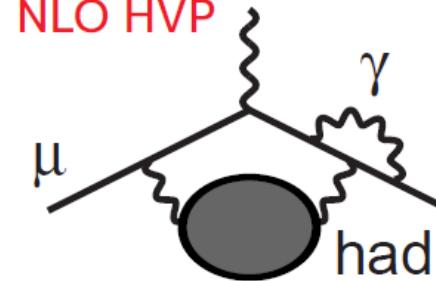
Hadronic contribution

There are several hadronic contributions:

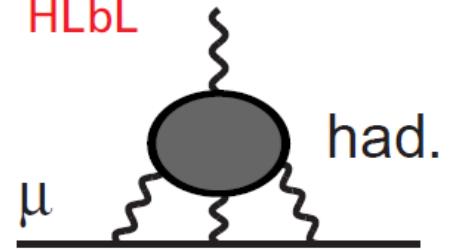
LO HVP



NLO HVP



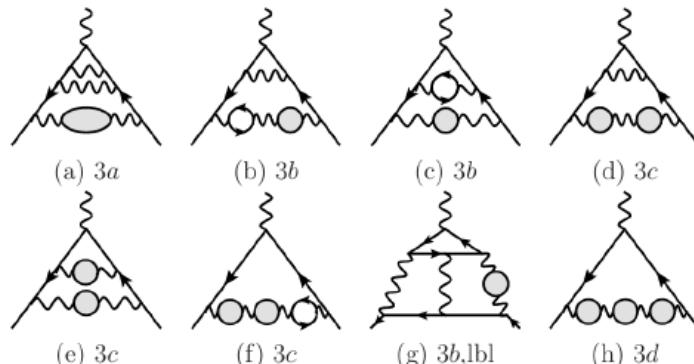
HLbL



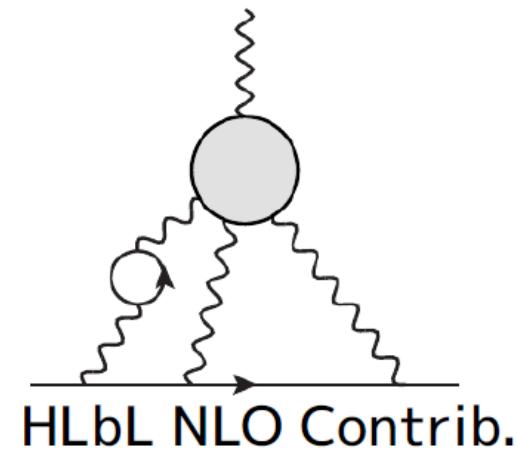
LO HVP: Leading Order Hadronic Vacuum Polarization Contribution

NLO HVP: Next-to-Leading Order HVP Contribution

HLbL: Hadronic Light-by-Light Scattering Contribution

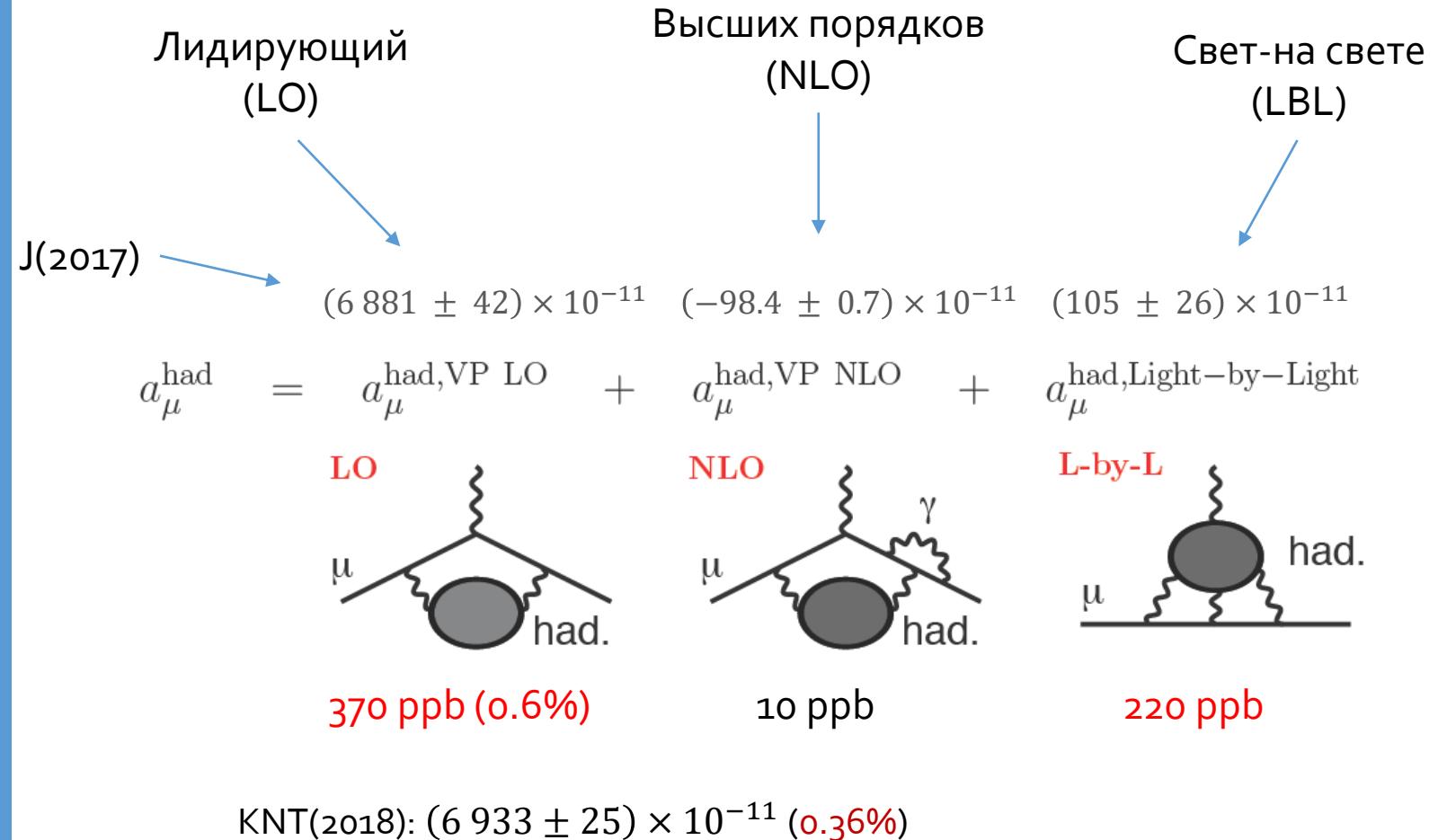


NNLO HVP Contributions



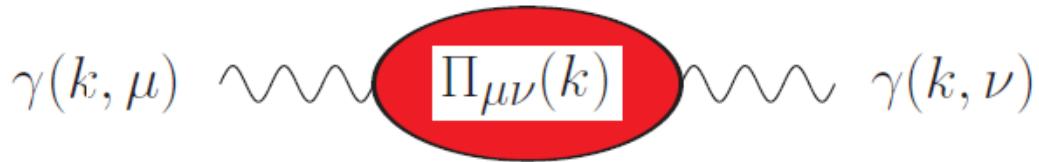
HLbL NLO Contrib.

Вклад сильных взаимодействий



HVP structure

- photon two-point function:



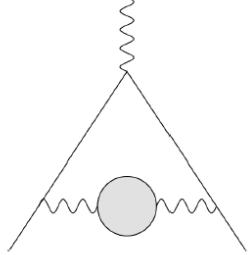
- ▷ one single independent momentum k
- ▷ symmetric rank-2 tensor: two structures $g_{\mu\nu}$, $k_\mu k_\nu$
- ▷ scalar invariant can depend on one single invariant k^2
- gauge invariance: $k^\mu \Pi_{\mu\nu}(k) = 0 = k^\nu \Pi_{\mu\nu}(k)$

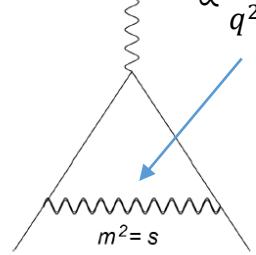
$$\boxed{\Pi_{\mu\nu}(k) = (k^2 g_{\mu\nu} - k_\mu k_\nu) \Pi(k^2)}$$

→ Lorentz + gauge invariance reduce HVP to
one single function of a single variable!

HVP: what do we need to measure

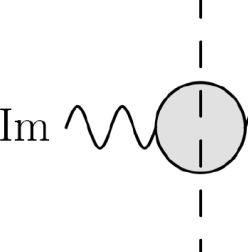
Analyticity
(dispersion relation):



$$= \int_0^\infty \frac{ds}{s} \frac{1}{\pi} \text{Im} \Pi'(s) \times$$


$$\alpha \frac{1}{\pi} K_\mu(s)$$

Optical theorem:



$$2 \text{Im} \langle \text{wavy line} \rangle = \left| \text{wavy line} \right|^2$$

$$\text{Im} \Pi'^{(s)} = \frac{s}{4\pi\alpha} \sigma^0(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons} + \dots)$$

Lets put everything together:

$$a_\mu^{\text{had}}(\text{LO}) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} R(s) K_\mu(s)$$

This is what we need to measure

$$R(s) = \frac{\sigma^0(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons})}{4\pi\alpha^2/3s}$$

$$\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)$$

$$s = (\text{c.m. energy})^2$$

R(s)

$$R(s) = \frac{\sigma^0(\text{q-qbar})}{\sigma^0(\mu^+\mu^-)}$$

In the zeroth order of QCD and zero quark masses:

$$R^{(0)}(s) = 3 \sum_f q_f^2$$

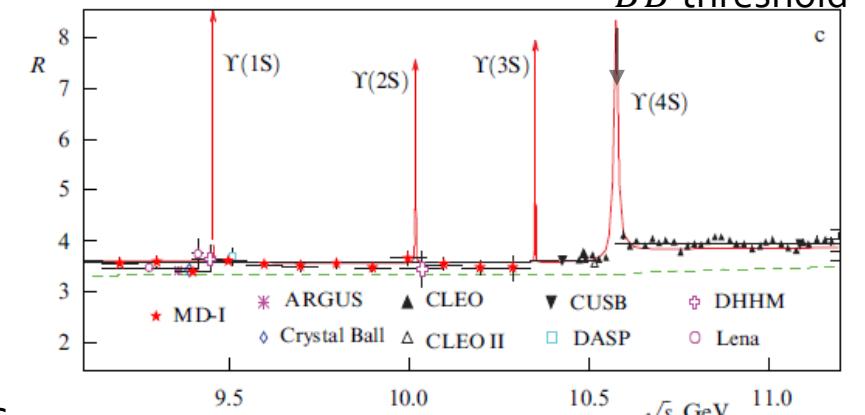
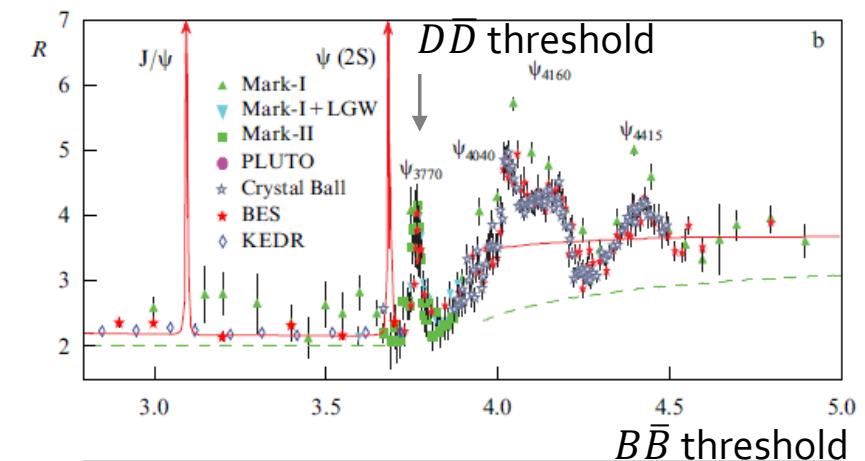
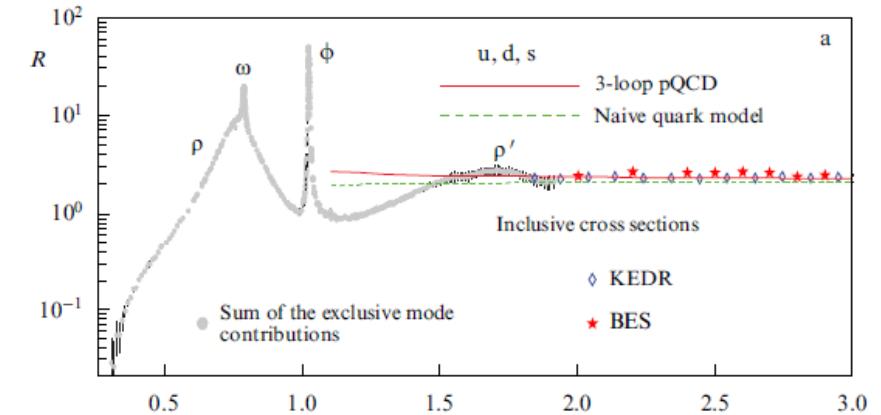
$$R(u, d, s) = \frac{6}{3}$$

$$R(u, d, s, c) = \frac{10}{3}$$

$$R(u, d, s, c, b) = \frac{11}{3}$$

Full pQCD calculation includes NNLO contribution, quark masses, running α_s , ...

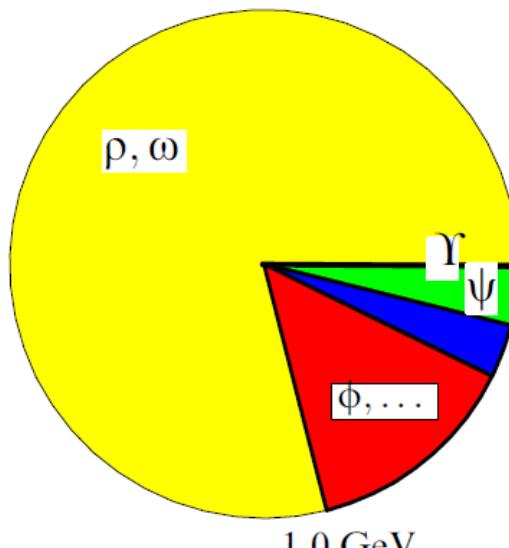
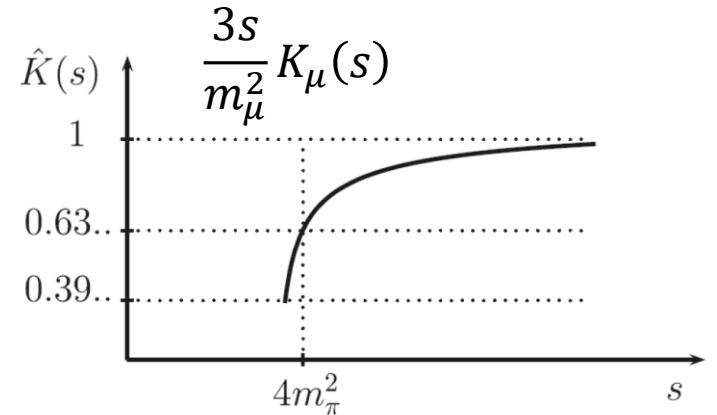
Good agreement of data vs pQCD at $\sqrt{s} > 2 \text{ GeV}$ and away from resonances



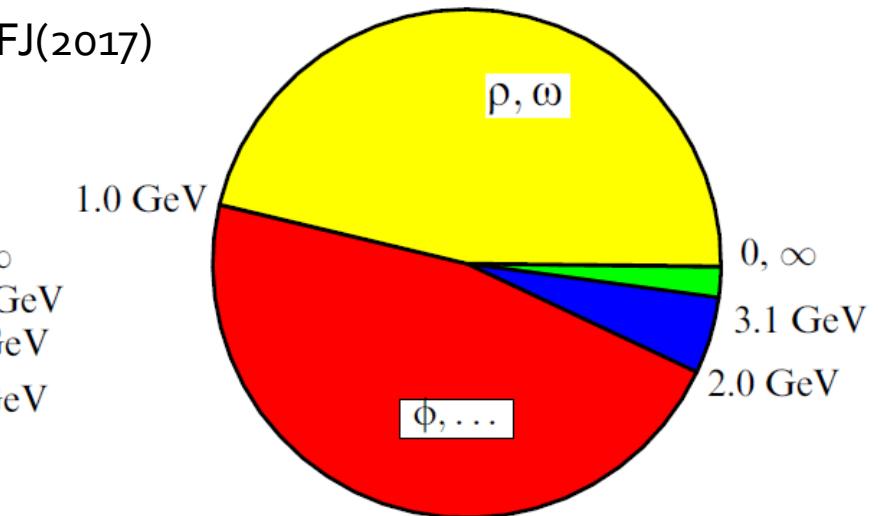
Contribution of various energies

In a_μ^{had} integral, the main contribution comes from low energies

$$a_\mu^{had}(LO) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} R(s) K_\mu(s) \sim \int \frac{R(s)}{s^2} ds$$



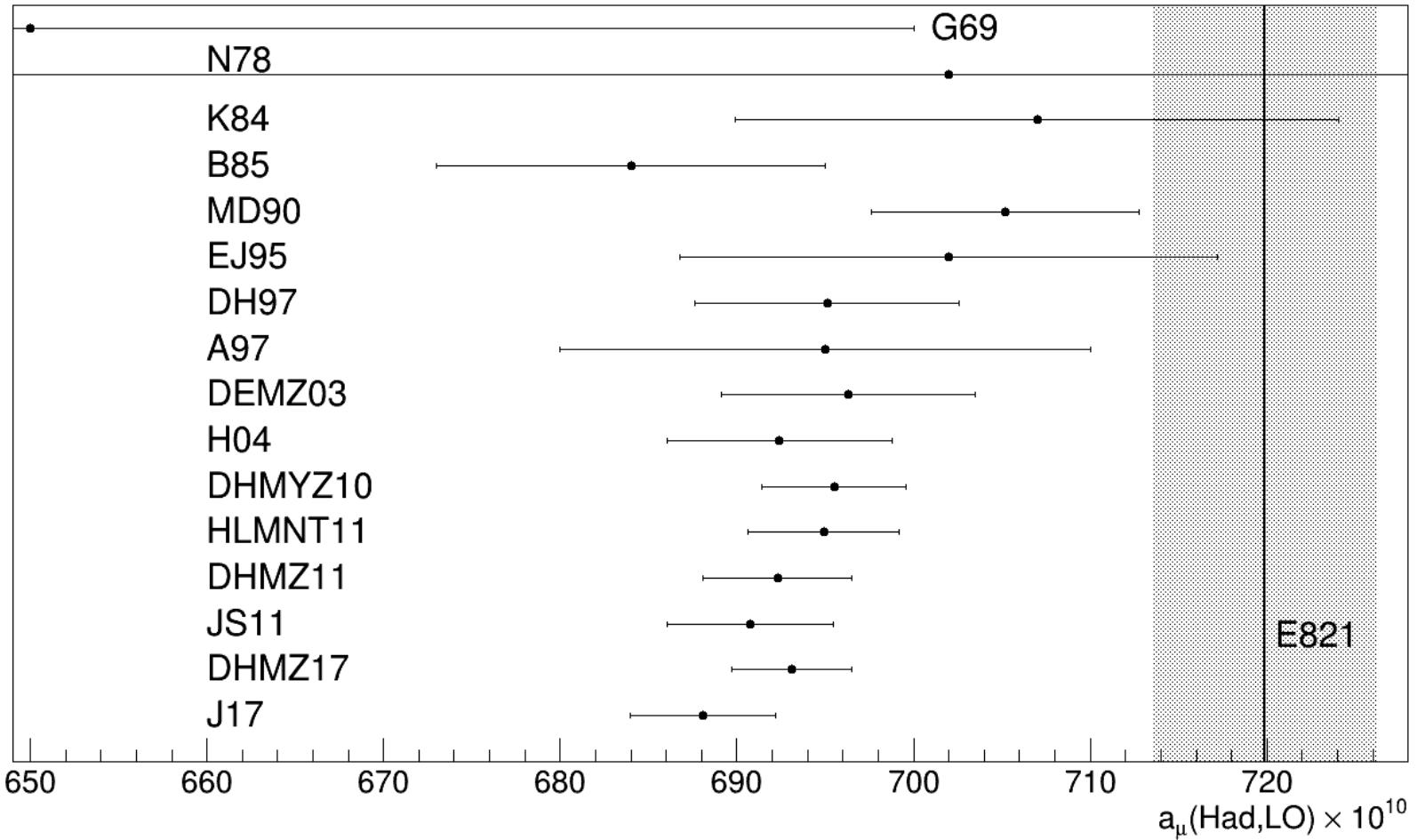
Contribution to the integral



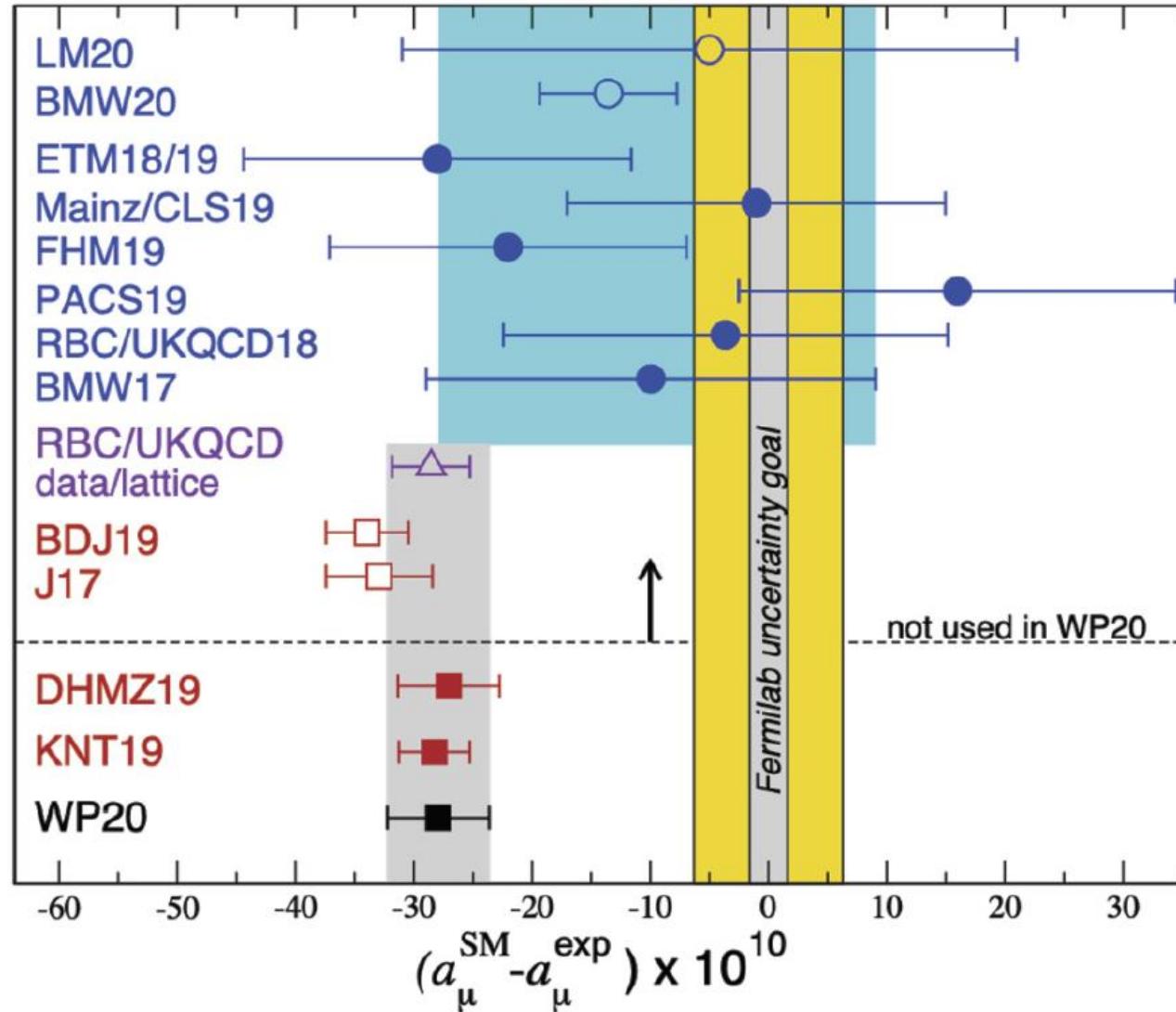
Contribution to the error of integral

When we measure $R(s)$ in order to calculate hadronic contribution to a_μ , we are focused at low energies $\sqrt{s} \lesssim 2$ GeV

Hadronic vacuum polarization contribution

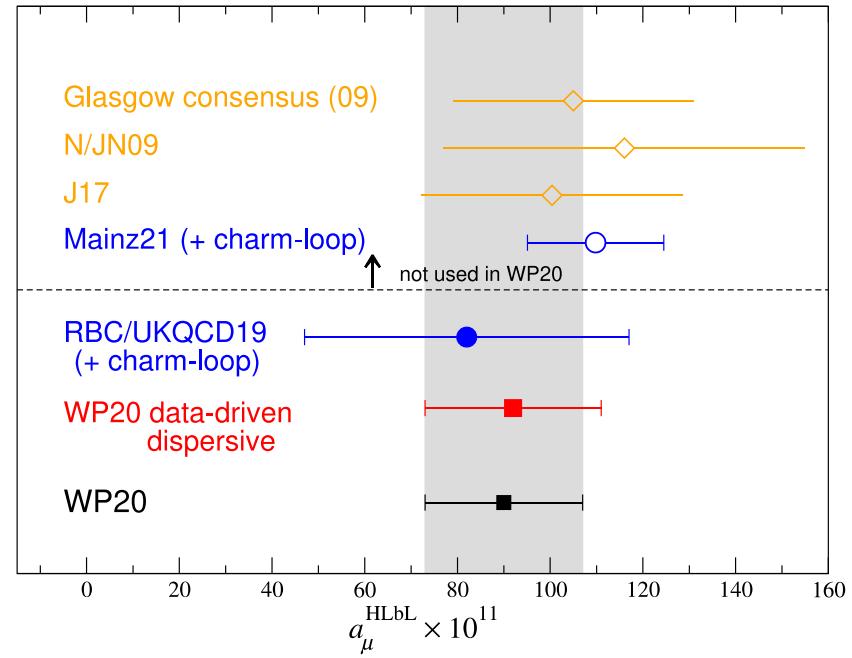
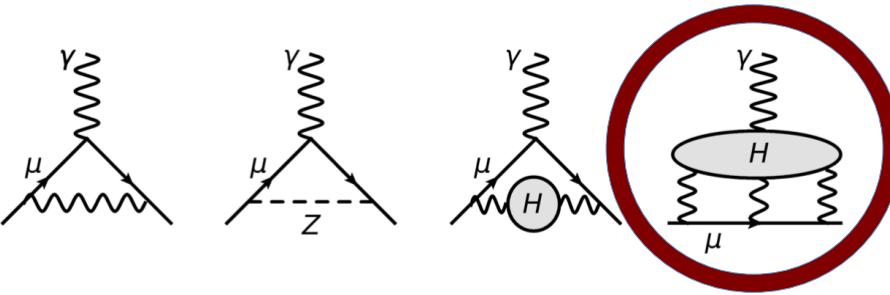


Hadronic vacuum polarization contribution



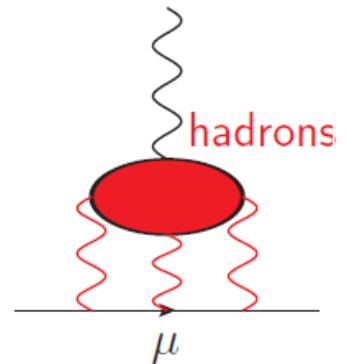
Hadronic light-by-light contributions

- Hadronic light-by-light has been particularly difficult in the past
 - Not calculable in QCD
 - Not directly measurable
 - Relied on model-dependent calculations
- Two developments
 - advancement in lattice calculations
 - data-driven approaches to check the models
- All approaches are in good agreement

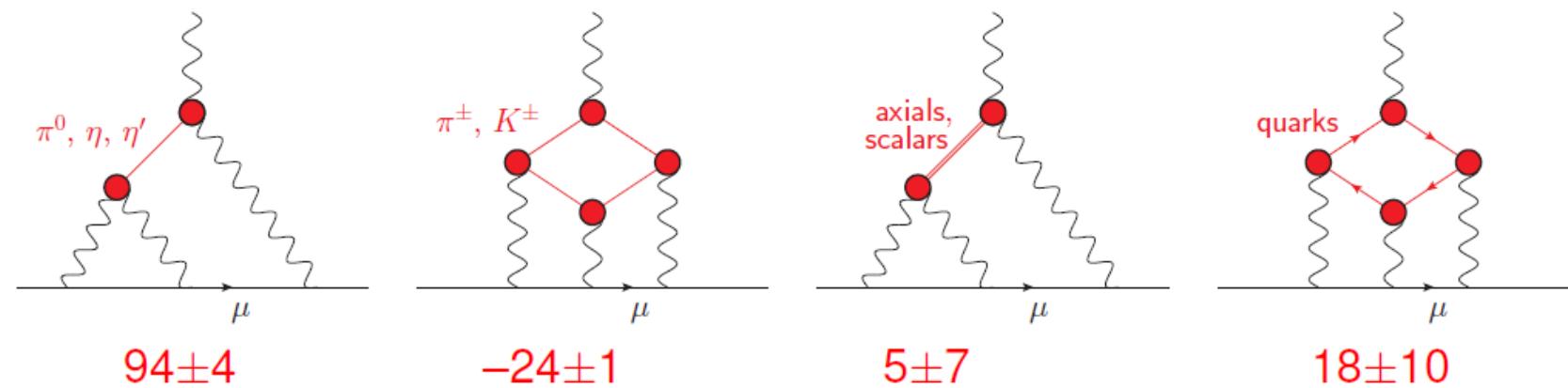


Вклады в HbL

$$\begin{aligned} a_{\mu}^{had;LBL} &= (9.545[6.468 + 1.487 + 1.590] \pm 1.240 + & (\pi^0, \eta, \eta') \\ &\quad 0.755[0.189 + 0.519 + 0.047] \pm 0.271 + & (a_1, f_1, f'_1) \\ &\quad -0.598[-0.017 - 0.296 - 0.285] \pm 0.120 + & (a_0, f_0, f'_0) \\ &\quad 0.11[0.079 + 0.007 + 0.022 + 0.002] \pm 0.01 + & (f'_2, f_2, a'_2, a_2) \\ &\quad -2.0 \pm 0.5 + & (\pi-loop) \\ &\quad 2.23 \pm 0.4 + & (quark-loop) \\ &\quad 0.3 \pm 0.2) \times 10^{-10} & (NLO) \\ &= (10.34 \pm 2.88) \times 10^{-10}. \end{aligned}$$



- different contributions calculated or estimated (in 10^{-11}):



→ increasing systematic control over HbL using
dispersion-theoretical approach

Aoyama et al. 2020

Hlbl structure

Colangelo, Hoferichter, Procura, Stoffer 2014, 2015

- HLbL tensor $\Pi^{\mu\nu\lambda\sigma}$: Lorentz invariance
→ 138 (136) scalar functions Eichmann et al. 2014
- gauge invariance: Bardeen, Tung 1968; Tarrach 1975

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i$$

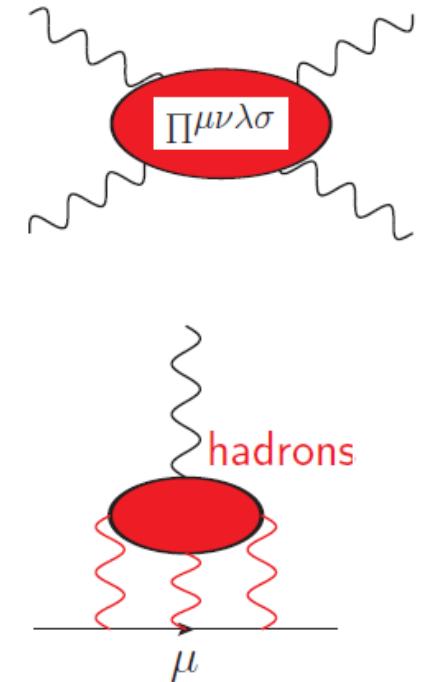
→ 7 distinct structures, 47 related by crossing

- master formula:

$$a_\mu^{\text{HLbL}} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{\sum_{i=1}^{12} \hat{T}_i(q_1, q_2; p) \hat{\Pi}_i(q_1, q_2, -q_1 - q_2)}{q_1^2 q_2^2 (q_1 + q_2)^2 [(p + q_1)^2 - m_\mu^2] [(p - q_2)^2 - m_\mu^2]}$$

- \hat{T}_i : known kernels

$\hat{\Pi}_i$: dispersively ↔ measurable form factors / scatt. amplitudes



a_μ в Стандартной Модели

From Table 1 of the White Paper

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	-98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^- , LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

HVP: Hadronic Vacuum Polarization contribution

HLbL: Hadronic Light-by-Light contribution

BSM contribution to muon ($g-2$)

Summary of main points

discrepancy $\approx 2 \times a_\mu^{\text{SM,weak}}$

but: expect $a_\mu^{\text{NP}} \sim a_\mu^{\text{SM,weak}} \times \left(\frac{M_w}{M_{\text{NP}}}\right)^2 \times \text{couplings}$

Many models involve enhancement mechanisms

but: experimental constraints!

Take-home message 3:

Which models can still accommodate large deviation?

Many models! General ideas still viable (SUSY, THDM, LQ, VLL, ...)

but: restricted parameter space! Specific scenarios excluded!

BSM contribution to muon ($g-2$)

Which models can still accommodate large deviation?

SUSY: **MSSM, MRSSM**

- MSugra... many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns

Two-Higgs doublet model

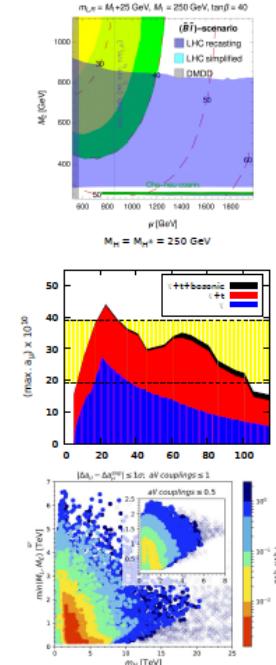
- Type I, II, Y, Type X(lepton-specific), flavour-aligned

Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R

Simple models (one or two new fields)

- Mostly excluded
- light N.P. (**ALPs, Dark Photon, Light $L_\mu - L_\tau$**)



Model	Spins	$20 f_{L_1} + 20 f_{L_2} + 4 f_{R_1} $	How
1	0	(1,A,0)	Excluded: $\Delta g < 8$
2	0	(1,A,-1)	Excluded: $\Delta g < 8$
3	0	(1,A,1)	Excluded: $\Delta g < 8$
4	0	(1,L,0)	Excluded: $\Delta g < 8$
5	0	(1,L,-1)	Excluded: $\Delta g < 8$
6	0	(1,L,1)	Excluded: $\Delta g < 8$
7	0	(1,R,0)	Excluded: $\Delta g < 8$
8	0	(1,R,-1)	Excluded: $\Delta g < 8$
9	0	(1,R,1)	Excluded: $\Delta g < 8$
10	1/2	(1,A,0)	Excluded: $\Delta g < 8$
11	1/2	(1,A,-1)	Excluded: $\Delta g < 8$ (not small)
12	1/2	(1,A,1)	Excluded: $\Delta g < 8$ (not small)
13	1/2	(1,L,0)	Excluded: $\Delta g < 8$
14	1/2	(1,L,-1)	Excluded: $\Delta g < 8$
15	1/2	(1,L,1)	Excluded: $\Delta g < 8$
16	1/2	(1,R,0)	Excluded: $\Delta g < 8$
17	1/2	(1,R,-1)	Excluded: $\Delta g < 8$ (not small)
18	1	(1,R,1)	Excluded: $\Delta g < 8$

[Athron,Balazs,Jacob,Kotarski,D5,Stöckinger-Kim, preliminary]



Адронные сечения для HVP

How well do we need to measure $R(s)$

From the White Paper (Physics Reports 887 (2020) 1):

$$a_\mu^{\text{had}}(LO) = 693.1(4.0) \times 10^{-10}$$

The expected final precision of the Fermilab measurement

$$\Delta a_\mu = 1.6 \times 10^{-10}$$

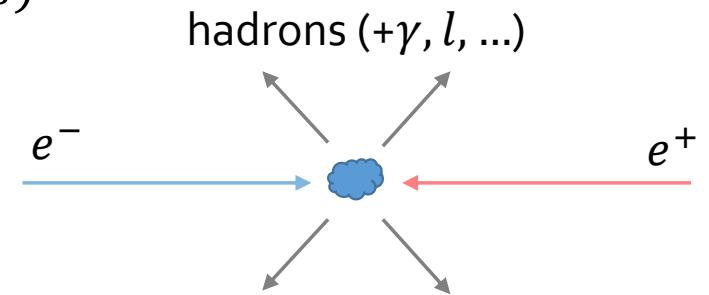
We need to know $R(s)$ to 0.23% to match Fermilab precision

Now the hadronic contribution is known to 0.57%

Energy scan approach

Direct measurement of $\sigma(e^+e^- \rightarrow \text{hadrons})$
(energy scan approach):

- performed at electron-positron collider
- collect data at different beam energy
- at each energy point: select final states with hadrons, subtract background and normalize to luminosity



Number of signal events

Number of background events

$$\sigma = \frac{N_{obs} - N_{bg}}{\varepsilon \cdot \int \mathcal{L} dt}$$

Detection efficiency:

- kinematical limits of detector (fiducial volume) – detector never has 4π coverage
- **detector response**

Luminosity integral

- measured by selection of monitoring events with known cross section

Exclusive vs inclusive measurement

Detection efficiency is (usually) calculated using MC simulation

- In order to calculate ε , we need to know the energy and angular distributions of final particles (including all correlations)

For high energies, where multiplicity is large enough, there are effective models of hadronization, which describe data reasonably well

At low energy the detection efficiency varies significantly between different final states and different paths of hadronization (intermediate states)

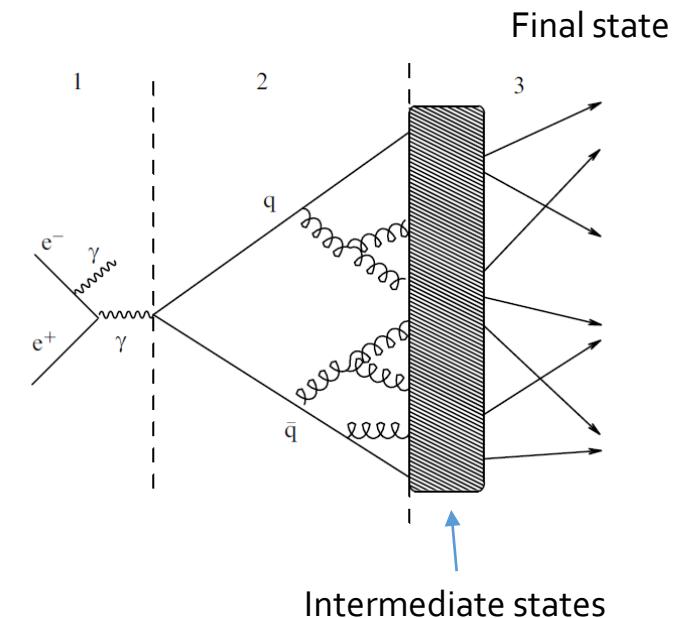
At low energies we have to measure cross section for each possible final state separately and then calculate sum to get R (**exclusive approach**)

At high energy we can measure total cross section directly (**inclusive approach**)

The practical boundary between two approaches in $\sqrt{s} = 2 \text{ GeV}$.

The $a_\mu^{\text{had}}(LO)$ calculation is mostly based on exclusive measurements.

$$\sigma = \frac{N_{\text{obs}} - N_{\text{bg}}}{\varepsilon \cdot \int \mathcal{L} dt}$$



The top exclusive hadronic cross sections in the world [of a_μ]

In exclusive approach, we calculate a_μ integral for each final state and sum them:

$$a_\mu^{had}(LO) = \sum_{X=\pi^0\gamma, \pi^+\pi^-, \dots} a_\mu^X(LO) = \sum_X \frac{1}{4\pi^3} \int \sigma^0(e^+e^- \rightarrow X) K_\mu(s) ds$$

Channel	$a_\mu^{had, LO} [10^{-10}]$
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$ (non- ω, ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \rightarrow \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega 2\pi$ ($\omega \rightarrow \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non- $3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$

From DHMZ'19

The larger the contribution, the better precision is required

$e^+e^- \rightarrow \pi^+\pi^-$ is by far the most challenging and has got the most attention (73% of total hadronic contribution!)



Luminosity measurement

We need to know luminosity integral in order to normalize the measured hadronic cross section.

For that we use *monitoring process* with known cross section

$$\int \mathcal{L} dt = \frac{N_{obs} - N_{bg}}{\varepsilon \cdot \sigma_{known}}$$

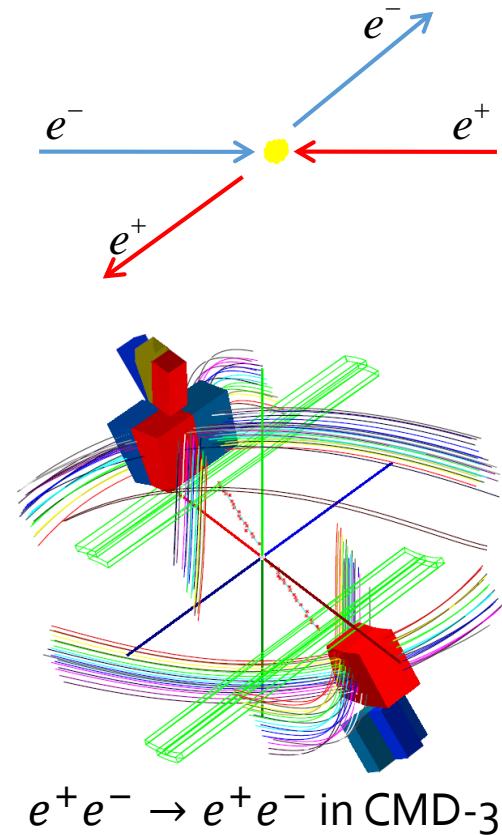
The most popular monitoring process is **large angle Bhabha scattering $e^+e^- \rightarrow e^+e^-$** : easily identifiable, large cross section

Other good processes for luminosity measurement:

- $e^+e^- \rightarrow \mu^+\mu^-$ *Has many advantages, but relatively small cross section and large background*
- $e^+e^- \rightarrow \gamma\gamma$ *Natural for final states with neutrals*
- $e^+e^- \rightarrow e^+e^-\gamma$
- $e^+e^- \rightarrow e^+e^-\gamma\gamma$ *Often used for online measurement*

All these are QED processes – the cross section can be calculated

MISF-2022. Muon anomalous magnetic moment



Radiative corrections



We want to measure $e^+e^- \rightarrow H$, but these events are accompanied by similar events where photons are emitted by any of the particles.

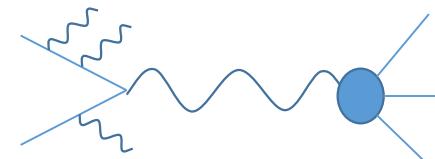
Radiation of high-energy γ is suppressed by α , but radiation of soft photons is enhanced.

Radiation changes both the cross-section and the kinematics of the final state:

$$\sigma = \frac{N_{obs} - N_{bg}}{\varepsilon(\delta) \cdot (1 + \delta) \cdot \int \mathcal{L} dt}$$

And we have to calculate radiative corrections to the cross section of monitoring process as well

Radiative processes



ISR

Initial
state radiation

FSR

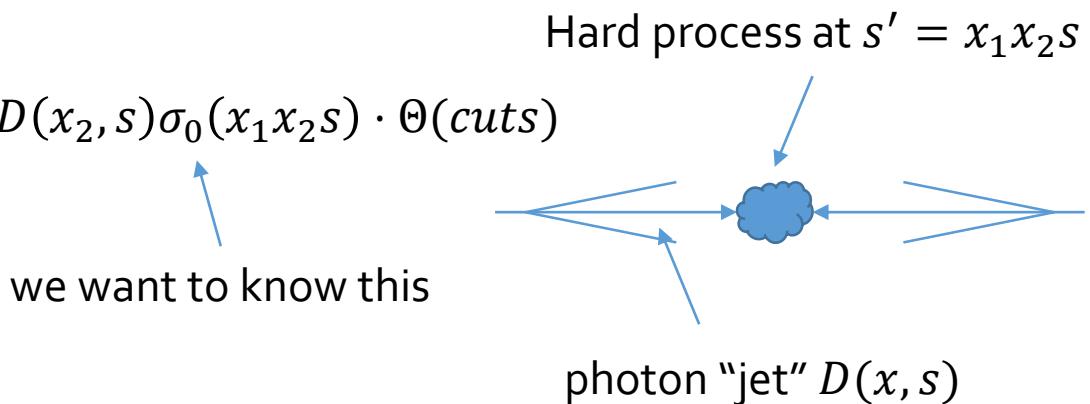
Final
state radiation

How to calculate radiative corrections

Main idea: allow each initial particle to emit any number of photons (jets).
The amount of energy carried by photons is described by structure function.

$$\sigma_{vis}(s) = \int_0^1 dx_1 dx_2 D(x_1, s) D(x_2, s) \sigma_0(x_1 x_2 s) \cdot \Theta(cuts)$$

we measure this



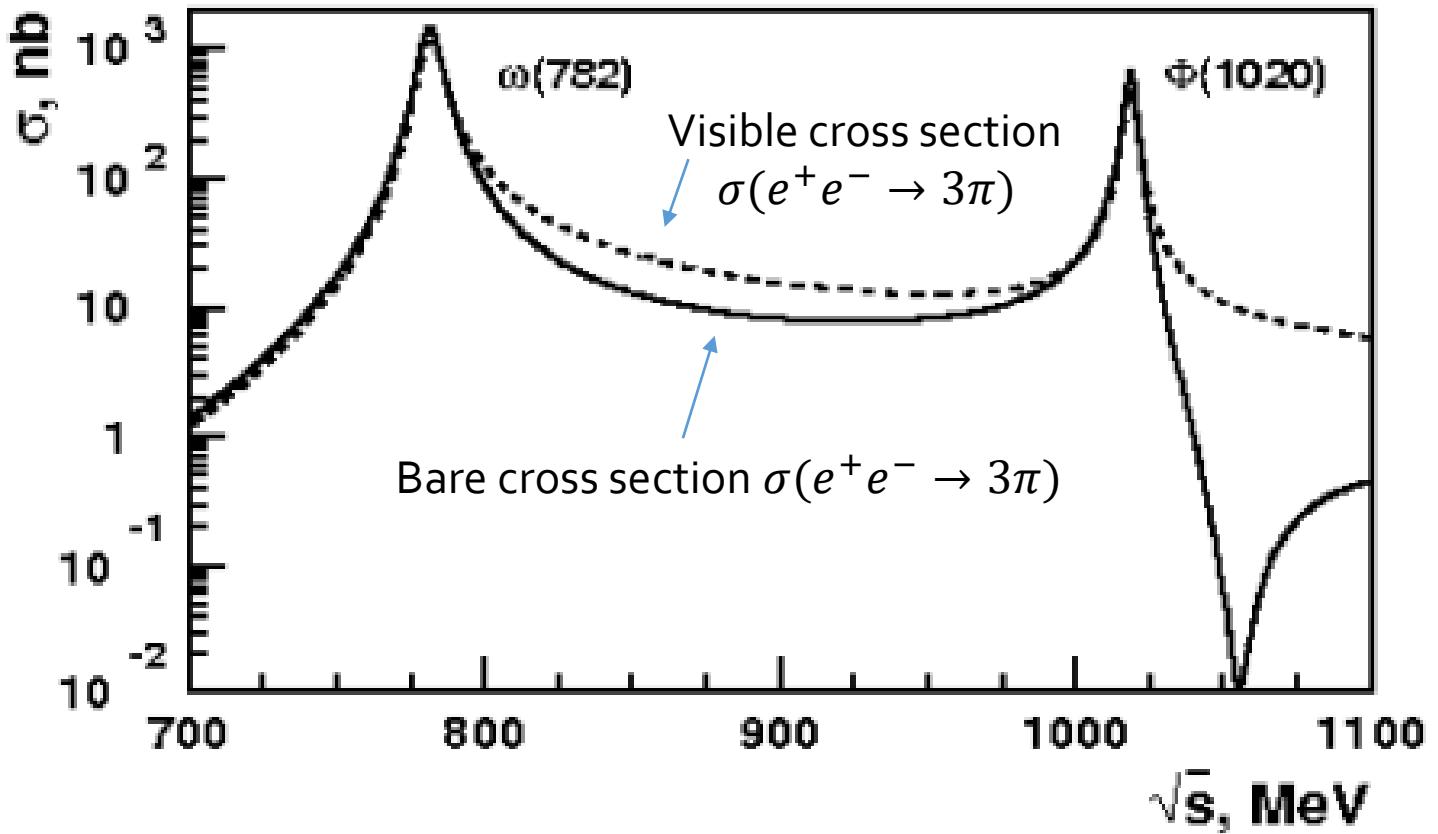
we want to know this

The radiative correction depends on the measured cross-section – need to use iterative procedure.

Structure functions are known to high precision (<0.1%). Main limitation is from kinematics: we don't take into account angular distribution of photons in the jet. This approach is ok for ~1% measurements and is typically used for multi-hadron events.

Typical value for radiative corrections is ~10% (can be much larger near narrow resonances)

Example:
 $e^+e^- \rightarrow \pi^+\pi^-\pi^0$



Radiative corrections for precise measurements

Calculation of radiative corrections for high-precision final states (e^+e^- , $\mu^+\mu^-$, $\pi^+\pi^-$, $\gamma\gamma$, ...) is much more complicated. Usually, it is implemented as MC generator and used together with the full detector simulation for proper evaluation of detector efficiency

Extensive review: Eur.Phys.J. C66 (2010) 585-686

MCGPJ (VEPP-2000)

1 real γ (from any particle) + jets along all particles

BABAYAGA (e^+e^-)

1 real γ + $n\gamma$ generated iteratively by emitting one γ at a time

PHOKHARA (KLOE, BABAR)

1 ISR γ + 1 real γ + soft

Many final states, intended for ISR measurements

These generators include ISR, FSR, virtual corrections, vacuum polarization and (partially) interference between various contributions.

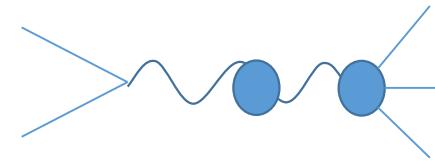
FSR from hadrons is model-dependent, e.g., assume point-like pions.

Vacuum polarization



$$\sigma^0(e^+e^- \rightarrow \gamma \rightarrow X)$$

In a_μ calculation



$$\sigma(e^+e^- \rightarrow \gamma^* \rightarrow X)$$

In experiment

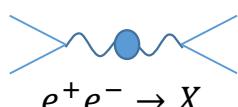
In the calculation of a_μ , we assume the lowest order photon propagator $1/q^2$. But the real propagator includes higher order effects (loop corrections): $1/(q^2 - \Pi(q^2))$. Therefore the measured cross section have to be corrected:

$$\sigma^0(e^+e^- \rightarrow X) = \sigma(e^+e^- \rightarrow X) \times \frac{|\alpha(s)|^2}{\alpha^2}$$

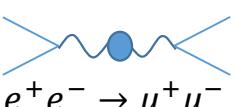
The running fine structure constant is also calculated via dispersion relation based on $R(s)$:

$$\Delta\alpha_{had}(s) = -\frac{\alpha s}{3\pi} \int_0^\infty \frac{R(s')}{s'(s-s'-i0)} ds'$$

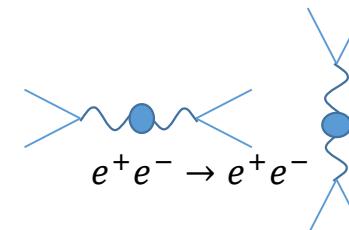
Nice way to avoid this correction is to use $e^+e^- \rightarrow \mu^+\mu^-$ for luminosity measurement



$$e^+e^- \rightarrow X$$

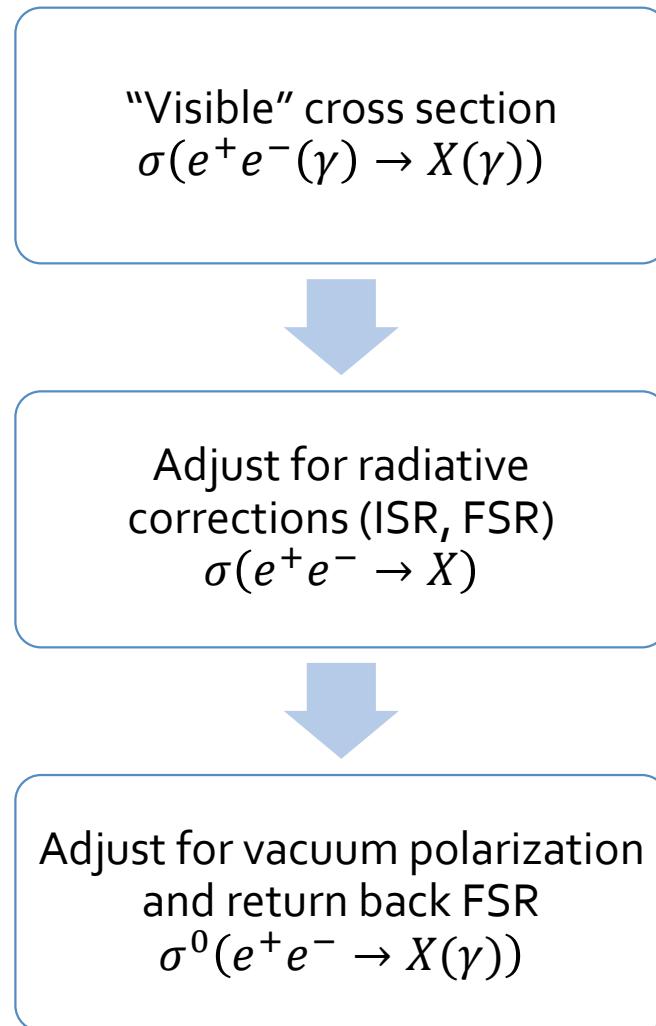


$$e^+e^- \rightarrow \mu^+\mu^-$$



$$e^+e^- \rightarrow e^+e^-$$

From measured cross section to input to a_μ calculation

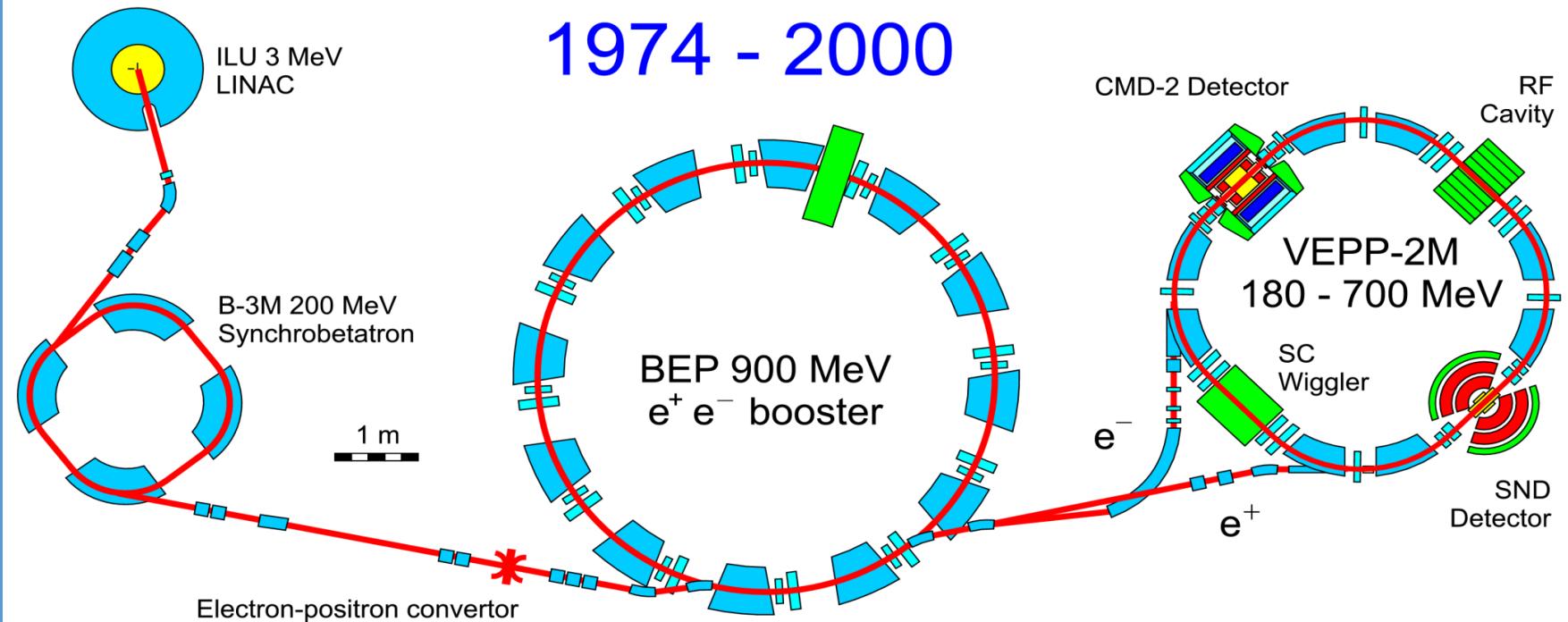


Here we correct for all detector effects

This one is used to get parameters of the resonances (mass, width,...)

This one is used in the a_μ integral

VEPP-2M (1993-2000)



1974 - 2000

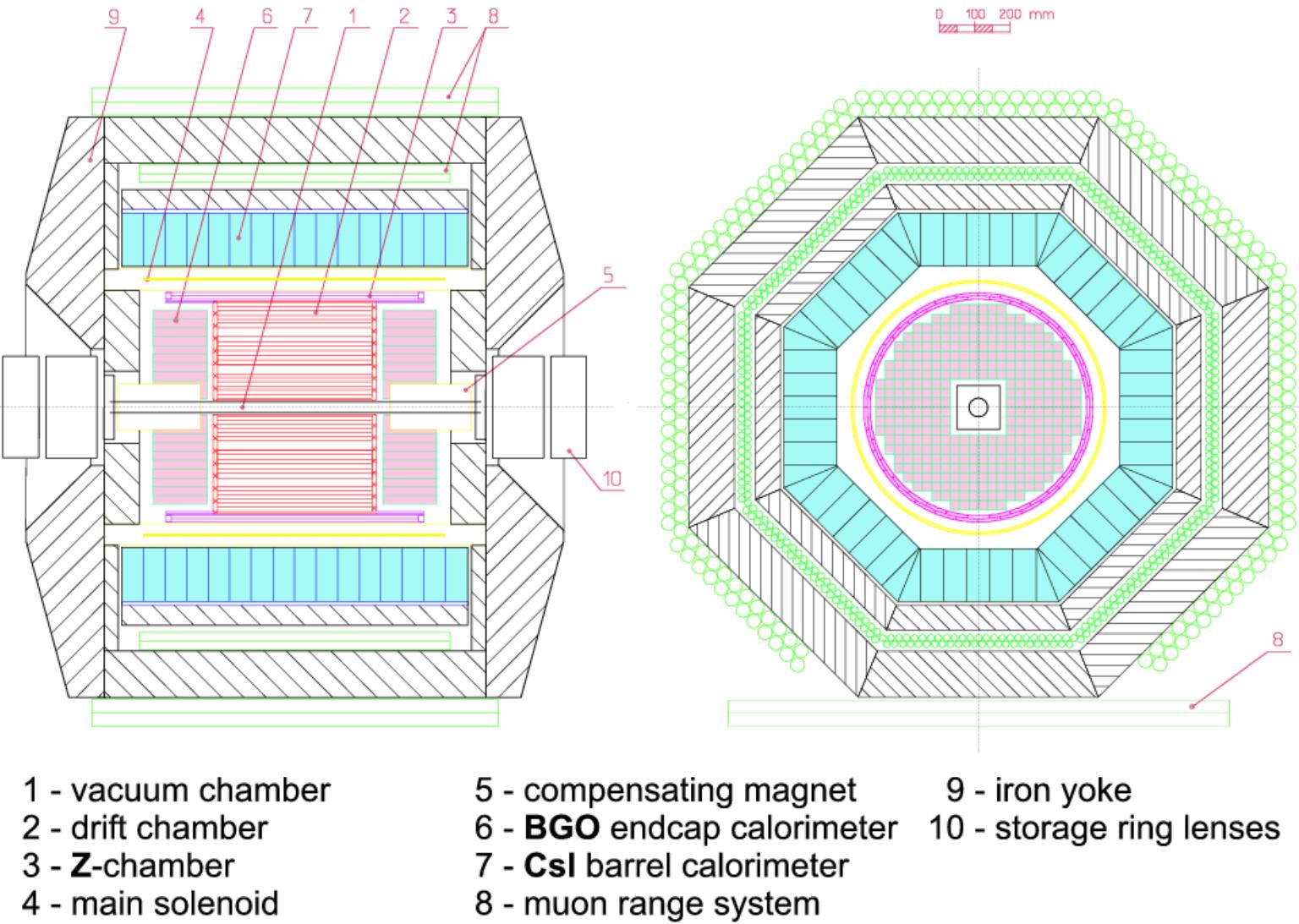
Energy range: 0.36 – 1.4 GeV

Luminosity up to $5 \times 10^{30} 1/\text{cm}^2\text{s}$

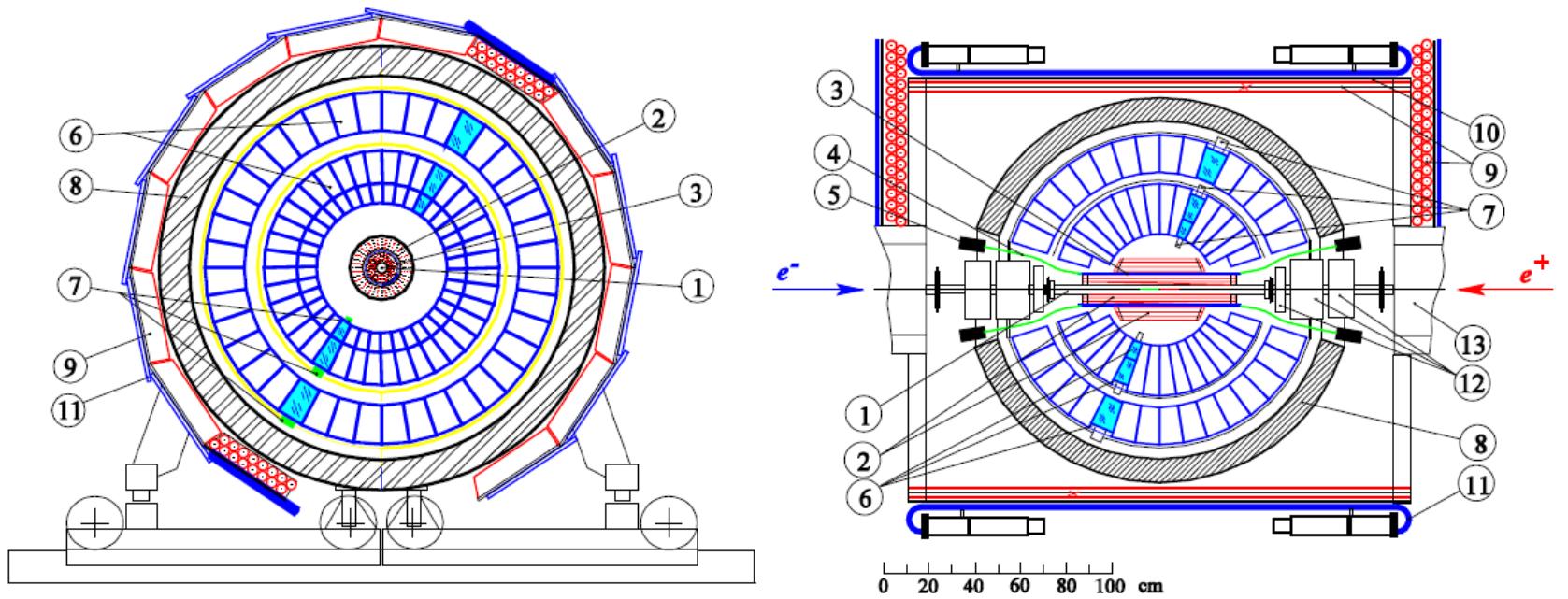
Lets set the scale:

$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ at ρ peak (0.77 GeV) $\sim 1000 \text{ nb}$
 $L = 10^{30} \text{ cm}^{-2}\text{s}^{-1}$ corresponds to 1 Hz for $\sigma = 1000 \text{ nb}$

CMD-2



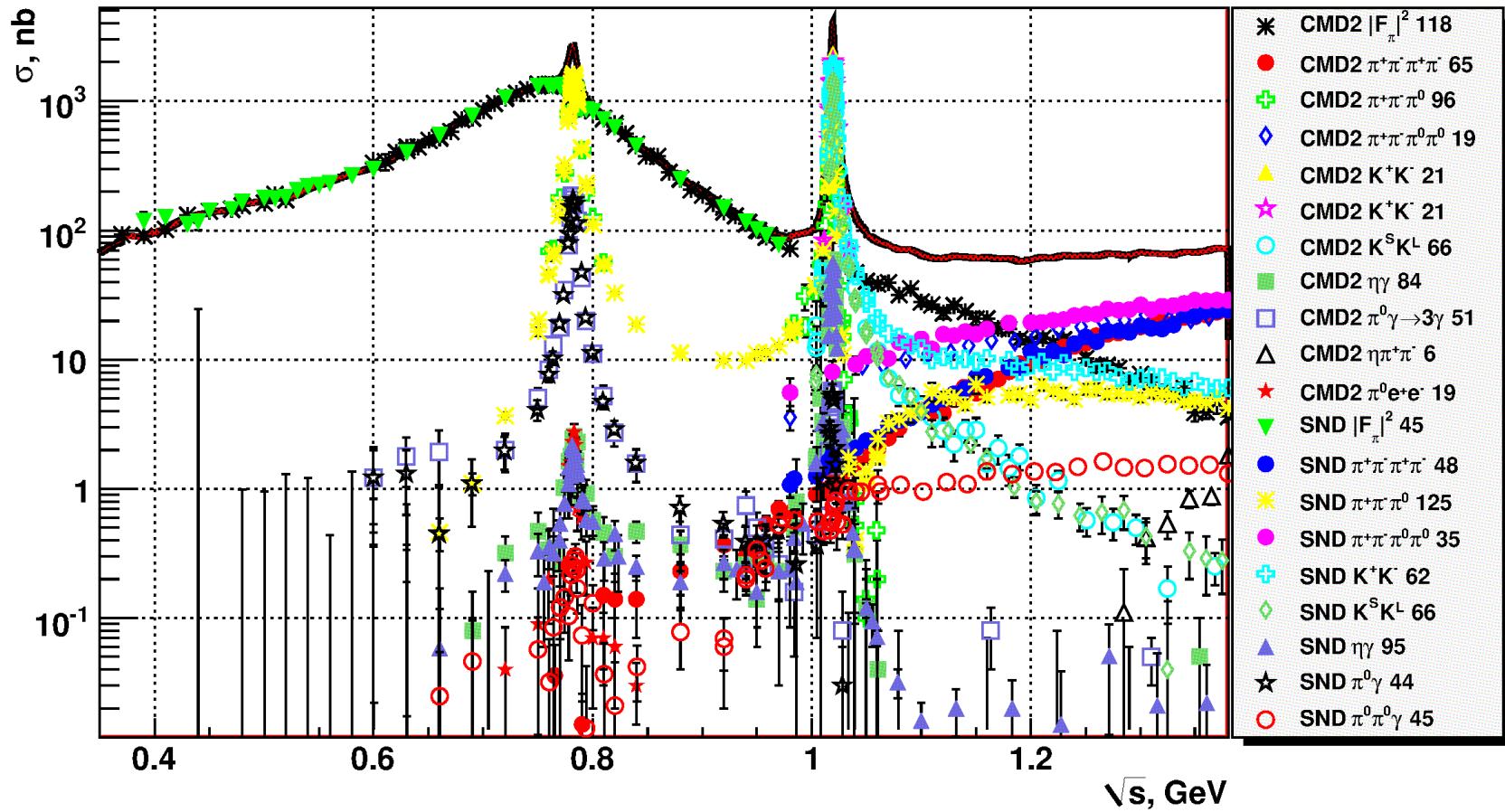
SND



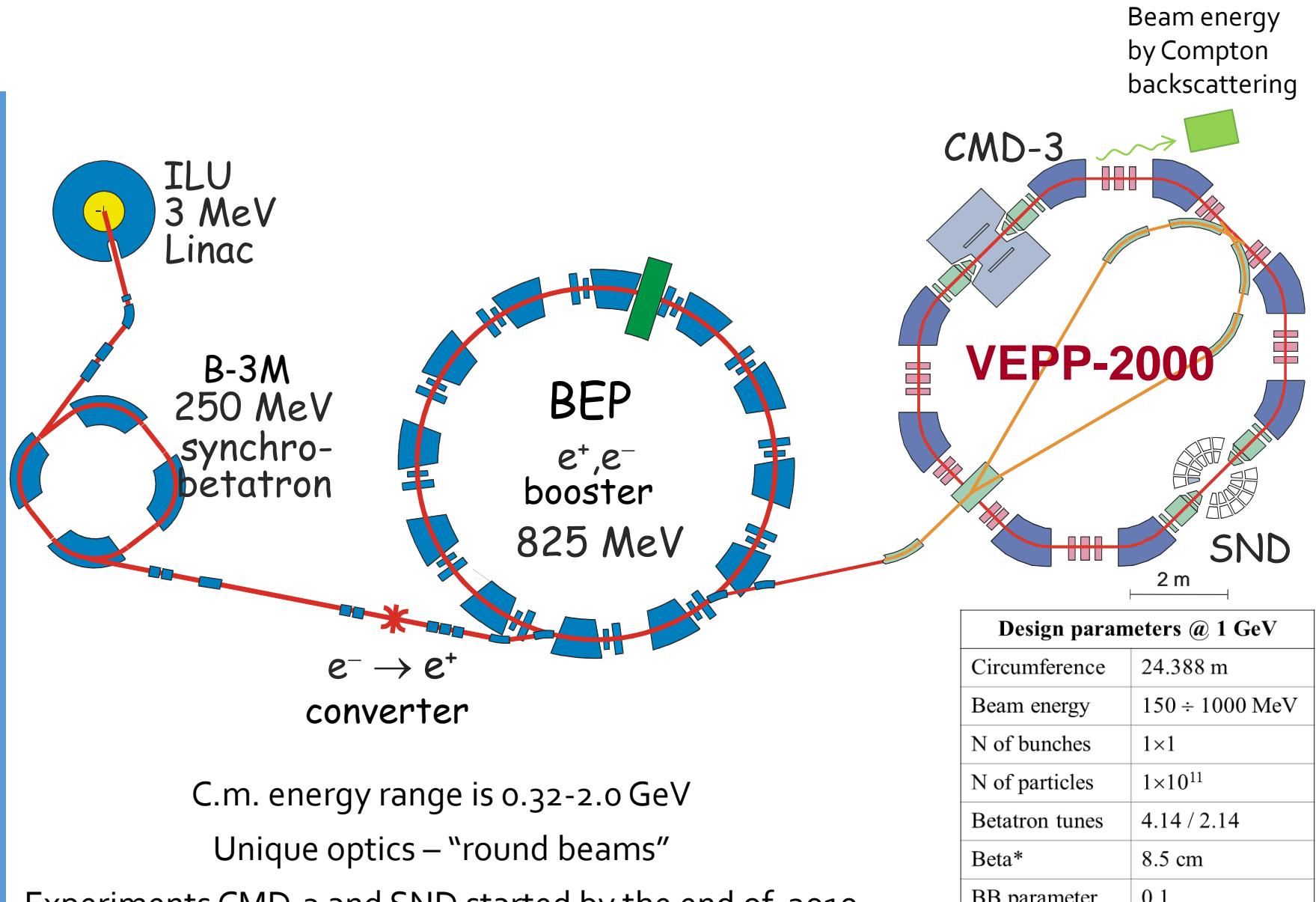
- No magnetic field
- Spherical three-layer NaI calorimeter
- Small drift chamber around interaction point

Optimized for neutral processes (e.g., $\pi^0\gamma$)

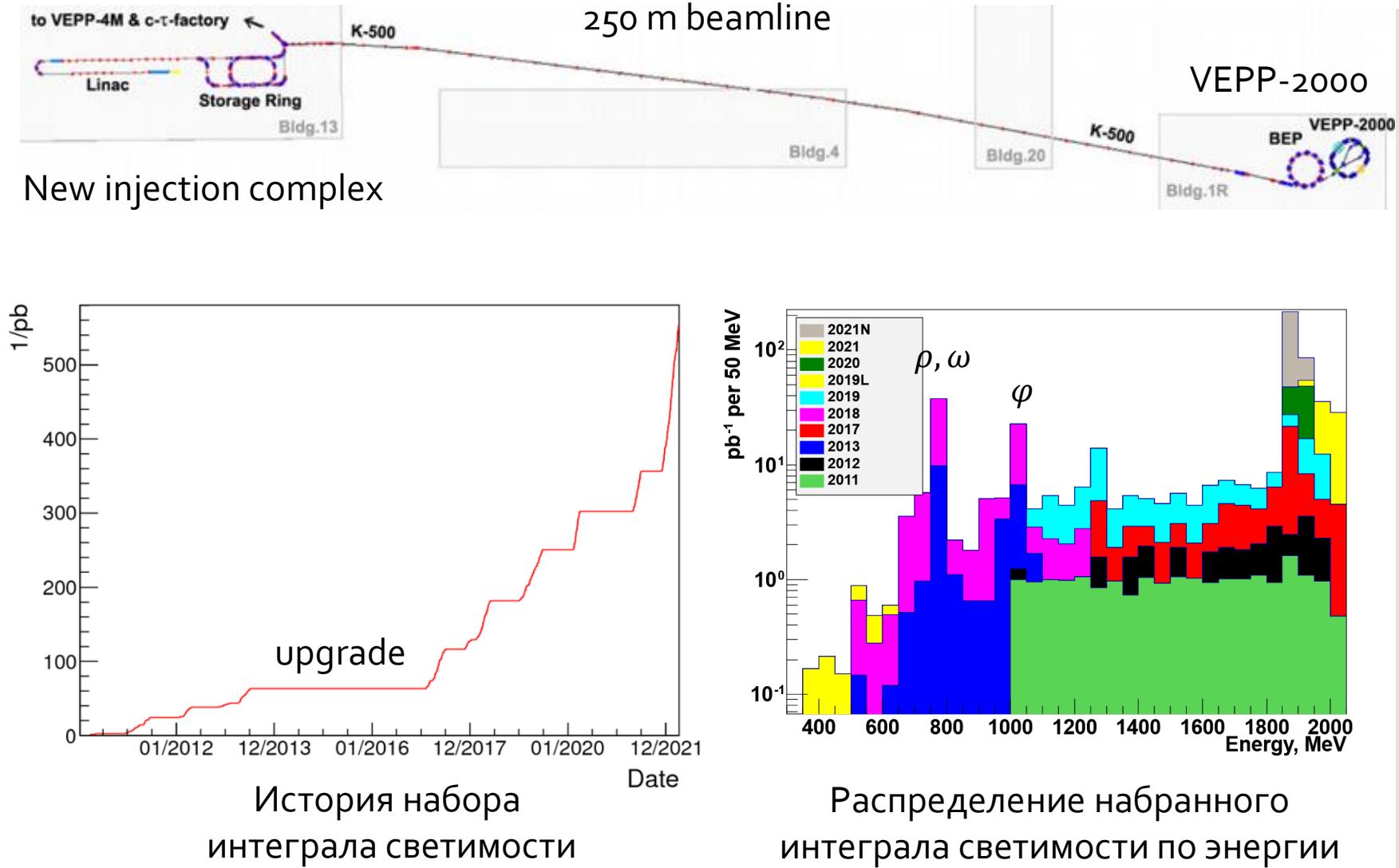
Overview of VEPP-2M measurements



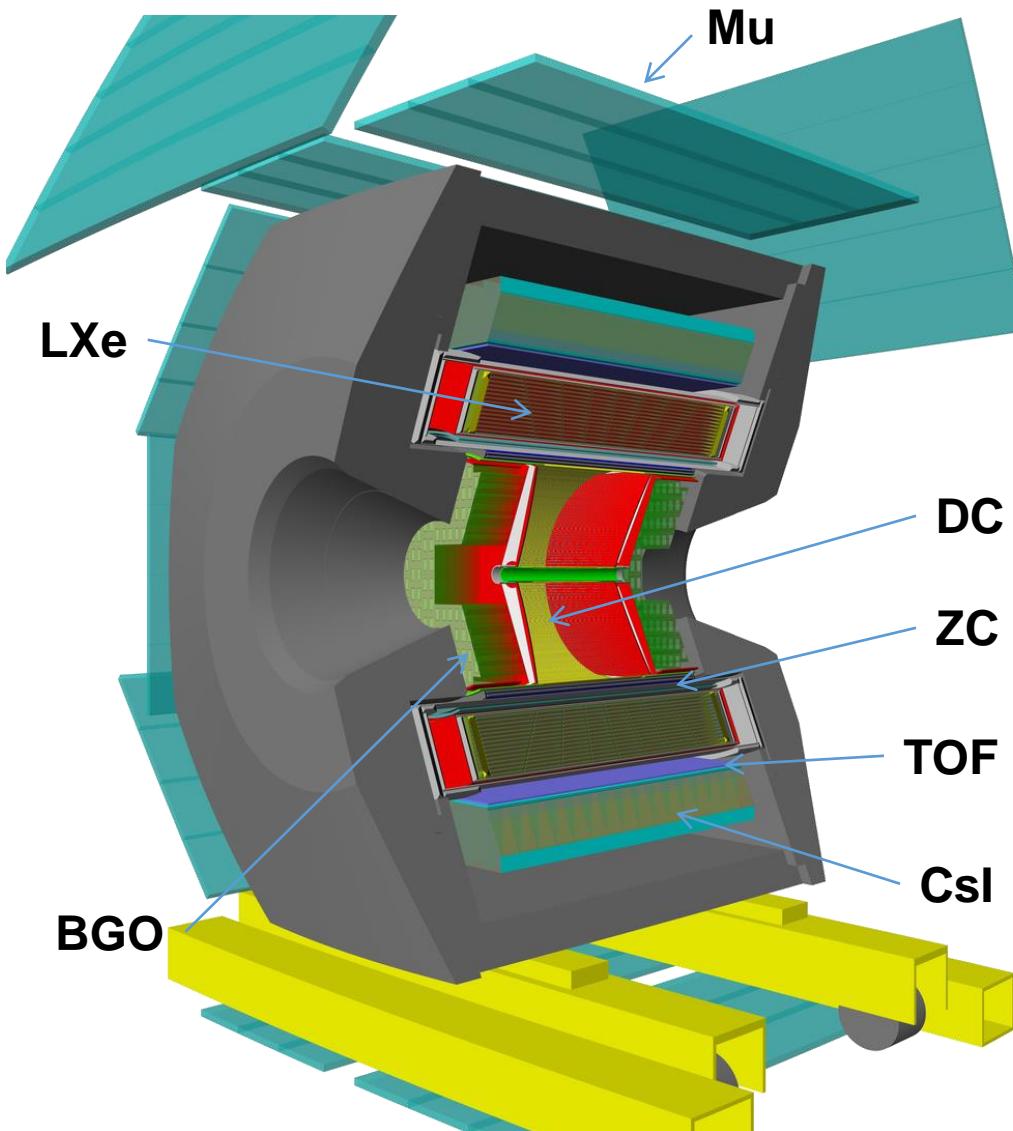
VEPP-2000 (2011-2013)



VEPP-2000 (2017-)



CMD-3

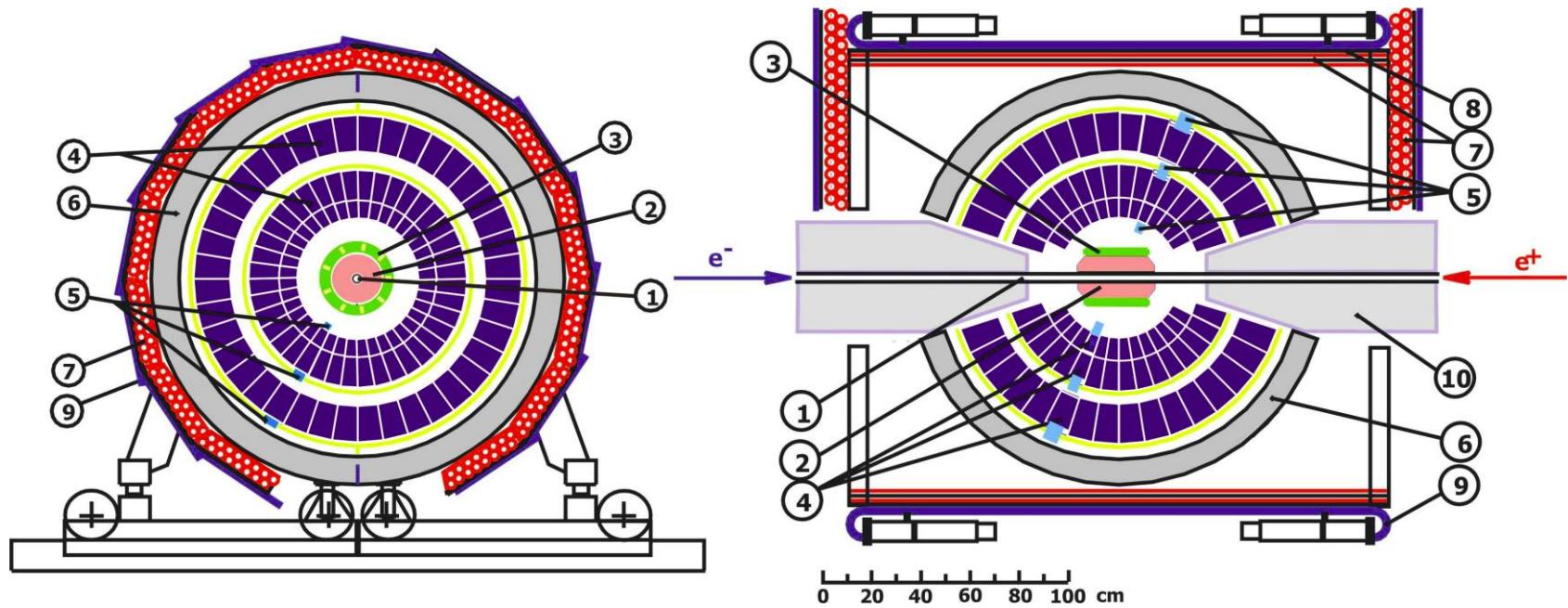


MISP 2022. Muon anomalous magnetic moment

Advantages compared to CMD-2:

- new drift chamber with two times better resolution, higher B field
better tracking
better momentum resolution
- thicker barrel calorimeter ($8.3X_0 \rightarrow 13.4 X_0$)
better particle separation
- LXe calorimeter
measurement of conversion point for γ 's
measurement of shower profile
- TOF system
particle id (mainly p, n)

SND

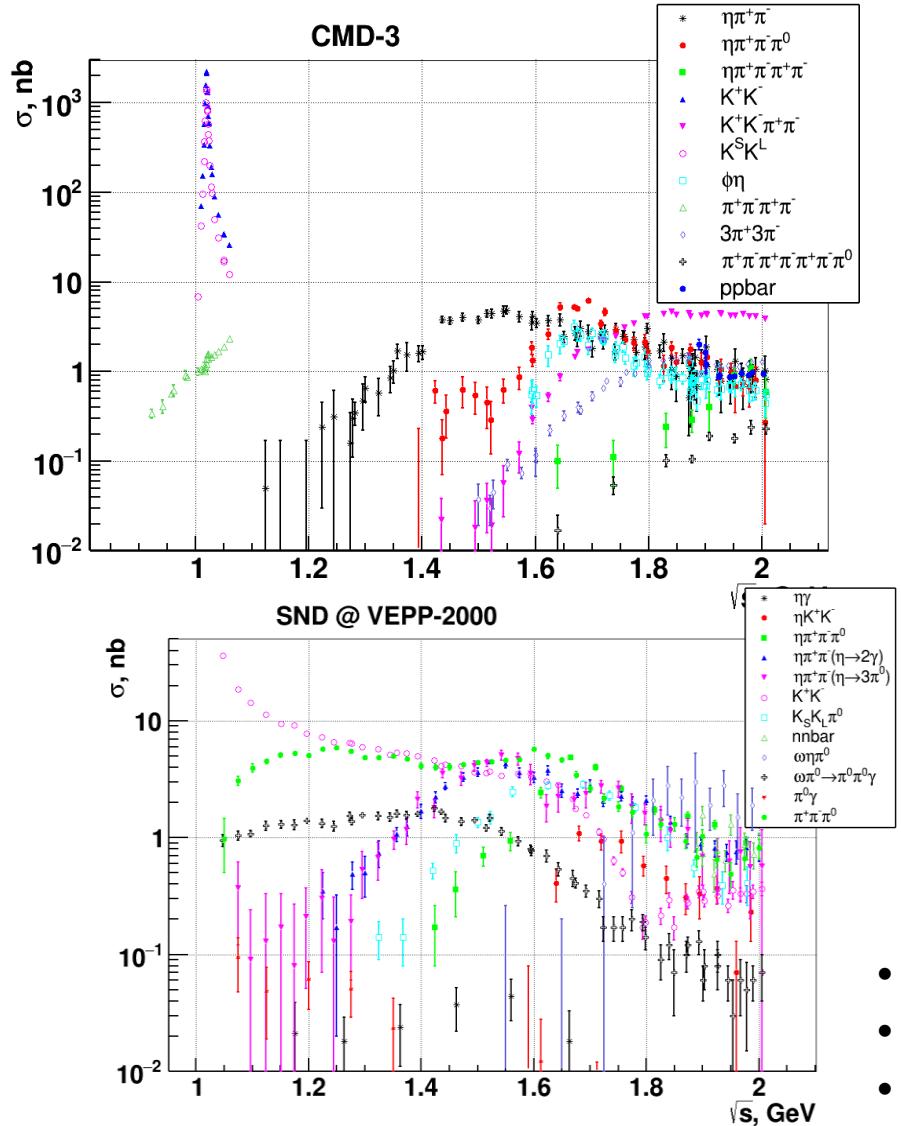


- 1 – beam pipe
- 2 – tracking system
- 3 – aerogel
- 4 – NaI(Tl) crystals
- 5 – phototriodes
- 6 – muon absorber
- 7-9 – muon detector
- 10 – focusing solenoid

Advantages compared to previous SND:

- new system - Cherenkov counter ($n=1.05, 1.13$)
e/ π separation $E < 450$ MeV
 π/K separation $E < 1$ GeV
- new drift chamber
better tracking
better determination of solid angle

Measurements at VEPP-2000

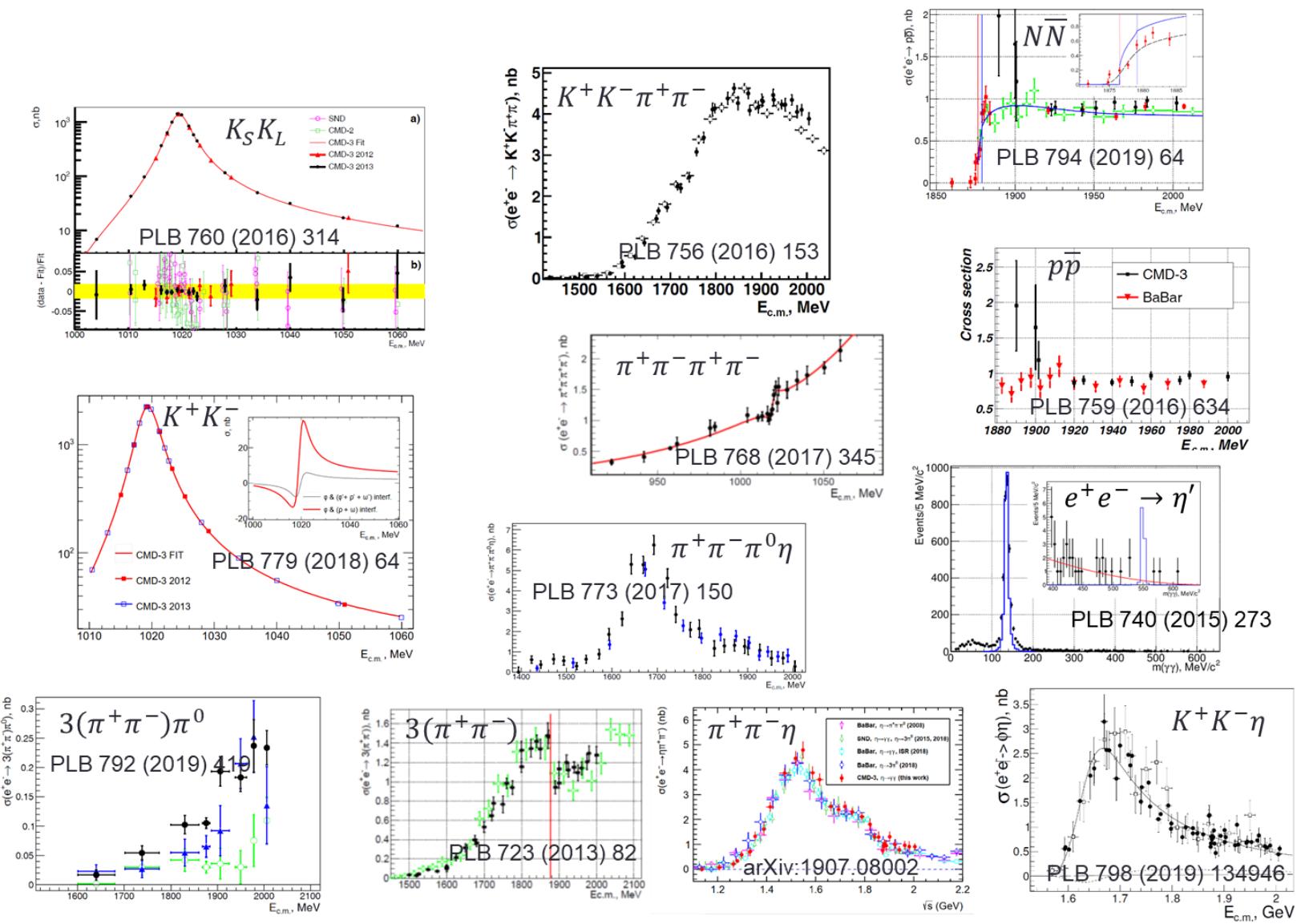


Final states under analysis at CMD-3

Signature	Final states (preliminary, published)
2 charged	$\pi^+\pi^-$, K^+K^- , $K_S K_L$, $p\bar{p}$
2 charged + γ 's	$\pi^+\pi^-\gamma$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-2\pi^0$, $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $\pi^+\pi^-\eta$, $\pi^+\pi^-\pi^0\eta$, $\pi^+\pi^-2\pi^0\eta$, $K^+K^-\pi^0$, $K^+K^-2\pi^0$, $K^+K^-\eta$, $K_S K_L \pi^0$, $K_S K_L \eta$
4 charged	$2(\pi^+\pi^-)$, $K^+K^-\pi^+\pi^-$, $K_S K^\pm \pi^\mp$
4 charged + γ 's	$2(\pi^+\pi^-)\pi^0$, $2\pi^+2\pi^-2\pi^0$, $\pi^+\pi^-\eta$, $\pi^+\pi^-\omega$, $2\pi^+2\pi^-\eta$, $K^+K^-\omega$, $K_S K^\pm \pi^\mp \pi^0$
6 charged	$3(\pi^+\pi^-)$, $K_S K_S \pi^+\pi^-$
6 charged + γ 's	$3(\pi^+\pi^-)\pi^0$
Neutral	$\pi^0\gamma$, $2\pi^0\gamma$, $3\pi^0\gamma$, $\eta\gamma$, $\pi^0\eta\gamma$, $2\pi^0\eta\gamma$
Other	$n\pi$, $\pi^0e^+e^-$, ηe^+e^-
Rare decays	η' , $D^*(2007)^0$

- More final states compare to VEPP-2M
- 1-2 order of magnitude more data
- The experiments are collecting data

CMD-3 published results



Understanding of intermediate dynamics

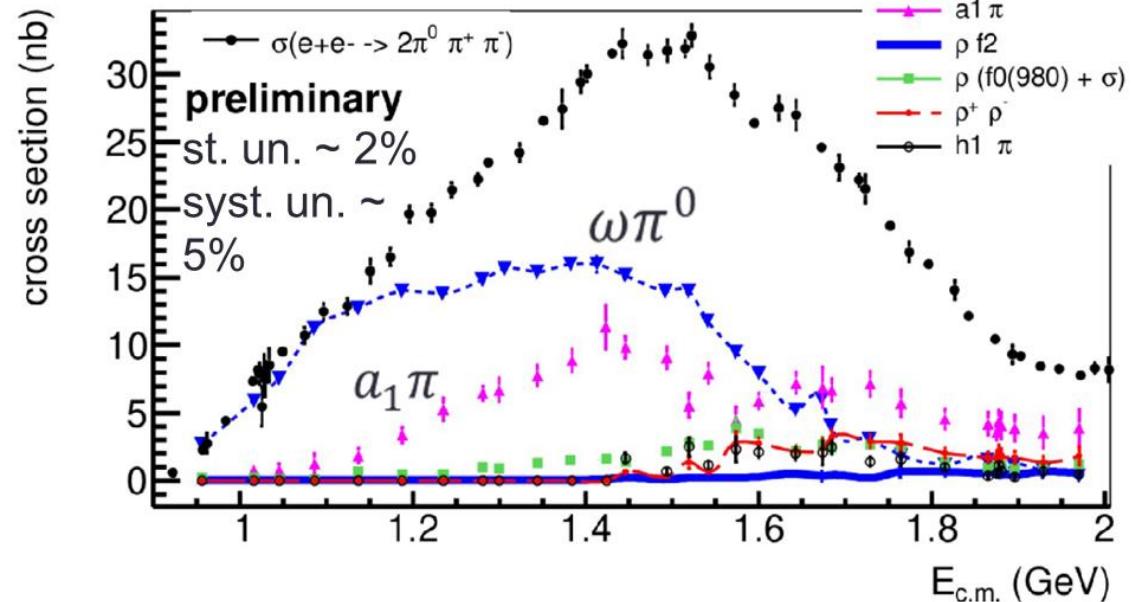
In order to measure hadronic cross section, you have to understand the dynamics of the process (to properly evaluate detector efficiency). **High statistics is crucial!**

Example: four pions at CMD-3

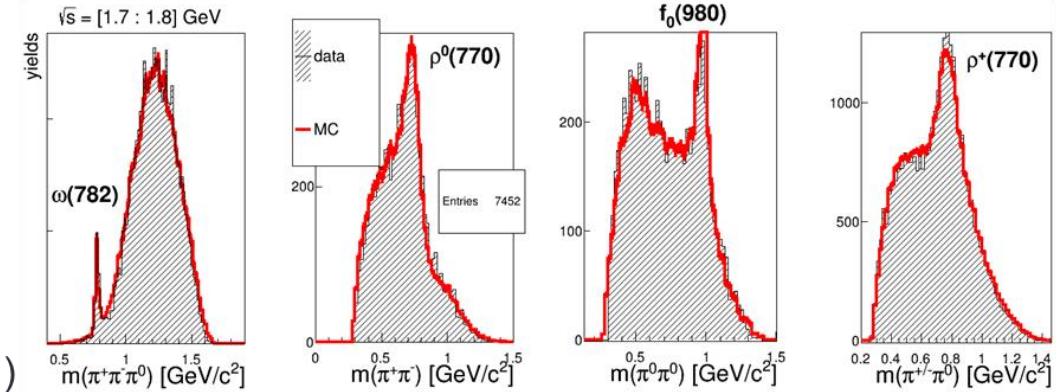
Simultaneous unbinned amplitude analysis of 150 000 $\pi^+\pi^-\pi^0\pi^0$ events and 250 000 $\pi^+\pi^-\pi^+\pi^-$ events.

Amplitudes accounted for in the likelihood function:

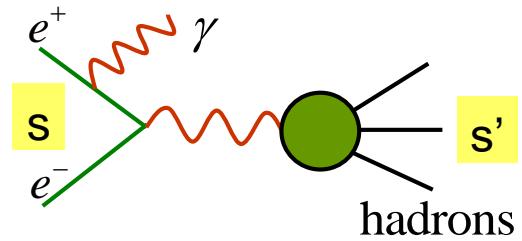
- $\omega[1^{--}]\pi^0[0^{++}]$ (only $\pi^+\pi^-2\pi^0$)
- $a_1(1260)[1^+]\pi[0^-]$
- $\rho[1^{--}]f^0/\sigma[0^{++}]$
- $\rho f_2(1270)[2^{++}]$
- $\rho^+\rho^-$ (only $\pi^+\pi^-2\pi^0$)
- $h_1(1170)[1^{+-}]\pi^0$ (only $\pi^+\pi^-2\pi^0$)



Intermediate resonances observed in data:



ISR approach



The cross section is extracted from the spectrum of $e^+e^- \rightarrow \gamma_{ISR} X$ events:

Effective luminosity

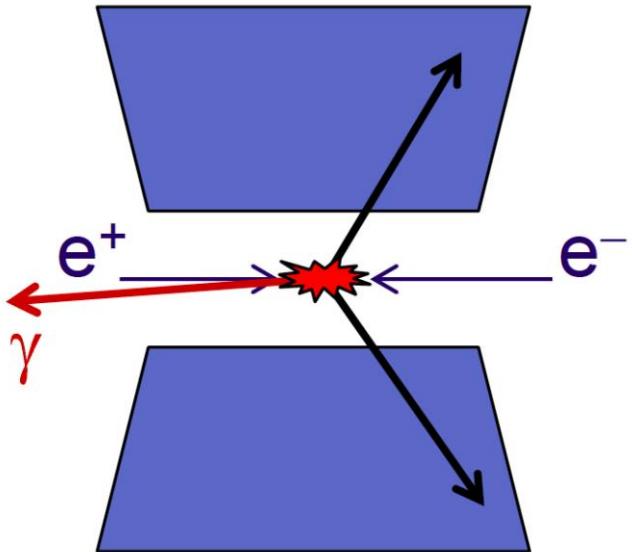
ISR luminosity is 2-3 orders of magnitude smaller than plain luminosity. Need high luminosity collider – “factory”.

The initial-state radiation (ISR) approach:
take data at single energy point and
identify $e^+e^- \rightarrow X + \gamma$ events to extract
cross-section $e^+e^- \rightarrow X$ in the wide
energy range.

$$\frac{dN_{X(\gamma)\gamma_{ISR}}}{d\sqrt{s'}} = \frac{dL_{ISR}^{eff}}{d\sqrt{s'}} \varepsilon_{X(\gamma)}(\sqrt{s'}) \sigma_{X(\gamma)}(\sqrt{s'})$$

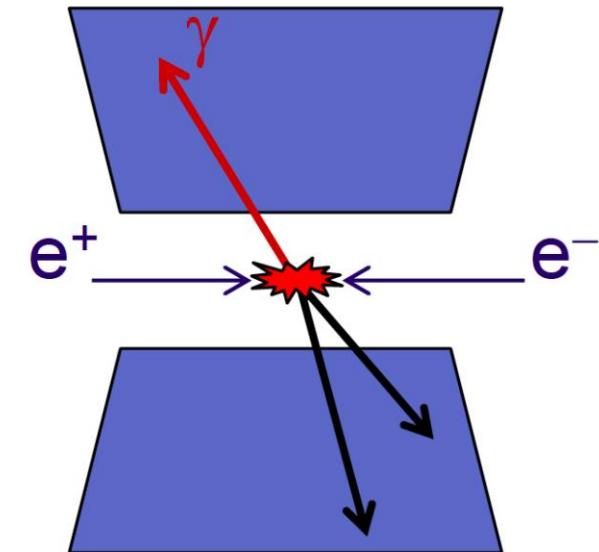
$$\frac{dL_{ISR}^{eff}}{d\sqrt{s'}} = L_{ee} \frac{dW}{d\sqrt{s'}} \quad \text{Radiator function – probability to radiate ISR photon (with radiative corrections)}$$

Small angle vs large angle ISR



Small angle (untagged) ISR

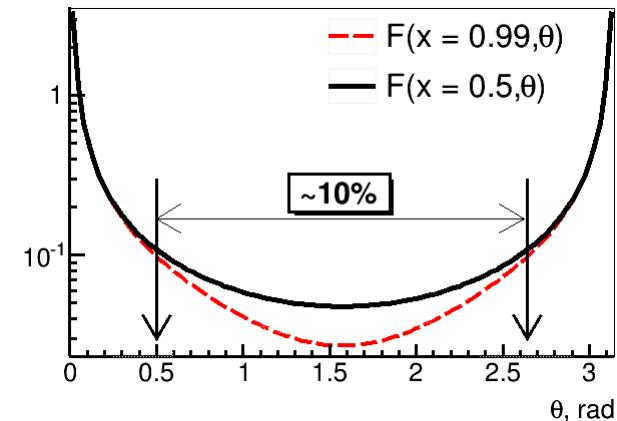
- ISR photon emitted along initial beam, undetected
- ISR photon is reconstructed from kinematics of the final state



Large angle (tagged) ISR

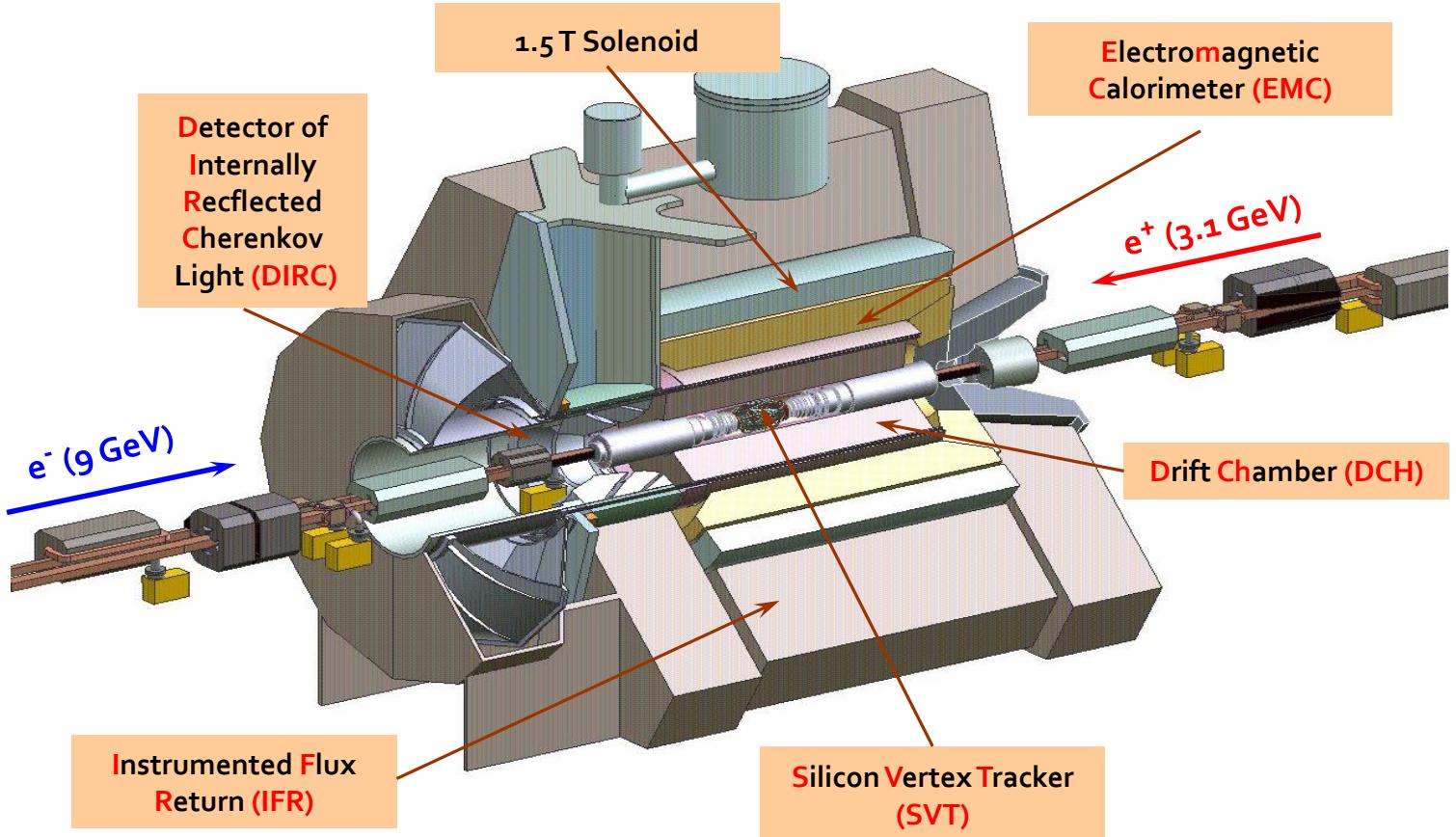
- ISR photon emitted at large angle and detected

Angular distribution of γ_{ISR}

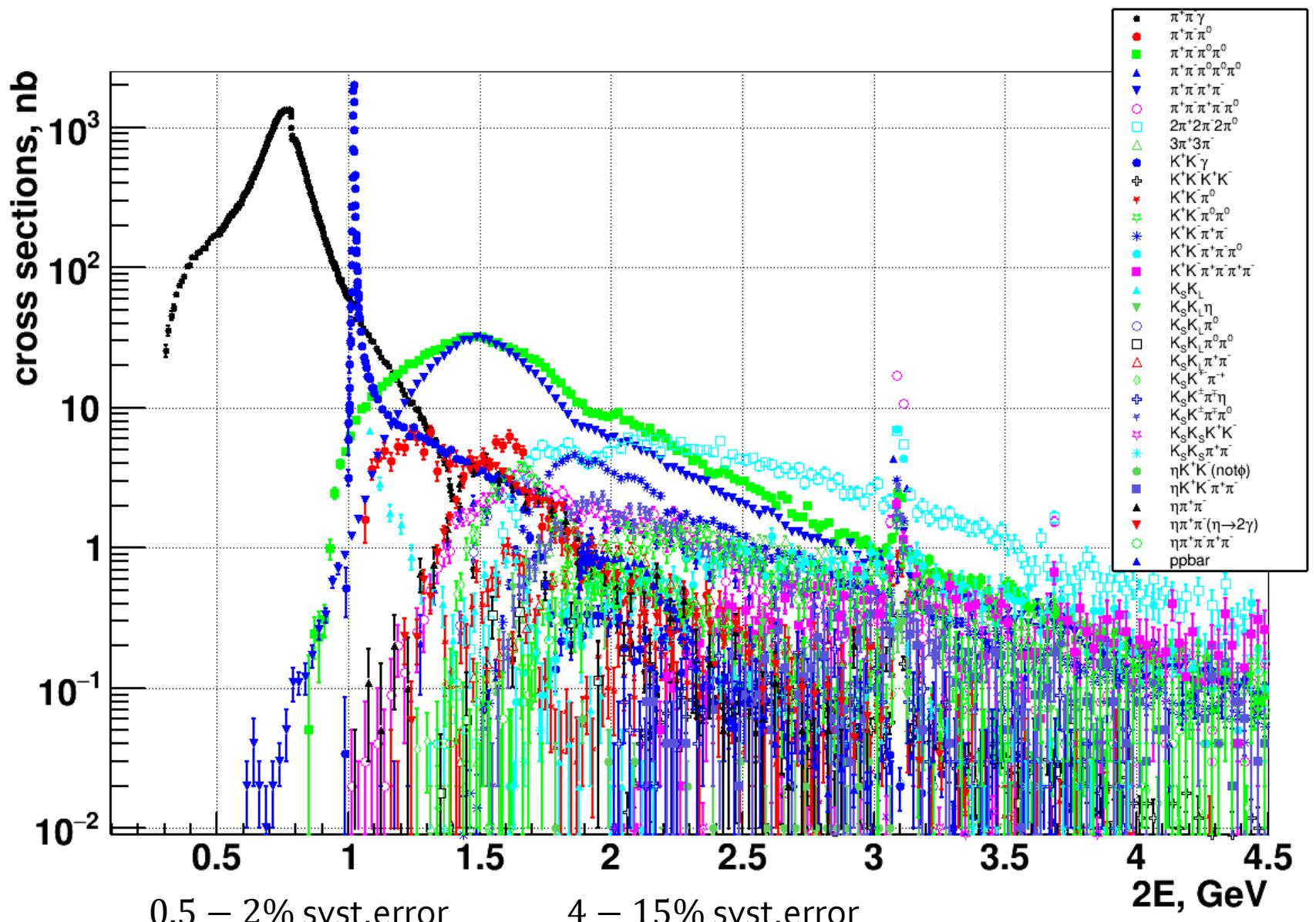


BABAR experiment (1999-2008)

PEP-II asymmetric e^+e^- collider at SLAC
9 GeV e^- and 3.1 GeV e^+
About 500 fb^{-1} collected in 1999-2008
Comprehensive program of ISR measurements, using a data sample of 469 fb^{-1} collected at and near $\Upsilon(4S)$ (10.58 GeV)



BABAR



BABAR measurements are mostly tagged

Tagged ISR method at BABAR

Fully exclusive measurement

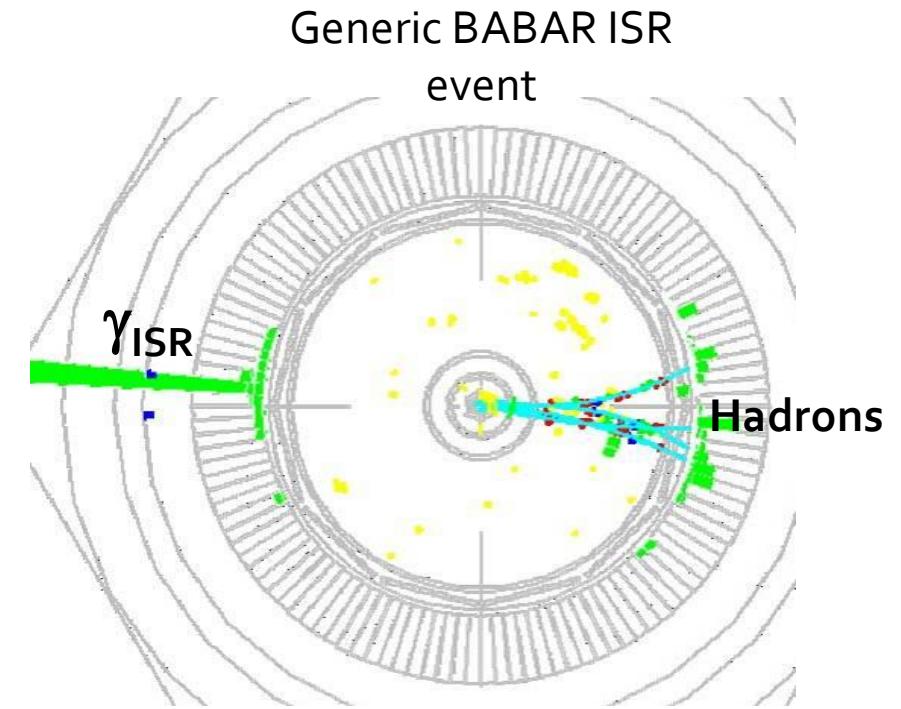
- ✓ Photon with $E_{CM} > 3$ GeV, which is assumed to be the ISR photon
- ✓ All final hadrons are detected and identified

Large-angle ISR forces the hadronic system into the detector fiducial region

- ✓ A weak dependence of the detection efficiency on dynamics of the hadronic system (angular and momentum distributions in the hadron rest frame)
⇒ smaller model uncertainty
- ✓ A weak dependence of the detection efficiency on hadron invariant mass ⇒ measurement near and above threshold with the same selection criteria.

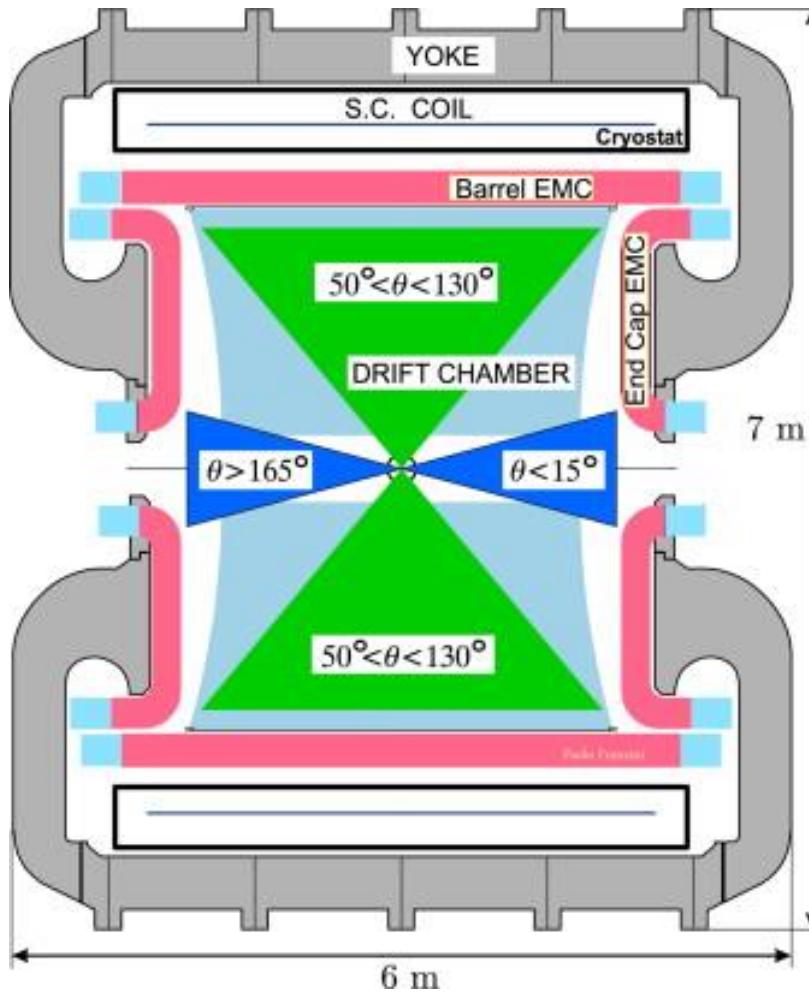
Kinematic fit with requirement of energy and momentum balance

- ✓ excellent mass resolution
- ✓ background suppression



Can access a wide range of energy in a single experiment: from threshold to ~5 GeV

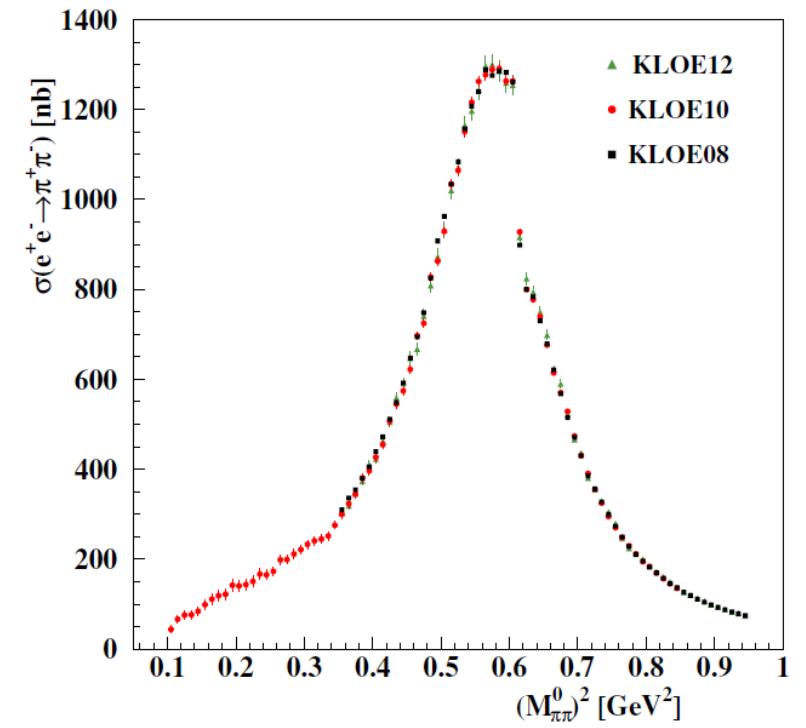
KLOE (2000-2006)



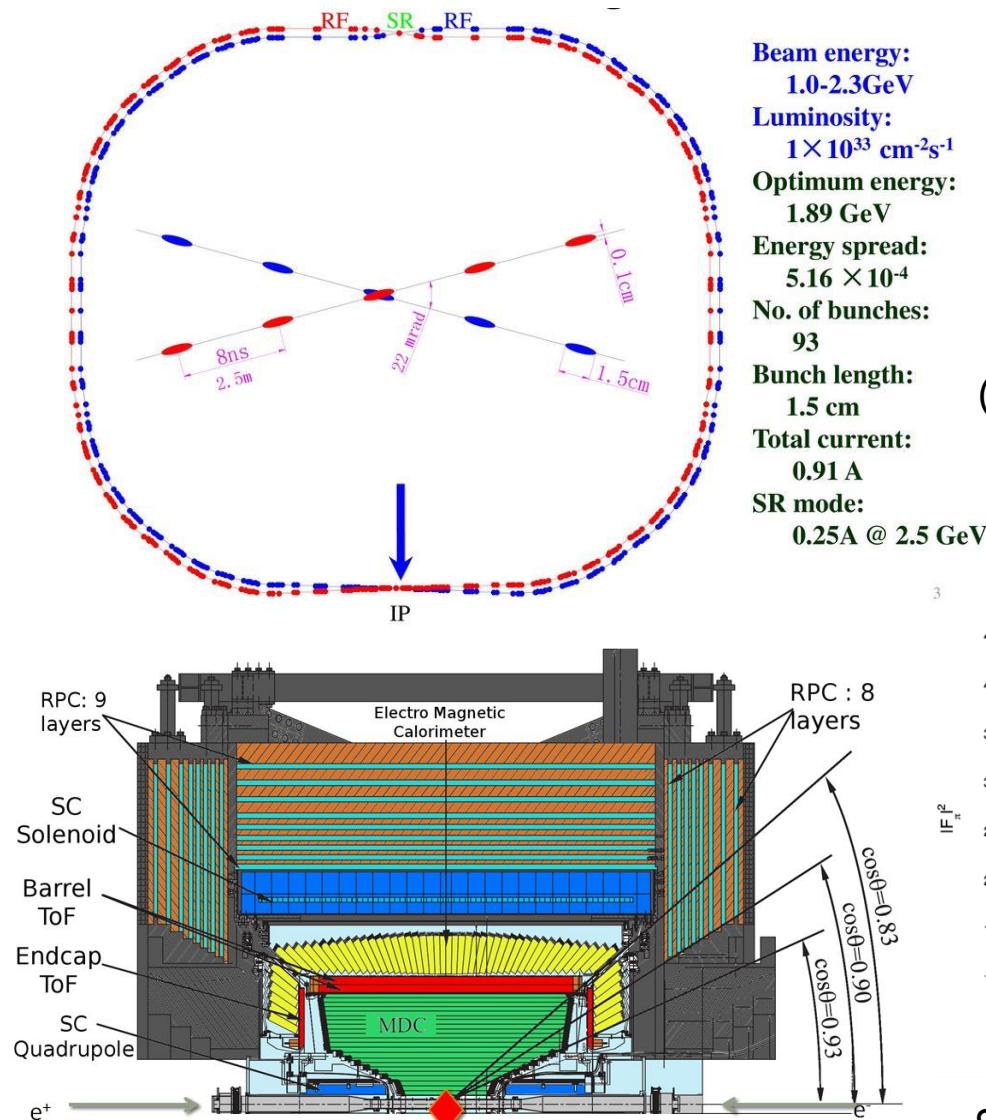
Installed at the DAFNE phi-factory

Mostly collected data at $\phi(1020)$ meson

ISR measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$,
both tagged and untagged



BES-III

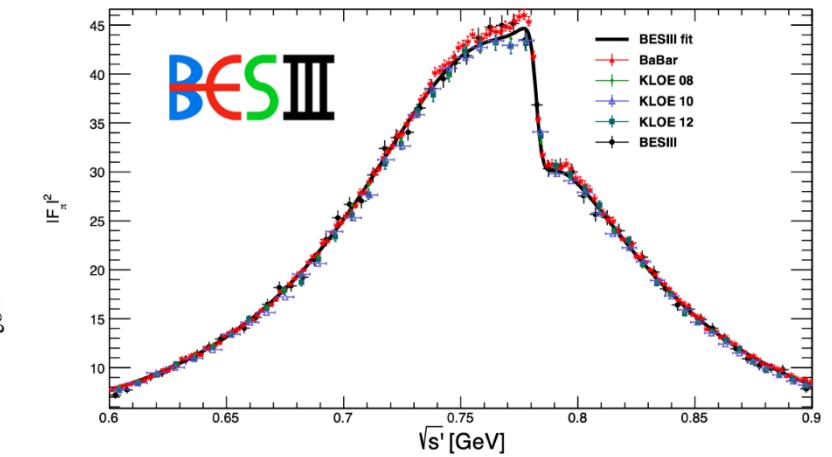


MISP 2022. Muon anomalous magnetic moment

BEPC-II collider covers c.m.energy range from 2 to 5 GeV
“ $c\tau$ -factory”

BES-III detector is taking data
(and there were BES and BES-II before)

Tagged ISR measurement
 $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$



Statistics is limited compare to BaBar

Variety of ISR approaches

	Tagged ISR	Untagged ISR
Normalization to e^+e^-	KLOE-2010 ($\pi^+\pi^-$) BABAR (most channels)	KLOE-2005 ($\pi^+\pi^-$) KLOE-2008 ($\pi^+\pi^-$) BABAR ($p\bar{p}$)
Normalization to $\mu^+\mu^-(\gamma)$	BABAR ($\pi^+\pi^-$)* BES-III ($\pi^+\pi^-$) CLEO-c ($\pi^+\pi^-$)	KLOE-2012 ($\pi^+\pi^-$)

ISR vs energy scan

- Energy scan analysis is generally simpler, but ISR measurements were done with superior detectors
- Before VEPP-2000, ISR measurements had more statistics
- In general, background is higher for ISR measurements
- ISR approach allows for larger detector coverage and smaller model-dependence
- In both approaches the visible cross-section is smeared and we need to unfold it:

Energy scan

The cross-section is smeared by ISR

$$\sigma_{vis}(s) = \int_0^1 dx_1 dx_2 D(x_1, s) D(x_2, s) \sigma_0(x_1 x_2 s)$$

The beam energy is known to high precision ($\sim 10^{-4} - 10^{-3}$)

The “unfolding” is done via radiative corrections

The “response” function is model-dependent, but it does not have unknown pieces

ISR

The cross-section is smeared by detector resolution

$$\frac{d\sigma_{vis}(s, s')}{ds'} = \frac{2s'}{s} W(s, s') \sigma_0(s')$$

The energy of the final state s' is reconstructed from the kinematics.

If the detector response function is known, the unfolding is the robust procedure.

But tails in the response function can lead to large effects.

Inclusive measurements

Inclusive measurements were systematically performed at $\sqrt{s} \gtrsim 2$ GeV

Signal events: one or more hadrons in the final state + any number of extra particles

Cuts on multiplicity, sphericity,...

With or without particle identification

$$\sigma_{\text{mh}}^{\text{obs}}(s) = \frac{N_{\text{mh}} - N_{\text{res.bg}}}{\int \mathcal{L} dt}$$

$$R = \frac{\sigma_{\text{mh}}^{\text{obs}}(s) - \sum \varepsilon_{\text{bg}}(s) \sigma_{\text{bg}}(s) - \sum \varepsilon_{\psi}(s) \sigma_{\psi}(s)}{\varepsilon(s)(1 + \delta(s)) \sigma_0^{\text{e}^+ \text{e}^- \rightarrow \mu^+ \mu^-}(s)}$$

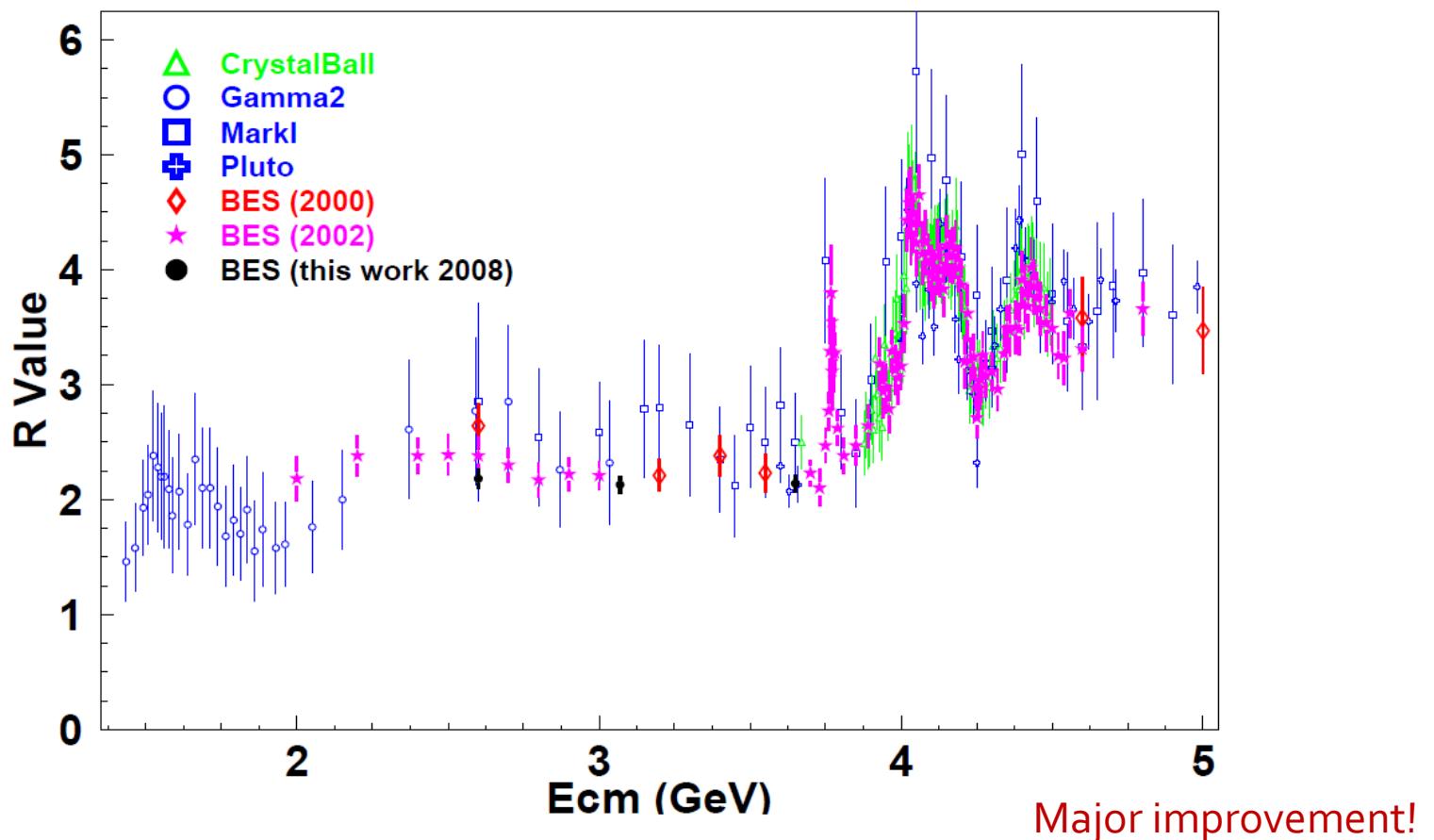
The analysis depends on the same ingredients as the exclusive measurement: event selection, luminosity measurement, calculation of radiative corrections, evaluation of detector efficiency

Key difficulty: to properly model hadronic events for evaluation of efficiencies and radiative corrections. There are dedicated MC generators: JETSET, LUARLW

“Typical” good precision: $\frac{\delta R}{R} \sim 3\%$, best achieved $\sim 2\%$.

Important to have large detection efficiency (now $\sim 75\%$)

BES-II

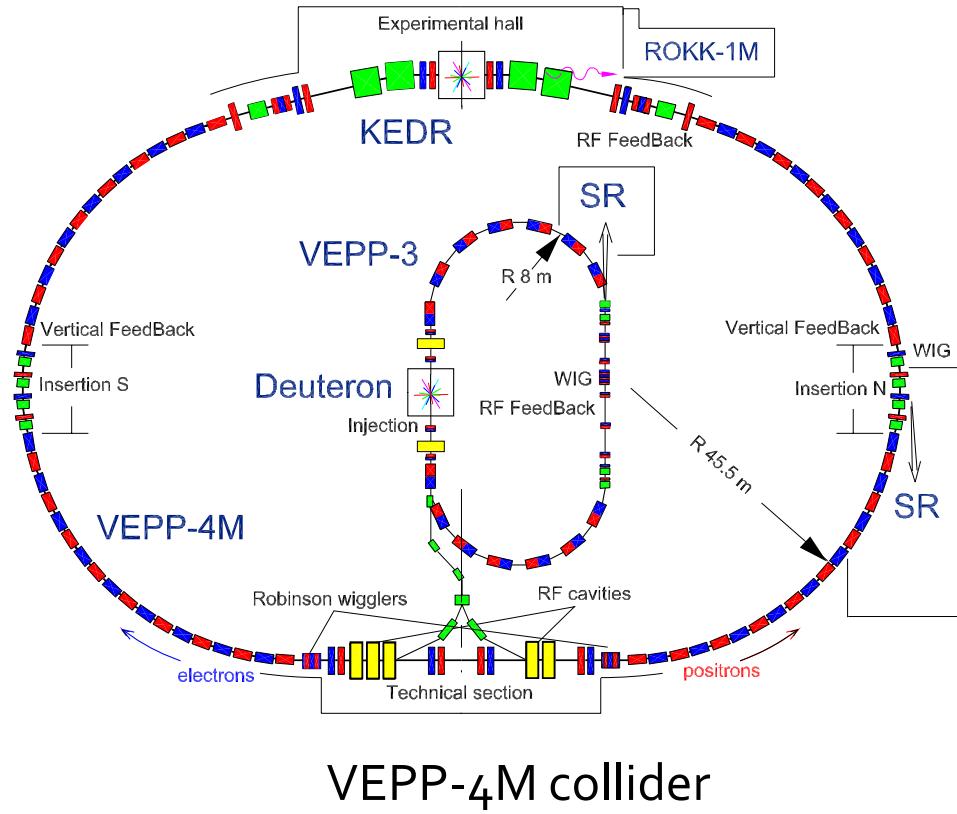


BES-II performed detailed $R(s)$ scan between 2 and 5 GeV

- 3 – 5% statistical error per point
- 5 – 8% systematical error

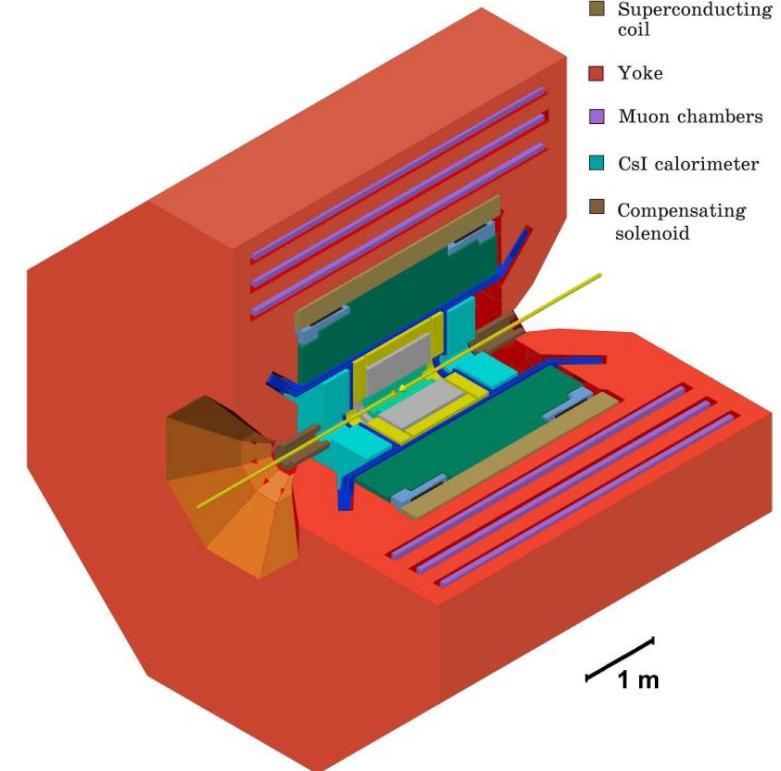
BES-III collected a lot of $R(s)$ data (125 points), not published yet

KEDR

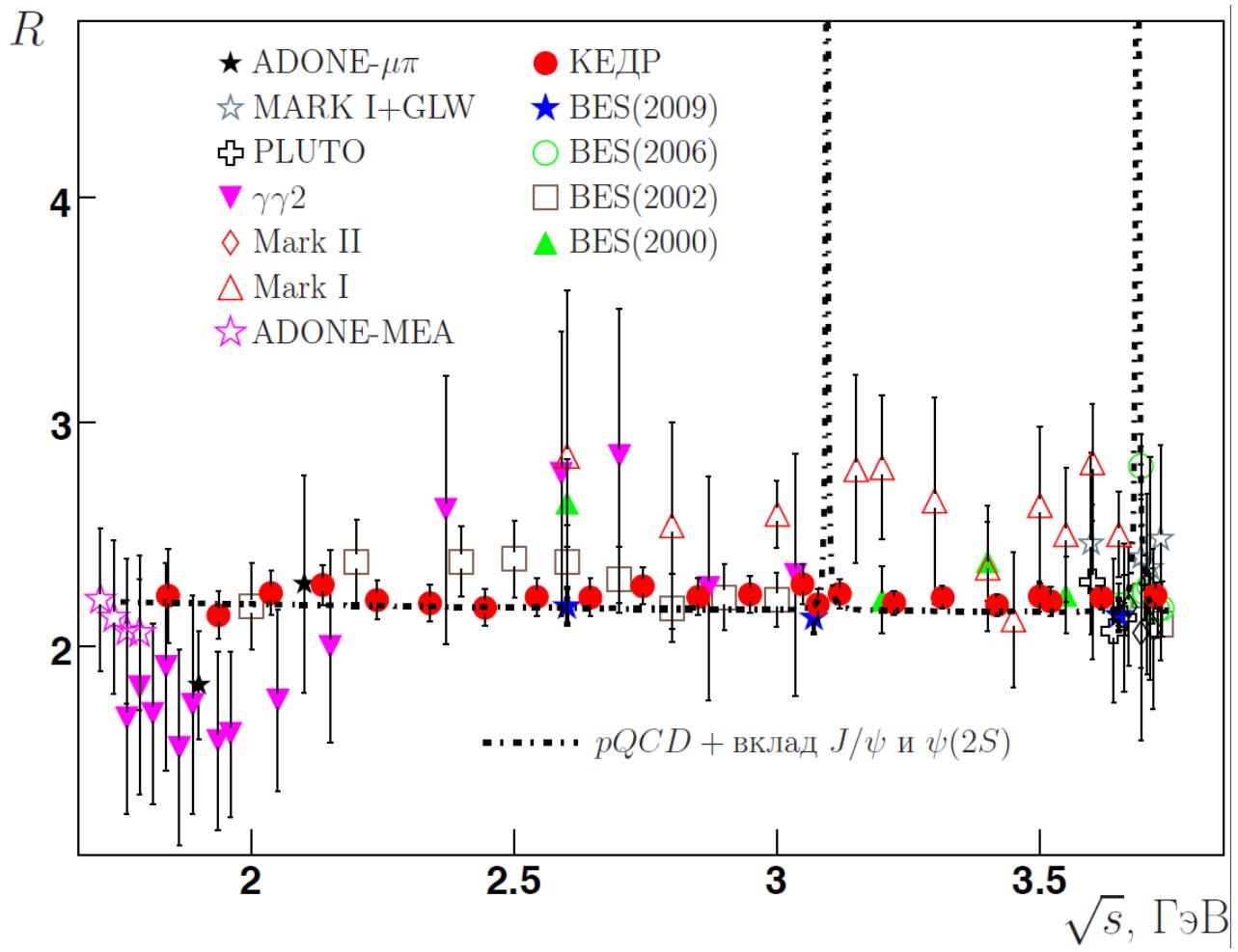


Beam energy range 0.925-5.3 GeV
Luminosity $\sim 4 \cdot 10^{31} \text{ cm}^{-2}\text{s}^{-1}$
Beam energy is determined to 20-30 keV
(using Compton backscattering and resonance depolarization)

KEDR detector



KEDR



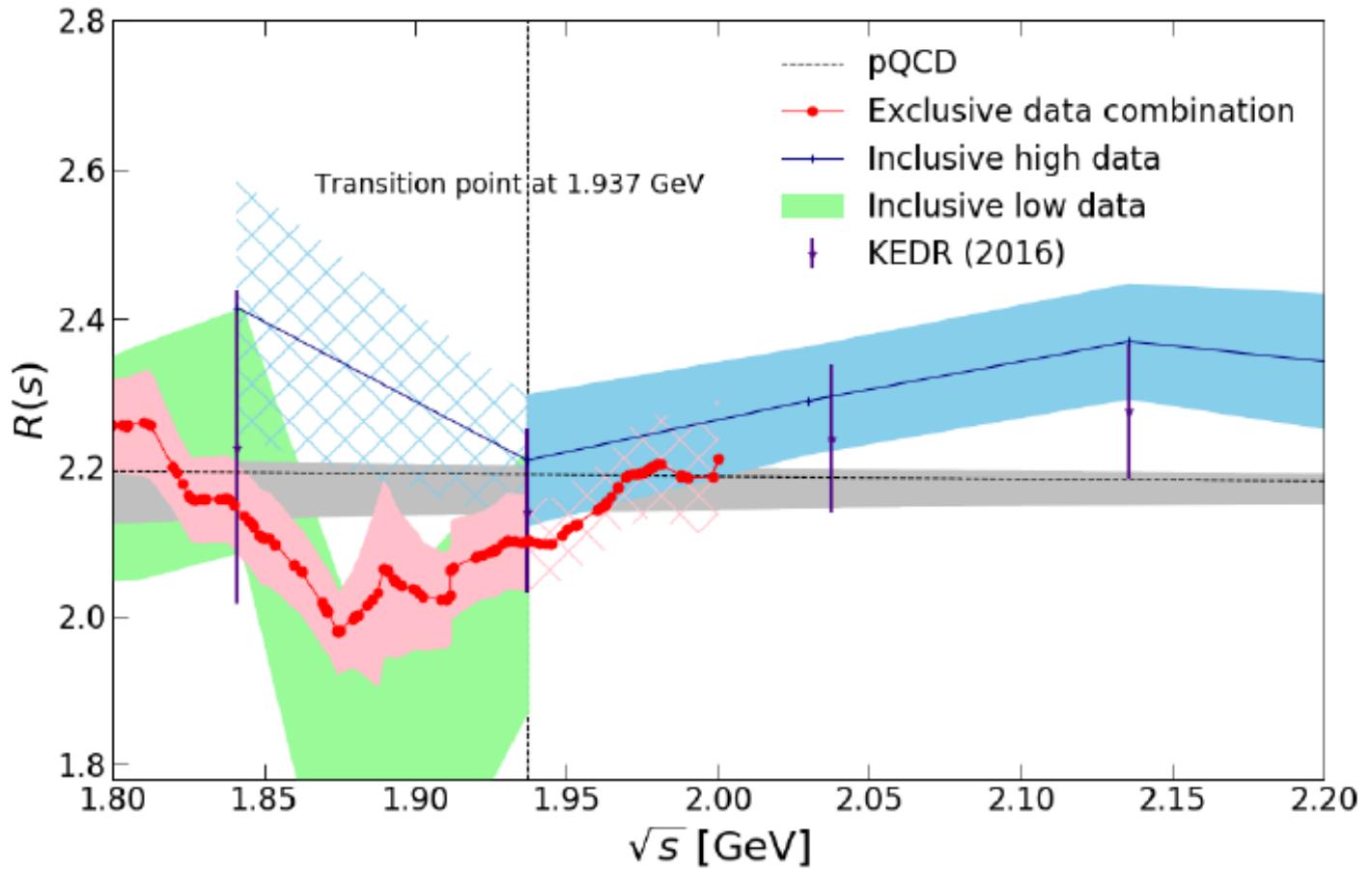
KEDR performed detailed $R(s)$ scan
between 1.8 and 3.7 GeV

- 2 – 3% statistical error per point
- 2 – 3% systematical error

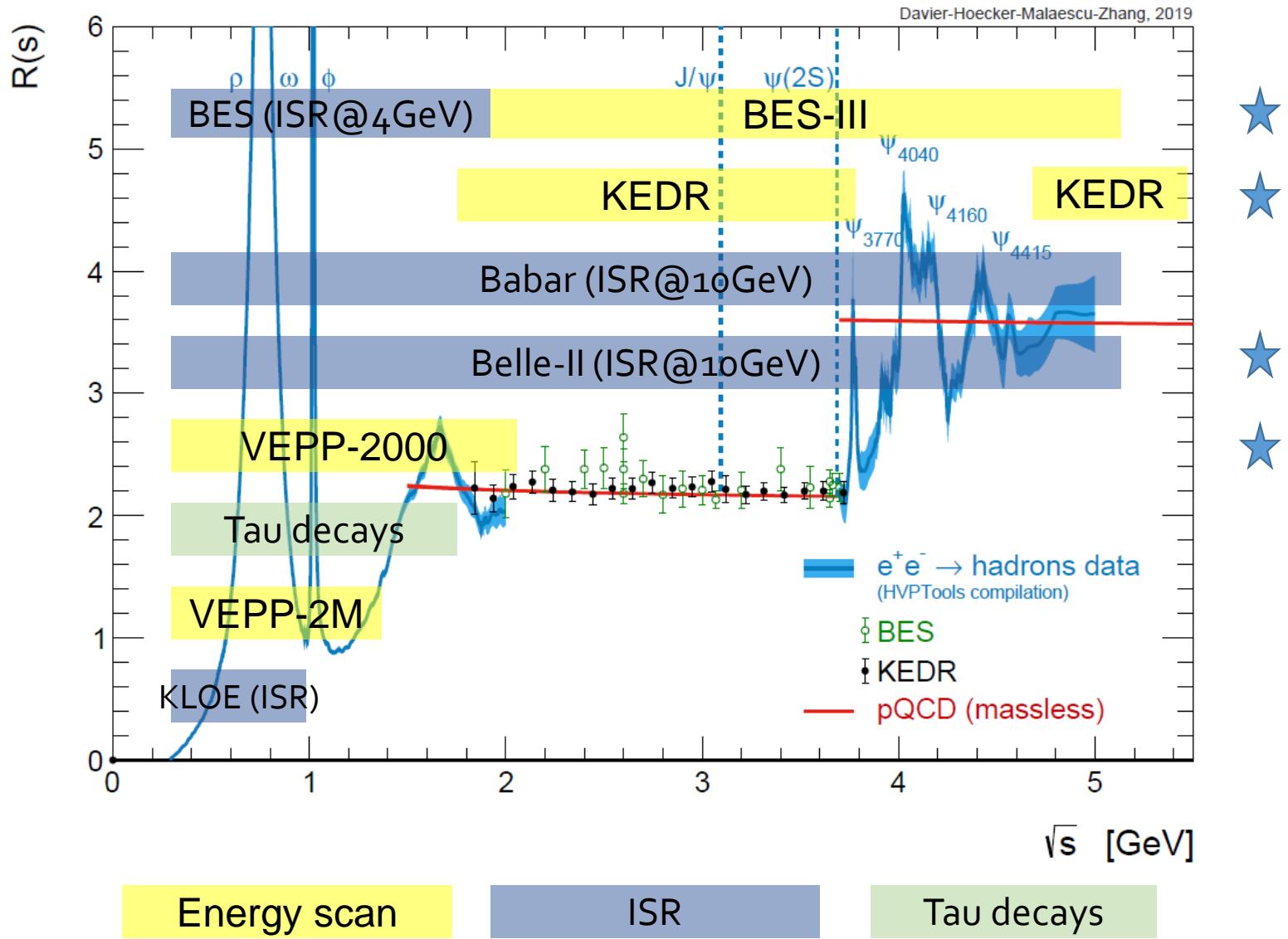
Most precise measurement

KEDR collected $R(s)$ data between 4.7 and 7.0 GeV (17 points)

Is there
agreement
between
inclusive and
exclusive?



Where the measurements are done

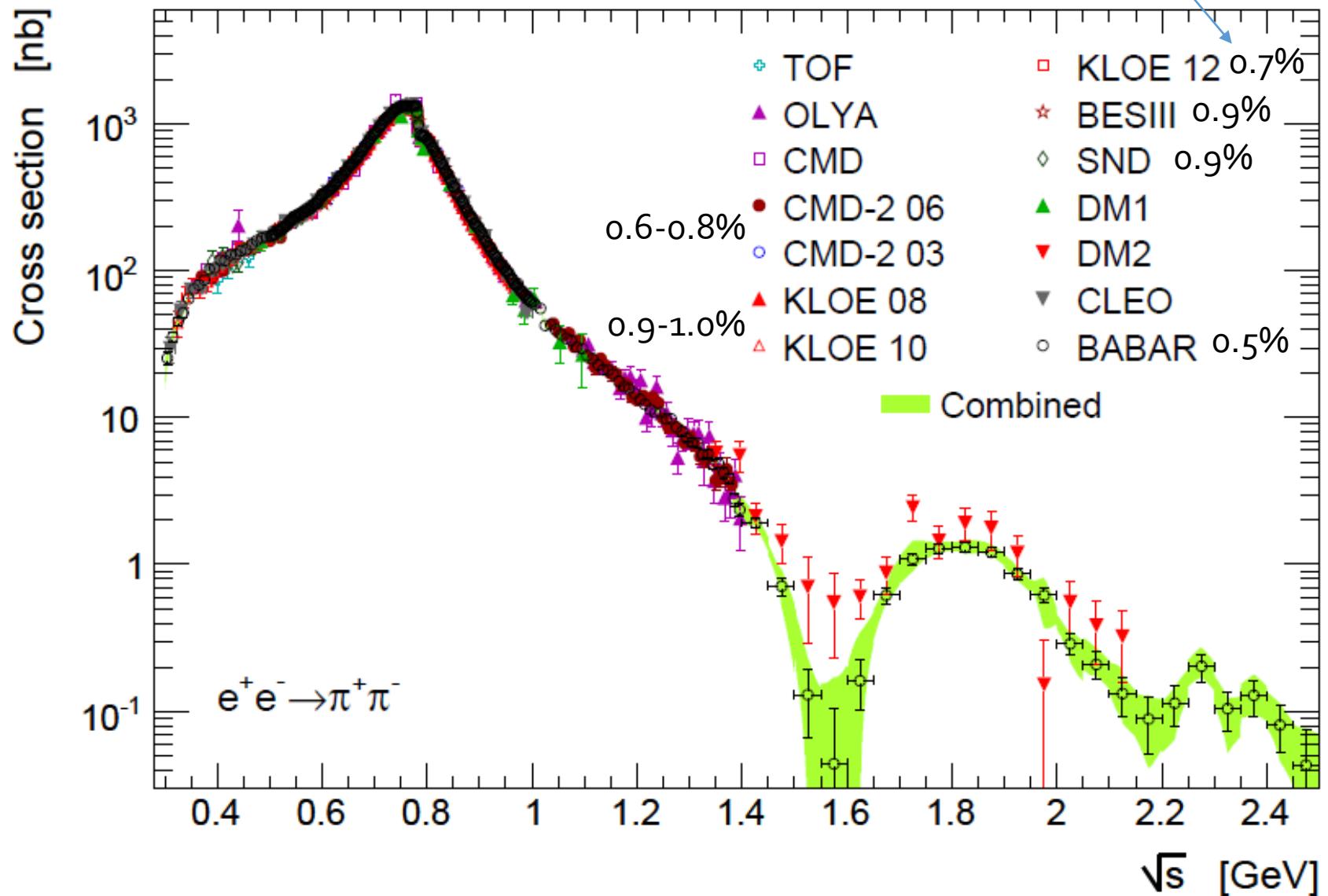


What to expect in near future

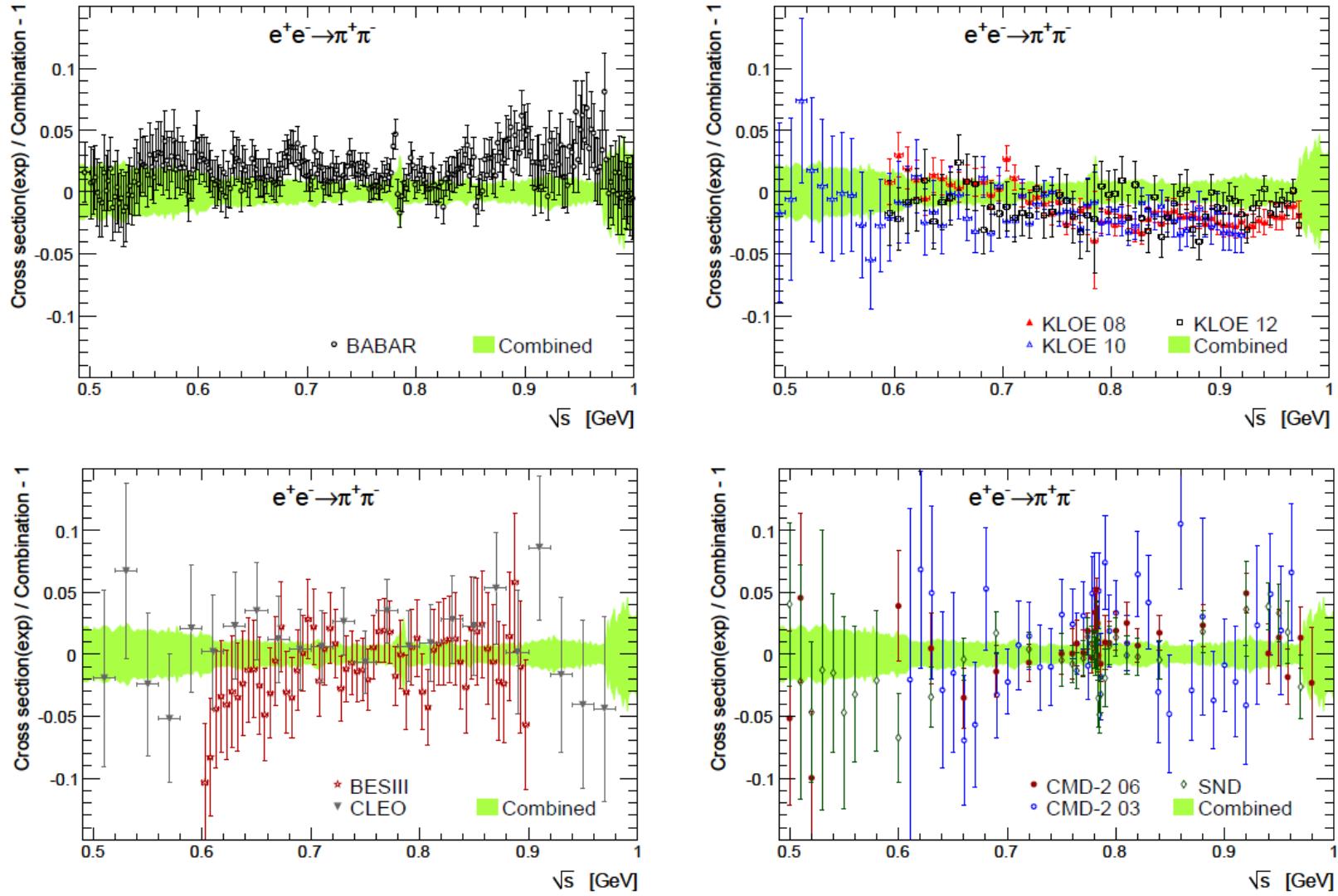
- VEPP-2000 has collected $>500 \text{ fb}^{-1}$ per detector. The ultimate goal is 1000 fb^{-1} per detector – many more data! Possibility to study intermediate dynamics.
- $e^+e^- \rightarrow \pi^+\pi^-$ cross section is about to be published by CMD-3 – record statistical precision
- SND published $e^+e^- \rightarrow \pi^+\pi^-$ cross section only using small portion of data - more results to be expected
- New analysis of BABAR $e^+e^- \rightarrow \pi^+\pi^-$ data based on angular distribution
- BELLE-II is taking data – expect new BABAR-like comprehensive ISR measurement
- BES-III plans to collect $\times 10$ of ISR data
- There is progress in development of new generators for radiative corrections calculations – very important for reaching higher accuracy (below 0.5%)
- With new high statistics measurements it will be possible to perform detailed comparison between ISR and energy scan

Status of $e^+e^- \rightarrow \pi^+\pi^-$

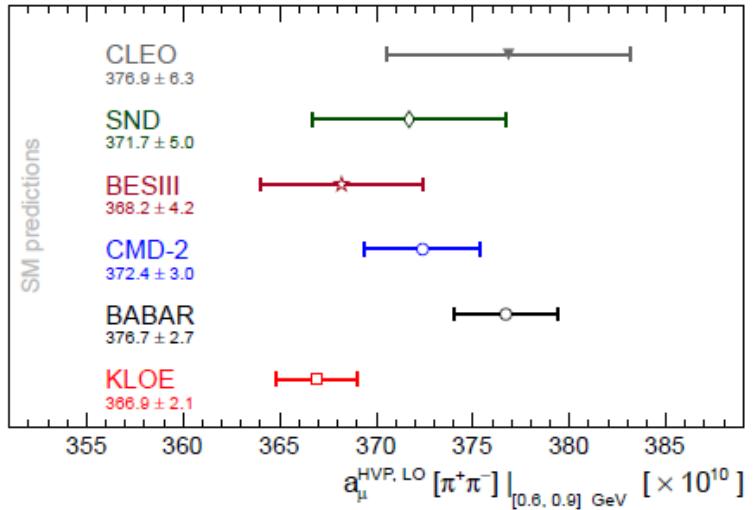
Systematic uncertainties



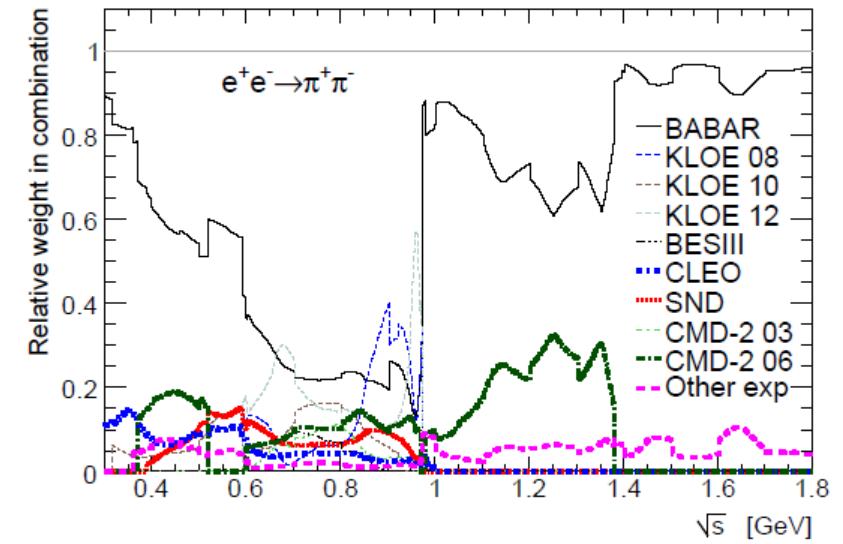
Status of $e^+e^- \rightarrow \pi^+\pi^-$



Status of $e^+e^- \rightarrow \pi^+\pi^-$

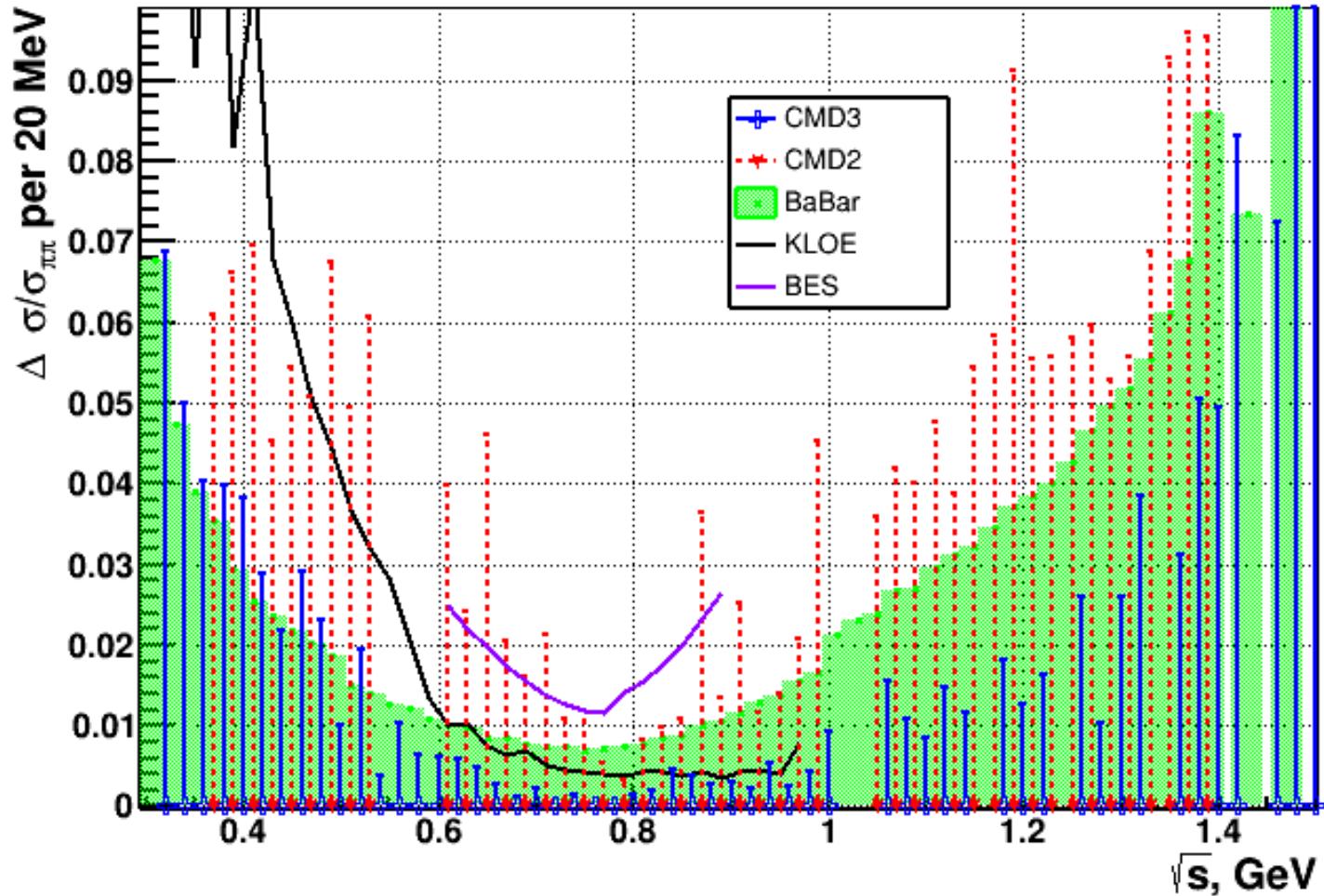


Infamous KLOE/BABAR tension
(more pronounced in the spectra)



a_μ calculation is BABAR
dominated outside of ρ energy
region (0.6-0.9 GeV)

Statistical precision of CMD-3 data



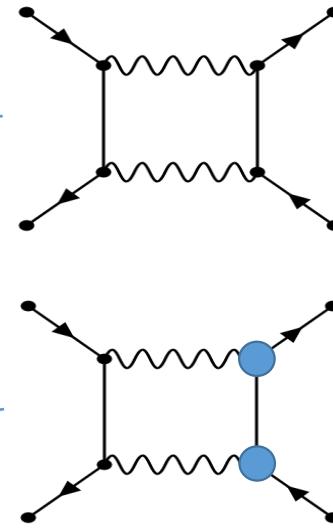
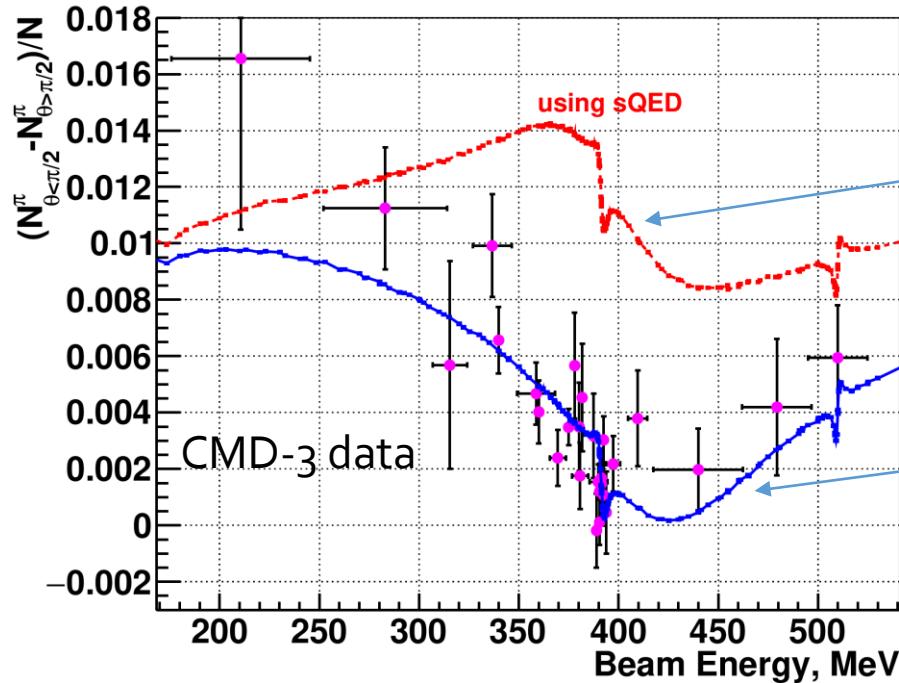
Relative statistical accuracy $\Delta\sigma/\sigma$ of various data sets in 20 MeV energy bins

That's all I can say about CMD-3 2π analysis at the moment 😊

Зарядовая асимметрия в $e^+e^- \rightarrow \pi^+\pi^-$

Проведено измерение зарядовой асимметрии в процессе $e^+e^- \rightarrow \pi^+\pi^-$ с детектором КМД-3

$$A = (N_{\Theta<\pi/2}^\pi - N_{\Theta>\pi/2}^\pi)/N$$



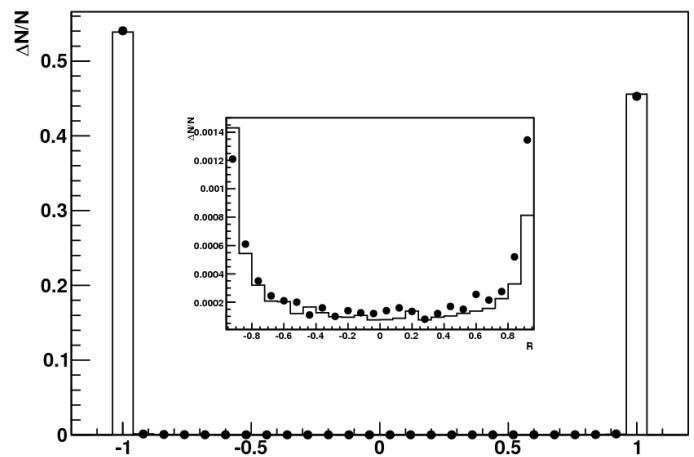
Экспериментальные данные не описываются обычно используемыми моделями, основанными на вычислениях в рамках sQED.
Результат важен при анализе и интерпретации измерений сечения рождения $\pi\pi$ (прямым методом и ISR).

$e^+e^- \rightarrow \pi^+\pi^-$
at SND (2021)

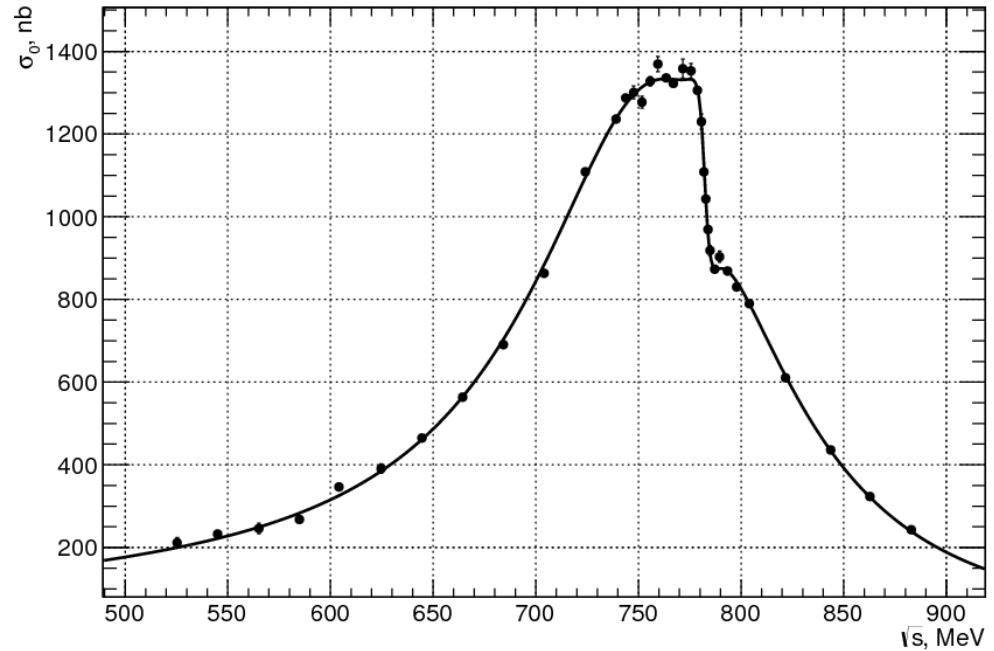
First measurement of
 $e^+e^- \rightarrow \pi^+\pi^-$
at VEPP-2000

The analysis is based on
4.7 pb⁻¹ data recorded in 2013
(1/10 full SND data set)

π/e separation using ML (BDT)



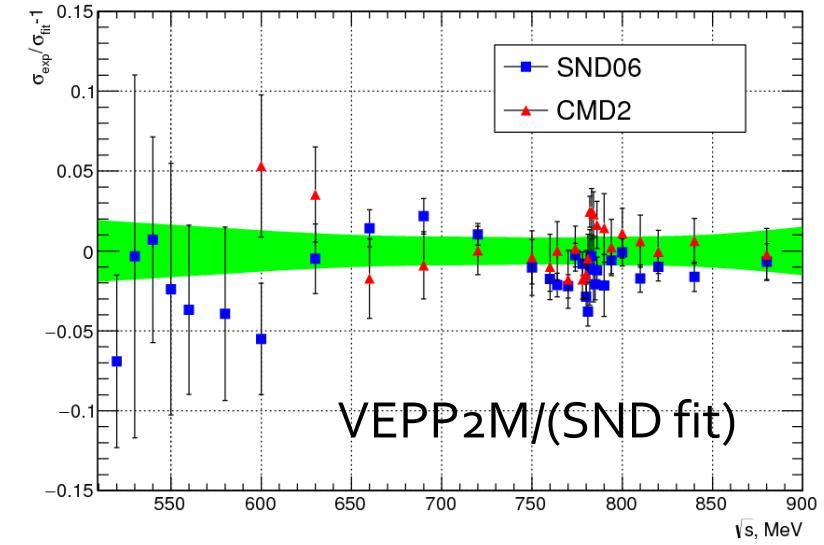
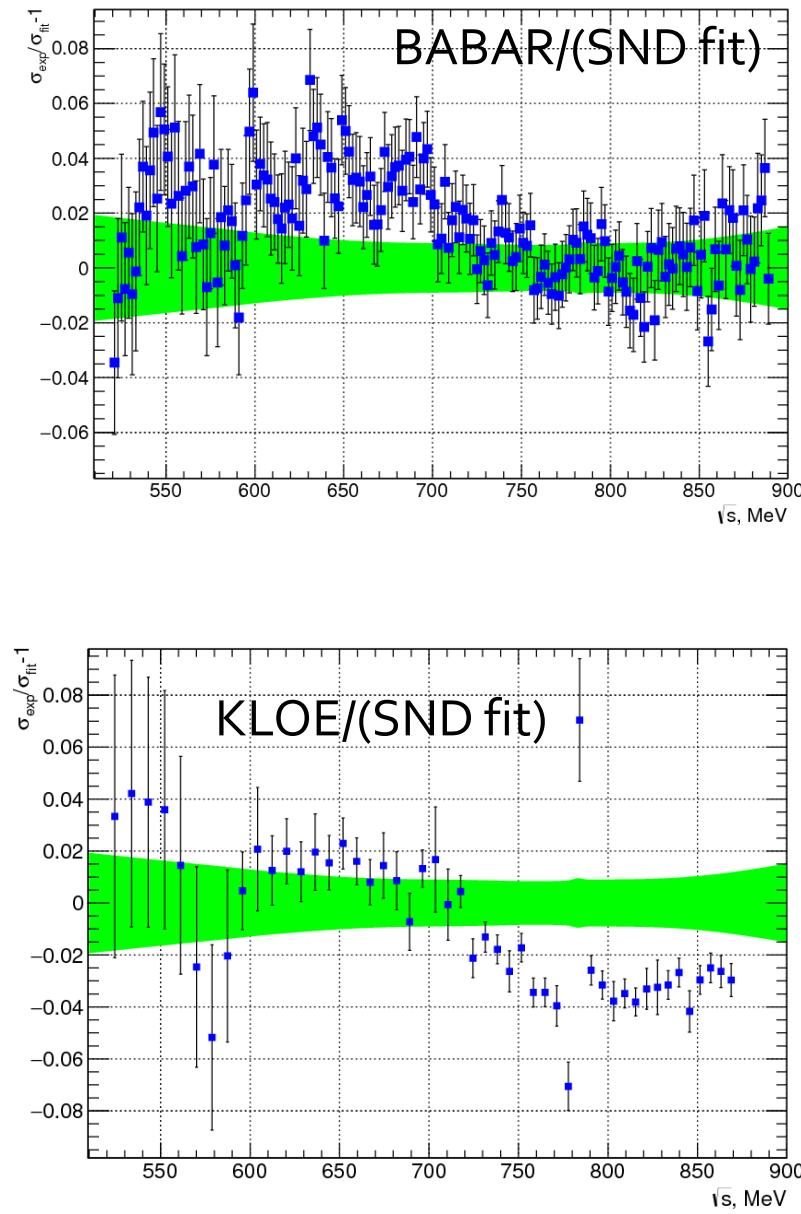
MISP 2022. Muon anomalous magnetic moment



Systematic uncertainty on the cross section (%)

Source	< 0.6 GeV	0.6 - 0.9 GeV
Trigger	0.5	0.5
Selection criteria	0.6	0.6
e/π separation	0.5	0.1
Nucl. interaction	0.2	0.2
Theory	0.2	0.2
Total	0.9	0.8

$e^+e^- \rightarrow \pi^+\pi^-$ at SND (2021): comparison to other measurements



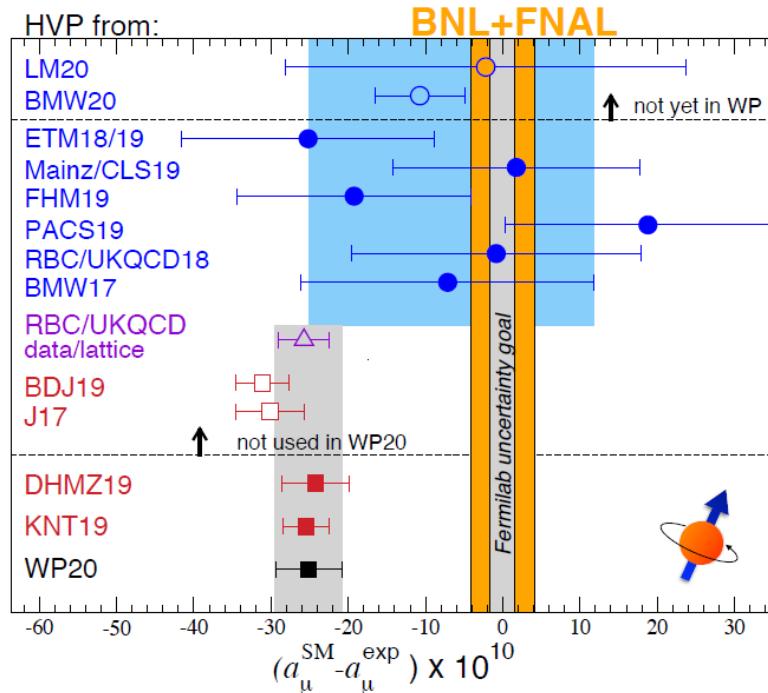
$0.53 < \sqrt{s} < 0.88 \text{ GeV}$

	$a_\mu(\pi^+\pi^-) \times 10^{10}$
SND & VEPP-2000	$409.8 \pm 1.4 \pm 3.9$
SND & VEPP-2M	$406.5 \pm 1.7 \pm 5.3$
BABAR	$413.6 \pm 2.0 \pm 2.3$
KLOE	$403.4 \pm 0.7 \pm 2.5$

Lattice calculations

Lattice calculations

Hadronic vacuum polarisation: Data-driven approach versus lattice QCD



Lattice calculations

Why computing the HVP contribution is a challenge

$$a_\mu^{\text{hvp, LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{K}(t) G(t), \quad G(t) = -\frac{a^3}{3} \sum_k \sum_{\vec{x}} \langle j_k^{\text{em}}(\vec{x}, t) j_k^{\text{em}}(0) \rangle$$

Sub-percent statistical precision;
exponentially growing signal-to-noise in
 $G(t)$ as $t \rightarrow \infty$

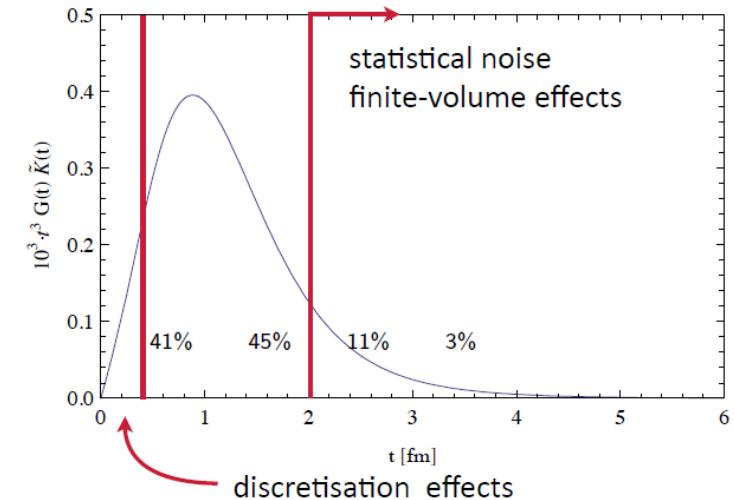
Correct for finite-volume effects

Control discretisation effects

Quark-disconnected diagrams:

control statistical & stochastic noise

Isospin breaking: $m_u \neq m_d$ and QED



From overview by H.Wittig at Schwinger Fest 2022

Lattice calculations

Window observables

Restrict integration over Euclidean time to sub-intervals
→ reduce/enhance sensitivity to systematic effects

Short distance: $W^{\text{SD}}(t; t_0) = 1 - \Theta(t, t_0, \Delta)$

Intermediate distance: $W^{\text{ID}}(t; t_0, t_1) = \Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)$

Long distance: $W^{\text{LD}}(t; t_1) = \Theta(t, t_1, \Delta)$

“Standard” choice:

$$t_0 = 0.4 \text{ fm}, t_1 = 1.0 \text{ fm}, \Delta = 0.15 \text{ fm}$$

Intermediate window:

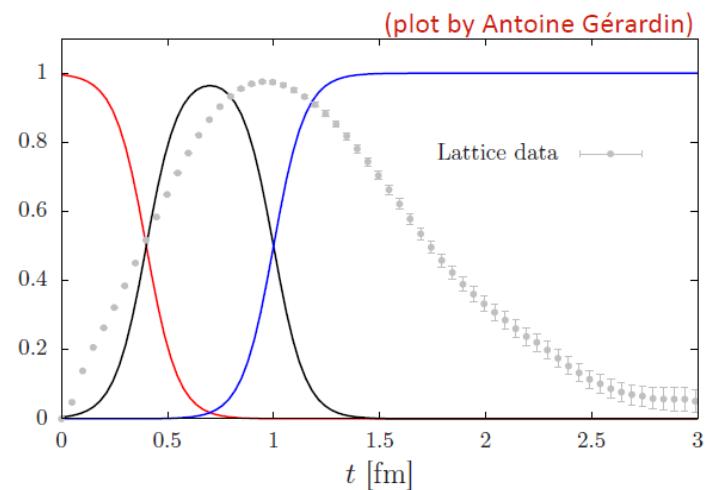
- Finite-volume correction reduced from 3% to 0.25%
- Uncertainty dominated by statistics
- ⇒ Precision test of different lattice calculations
- ⇒ Comparison with corresponding R -ratio estimate

From overview by H.Wittig at Schwinger Fest 2022

$$a_\mu^{\text{hyp, win}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{K}(t) G(t) W(t; t_0, t_1)$$

$$\Theta(t, t', \Delta) = \frac{1}{2} [1 + \tanh(t - t')/\Delta]$$

[RBC/UKQCD 2018]



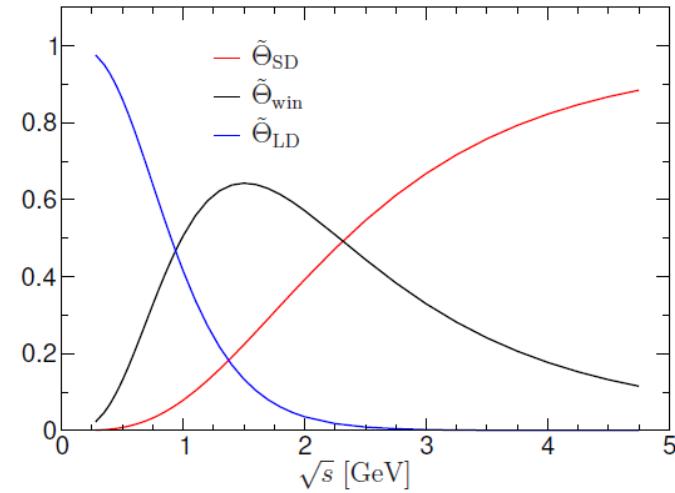
Lattice calculations

Window observables: Comparison with R -ratio

Starting point: $G(t) = \frac{1}{12\pi^2} \int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s}) R(s) s e^{-\sqrt{st}}$ [RBC/UKQCD 2018]

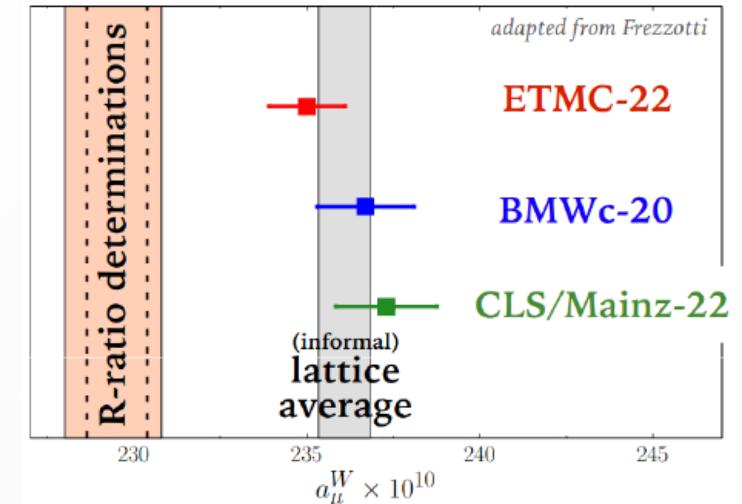
Insert $G(t)$ into expression for time-momentum representation:

$$a_{\mu}^{\text{hyp, ID}} = \left(\frac{\alpha}{\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s}) R(s) \frac{1}{12\pi^2} s \int_0^{\infty} dt \tilde{K}(t) W^{\text{ID}}(t; t_0, t_1) e^{\sqrt{st}}$$



[Colangelo et al., arXiv:2205.12963]

Latest lattice results in the intermediate window ($\sim 30\% a_{\mu}^{\text{HLO}}$):
Status and outlook on QCD predictions,
Gavin Salam ICHEP2022



MUONE

a_μ^{HLO} : space-like approach

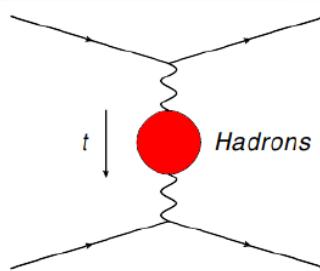


MUonE: a new independent evaluation of a_μ^{HLO}

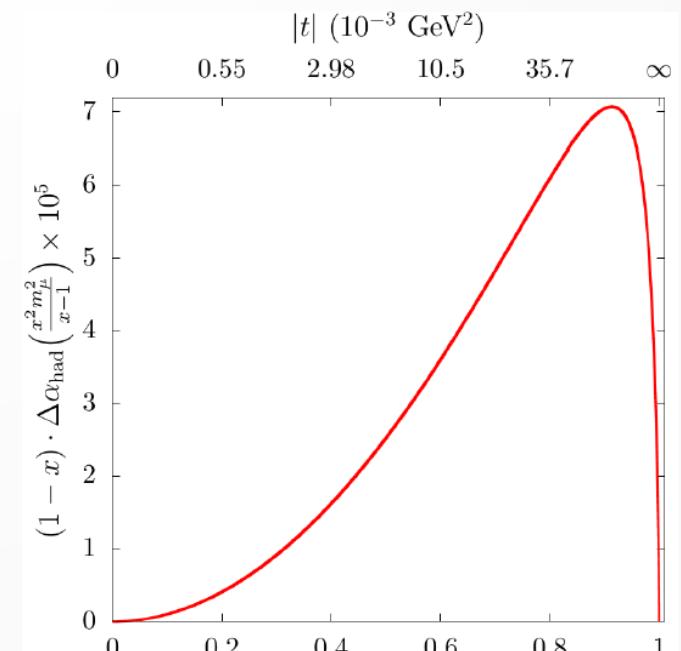
$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx (1-x) \Delta \alpha_{had}[t(x)]$$

Lautrup, Peterman, De Rafael, Phys. Rep. C3 (1972), 193

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$



Based on the measurement of $\Delta \alpha_{had}(t)$:
hadronic contribution to the running of the
electromagnetic coupling constant.



Carloni Calame, Passera, Trentadue, Venanzoni,
Phys. Lett. B 746 (2015), 325

4

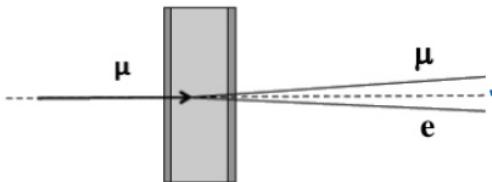
From talk by R.Pilato "Status of MUonE experiment"

MUONE

The MUonE experiment



Extraction of $\Delta\alpha_{\text{had}}(t)$ from the shape of the $\mu e \rightarrow \mu e$ differential cross section

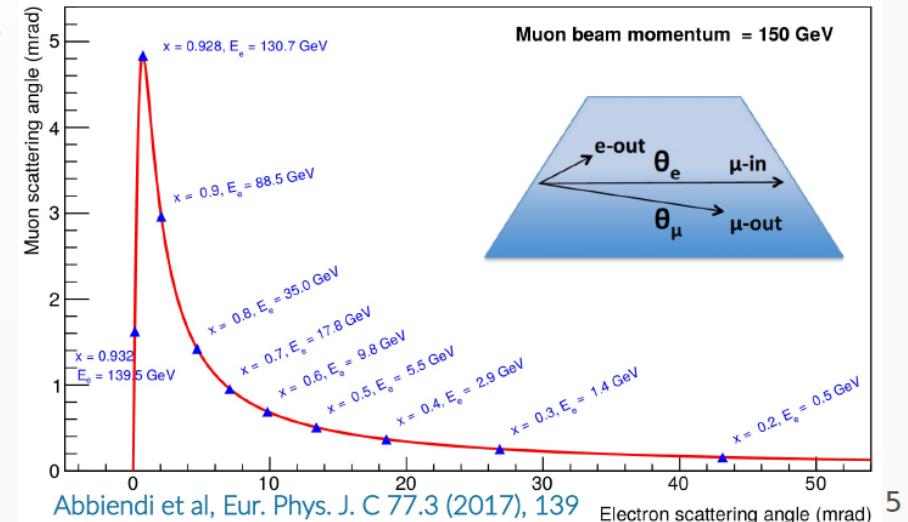


$$\frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}} = 0)} \sim 1 + 2\Delta\alpha_{\text{had}}(t)$$

To be measured

From theoretical calculation

- A beam of 160 GeV muons allows to get the whole a_μ^{HLO} (87% directly measured + 13% extrapolated).
- Correlation between muon and electron angles allows to select elastic events and reject background (e^+e^- pair production).
- Boosted kinematics:
 $\theta_\mu < 5 \text{ mrad}$, $\theta_e < 32 \text{ mrad}$.



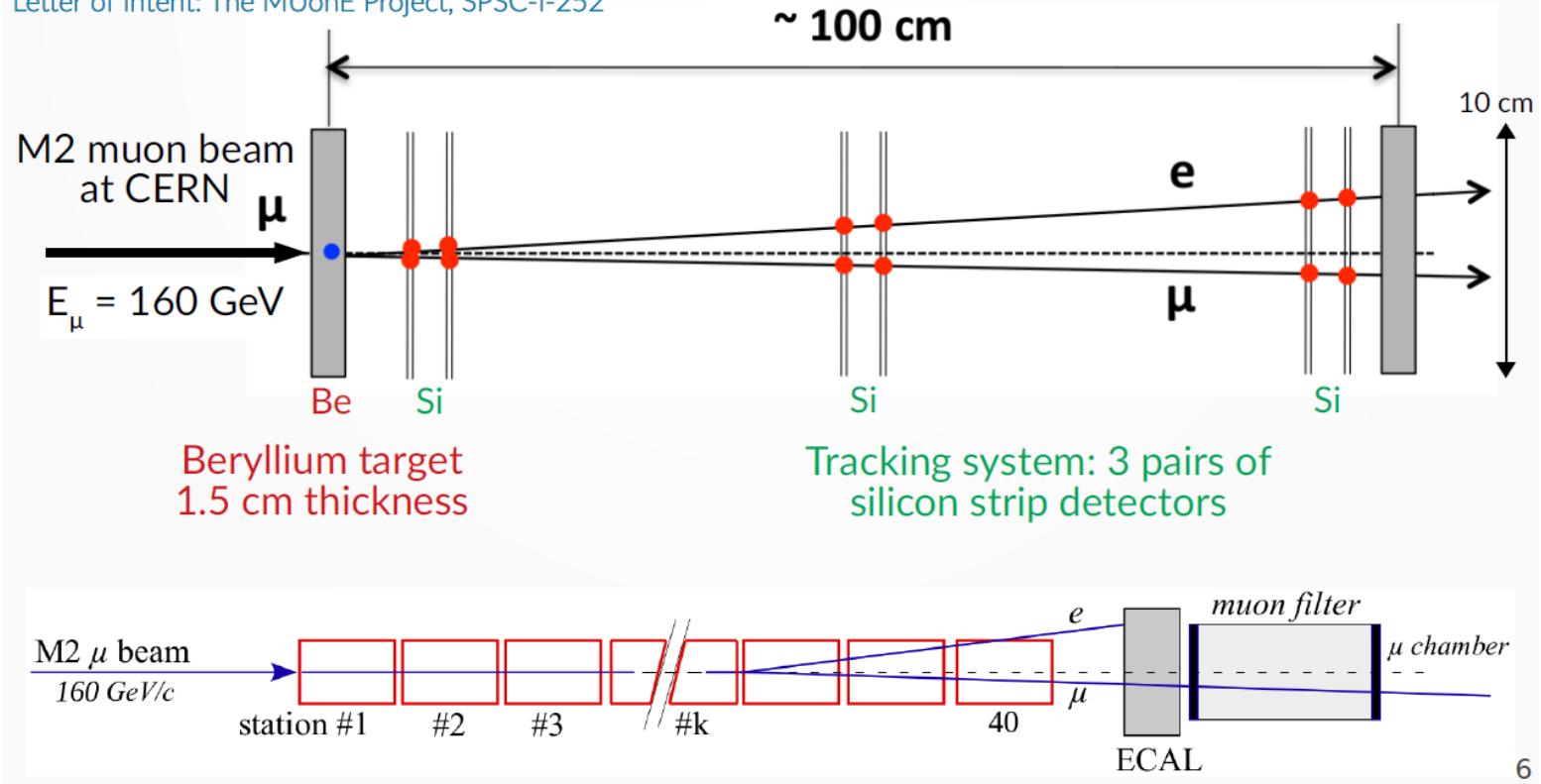
From talk by R.Pilato "Status of MUonE experiment"

MUONE

The experimental apparatus



Letter of Intent: The MUonE Project, SPSC-I-252



From talk by R.Pilato "Status of MUonE experiment"

Conclusions

MUonE
web site



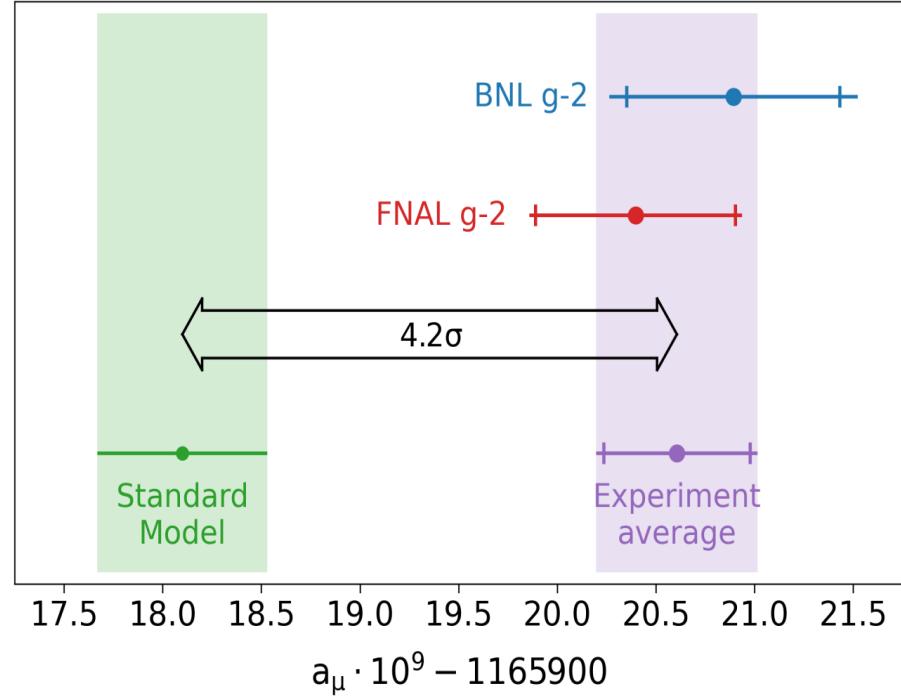
- The new method proposed by MUonE is independent and competitive with the latest evaluations.
- Intense Beam Test activities in 2021-2022: first experience with detector in real beam conditions.
- 3 weeks Test Run in 2023: proof of concept of the experimental proposal using 3 tracking stations + calorimeter.
- Towards the full experiment: 10 stations before LS3 (2026). Four months data taking: $\sim 2\%$ (stat) measurement of a_μ^{HLO} .
- Collaborators are welcome!



From talk by R.Pilato “Status of MUonE experiment”

Conclusion

- Фермилаб уже набрал статистику, достаточную для улучшения точности измерения в 4 раза
- Следующий результат из Фермилаб ожидается в 2022 году, с точностью, в 2 раза лучше, чем в 2021 году
- КМД-3 и СНД набрали большой объем статистики, идет анализ данных. Публикуются новые результаты от обоих экспериментов.
- В мире развиваются методы решеточных вычислений адронного вклада – независимый подход



Результаты из
Новосибирска
улучшают точность
теоретического
расчета

Lattice/ e^+e^- tension?

Результаты из
Фермилаб улучшают
точность
измерения

Extra slides

Магнитный момент

Suppose that there is a point particle f at rest in an external magnetic field \vec{B} . If the interaction Hamiltonian H_{mdm} between f and \vec{B} is given by

$$H_{\text{mdm}} = -\vec{\mu} \cdot \vec{B},$$

then $\vec{\mu}$ is called the **magnetic dipole moment** of f .

- If f has a non-zero spin \vec{s} , then $\vec{\mu} \propto \vec{s}$

- H_{mdm} is P-even and T-even

- Its cousins:

EDM \vec{d} : $H_{\text{EDM}} = -\vec{d} \cdot \vec{E}$ (P-odd, T-odd)
(EDM: electric dipole moment)

anapole \vec{a} : $H_{\text{ana}} = -\vec{a} \cdot (\nabla \times \vec{B})$ (P-odd, T-even)

Дипольные моменты частицы со спином 1/2

For a spin-1/2 particle f ,

$$\langle f(p') | J_\mu^{\text{em}} | f(p) \rangle = \bar{u}_f(p') \Gamma_\mu u_f(p) ,$$
$$\Gamma_\mu = F_1(q^2) \gamma_\mu + \frac{i}{2m_f} \color{red} F_2(q^2) \sigma_{\mu\nu} q^\nu - F_3(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 - F_4(q^2) (\gamma_\mu q^2 - 2m_f q_\mu) \gamma_5$$

There are no other independent form factors of a spin-1/2 particle other than $F_1(q^2), \dots, F_4(q^2)$ (See e.g., Nowakowski, Paschos, & Rodriguez, [physics/0402058](#))

$$F_1(0) = -eQ_f \quad (\text{electric charge}) \quad \text{по определению!}$$

$$\color{red} F_2(0) = -eQ_f a_f \quad (a_f : \text{anomalous magnetic moment})$$

$$F_3(0) = d_f \quad (\text{EDM})$$

$$F_4(0) = \tilde{a}_f \quad (\text{anapole moment})$$

If f is a Majorana particle, then $F_1(q^2) = F_2(q^2) = F_3(q^2) = 0$.

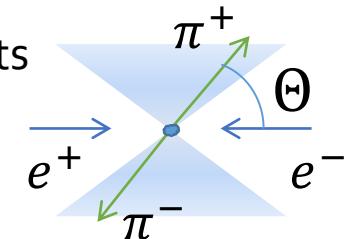
CMD-3 $e^+e^- \rightarrow \pi^+\pi^-$ analysis

Very simple kinematics, but the most challenging analysis due to high precision requirement: need to take into account many effects (which can affect result by 0.1% or more)

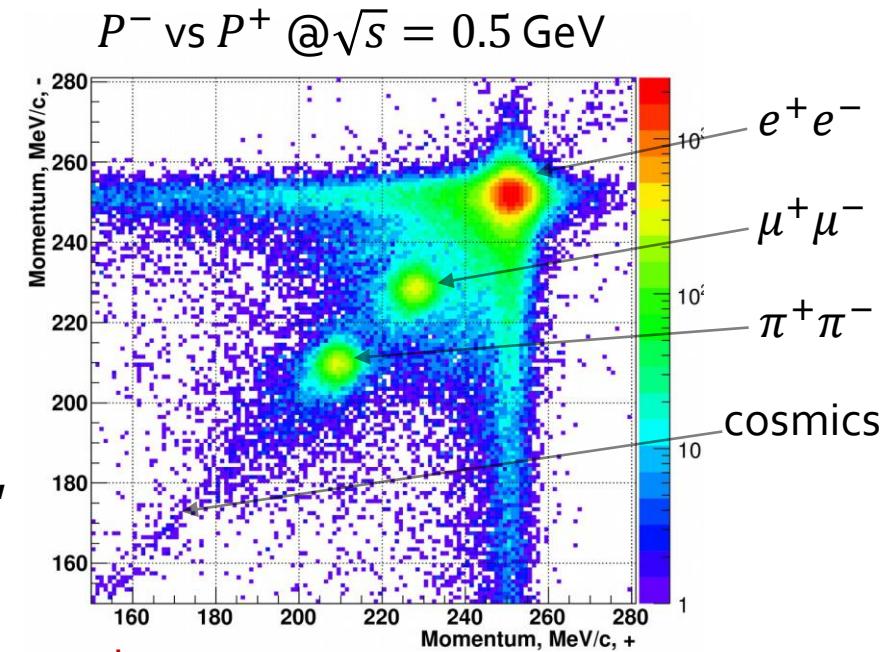
Measurement at CMD-3:

- several scans of the whole energy region below 2 GeV (took data in ρ region in 2013, 2018, 2020)
- employ correlations of the final particles: e^+e^- , $\mu^+\mu^-$, $\pi^+\pi^-$ separation **either**
 - by 2D **momentum** or
 - by 2D **energy deposition**independent measurements!
- many things to study: fiducial volume, pion decays, pions interactions in detector, backgrounds,...

High statistics is crucial! Goal: ~0.5% systematics



Main background:
 $e^+e^- \rightarrow e^+e^-, \mu^+\mu^-$



CMD-3 $e^+e^- \rightarrow \pi^+\pi^-$ analysis: radiative corrections

Measurement of $e^+e^- \rightarrow \pi^+\pi^-$
requires high precision calculation of
radiative corrections.

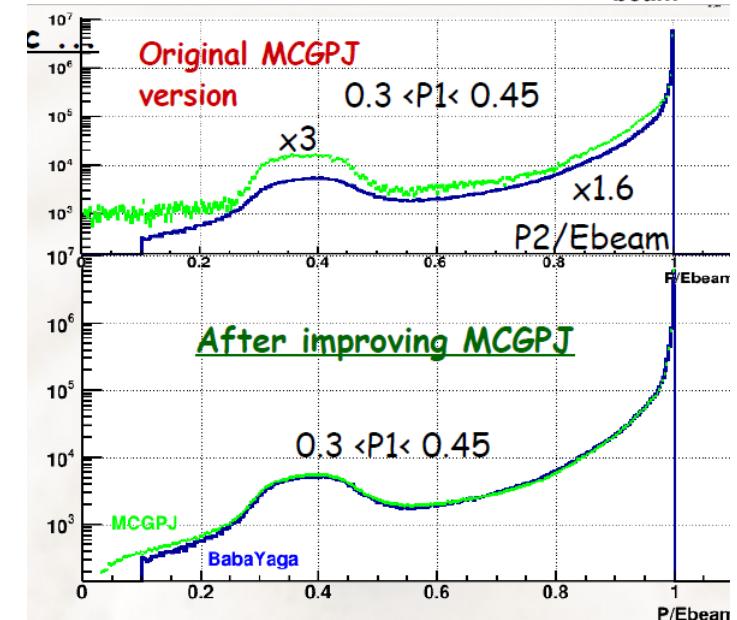
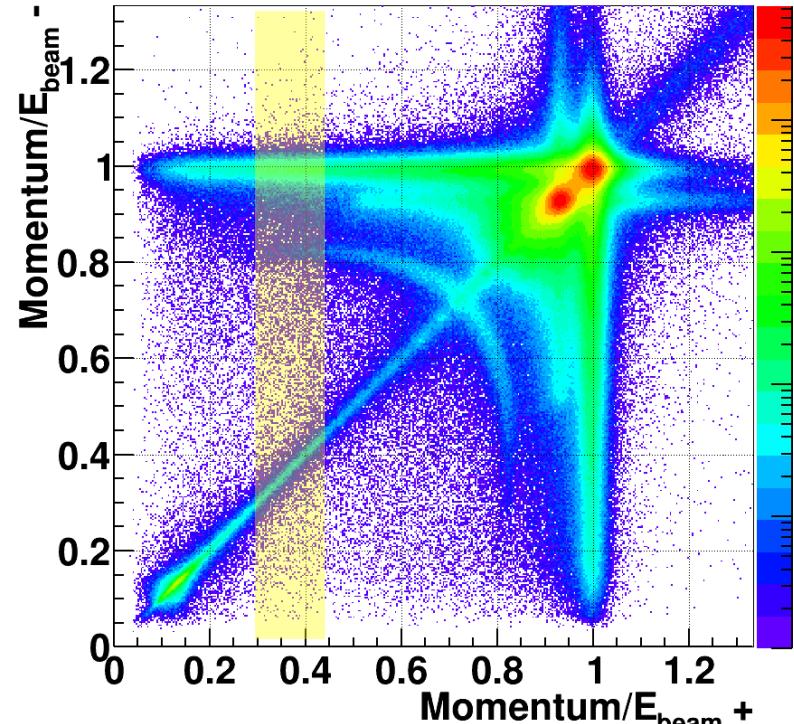
We use two high-precision MC
generators for $e^+e^- \rightarrow e^+e^-$:

- MCGPJ generator (0.2%)
- BaBaYaga@NLO (0.1%)

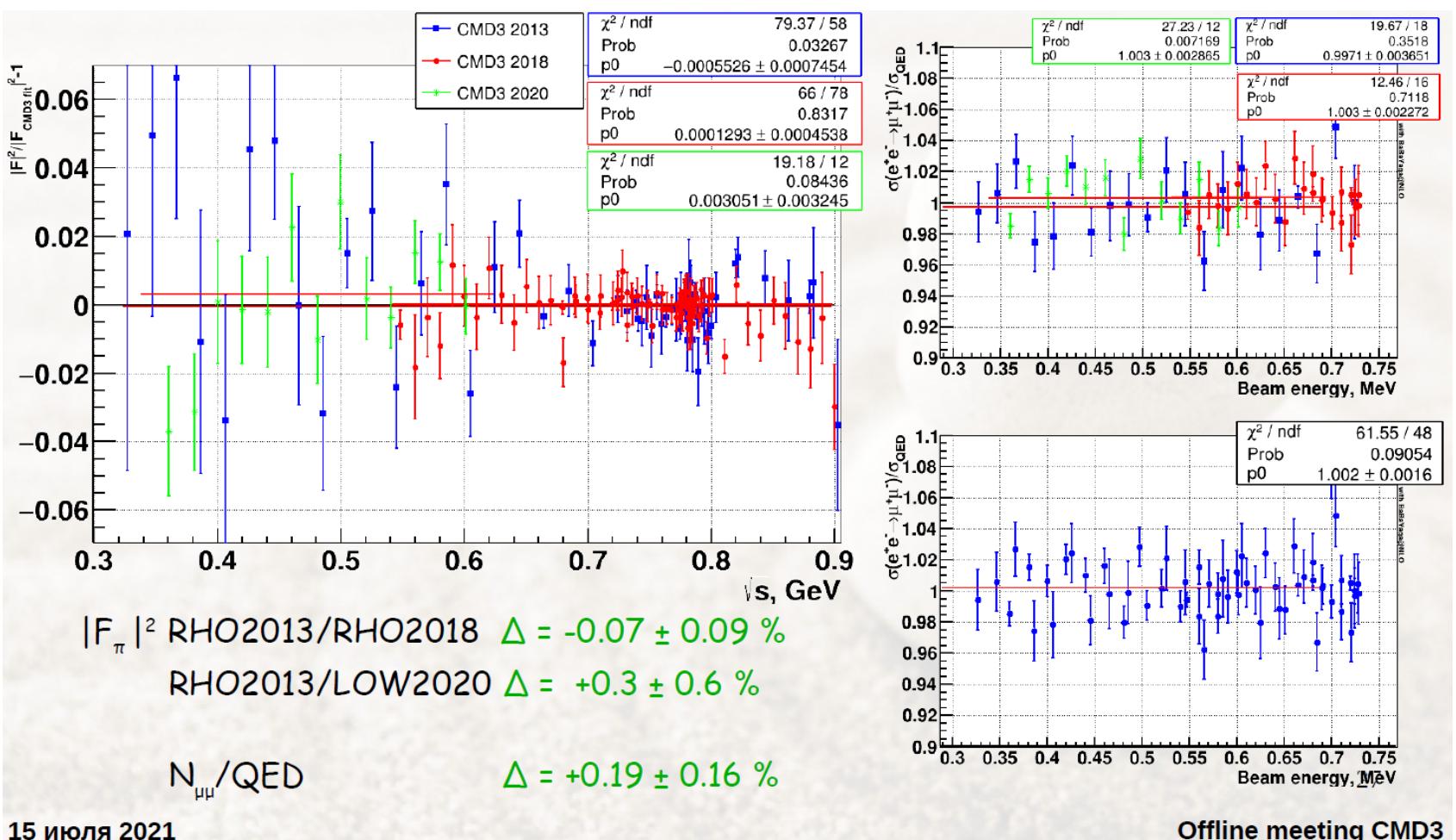
With high statistics we've observed
inconsistencies in tails of distributions,
which were traced to particulars of
MCGPJ generator

After improvements, tails of e^+e^-
spectra still differ by few %,
which limits the precision to O(0.1%)

NNLO MC generator for $e^+e^- \rightarrow e^+e^-$
is needed for higher precision



CMD-3 $e^+e^- \rightarrow \pi^+\pi^-$ analysis: internal checks



15 июля 2021

Comparison between different data sets

Comparison of measured
 $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ to QED