

# The Michel parameters $\xi'$ , $\xi''$ , $\eta''$ , $\alpha'$ , $\beta'$ measurement in the muonic tau-decays at SCTF

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# Michel parameters

- The most general expression for the decay matrix element:

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma = S, V, T \\ \varepsilon, \mu = R, L}} g_{\varepsilon\mu}^\gamma \langle \bar{\ell}_\varepsilon | \Gamma^\gamma | ((\nu_\ell)_\alpha) \rangle \langle (\bar{\nu}_\tau)_\beta | \Gamma_\gamma | \tau_\mu \rangle$$

$$\Gamma^S = 1, \Gamma^V = \gamma^\mu, \Gamma^T = \frac{1}{\sqrt{2}} \sigma^{\mu\nu} = \frac{i}{2\sqrt{2}} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$$

- Ten complex constants  $g_{\varepsilon\mu}^\gamma$  describe the Lorentz structure of the charged currents interaction in the theory of weak interaction
- The only nonzero term in the SM theory of weak interaction:  $g_{LL}^V = 1$
- It is convenient to express experimental observables in terms of Michel parameters (MP) which are bilinear combinations of  $g_{\varepsilon\mu}^\gamma$

# Status of the Michel parameters

MP (SM)	$\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$	$\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$	Tau spin	Daughter spin
$\rho(0.75)$	$0.747 \pm 0.010$	$0.763 \pm 0.020$	-	-
$\xi(1)$	$0.994 \pm 0.040$	$1.030 \pm 0.059$	+	-
$\eta(0)$	$0.013 \pm 0.020$	$0.094 \pm 0.073$	-	-
$\xi \cdot \delta(0.75)$	$0.734 \pm 0.028$	$0.778 \pm 0.037$	+	-
$\xi'(1)$	$2.6 \pm 4.8$	$-2.2 \pm 2.4$	-	+
$\xi''(1)$		$6.2 \pm 6.8$	+	+
$\eta''(0)$			+	+
$\alpha'/A(0)$			+	+
$\beta'/A(0)$			+	+

- Standard Model expectation for the Michel Parameters in brackets
- Green parameters are targets of this talk
- Not all listed parameters are independent!

# Motivation

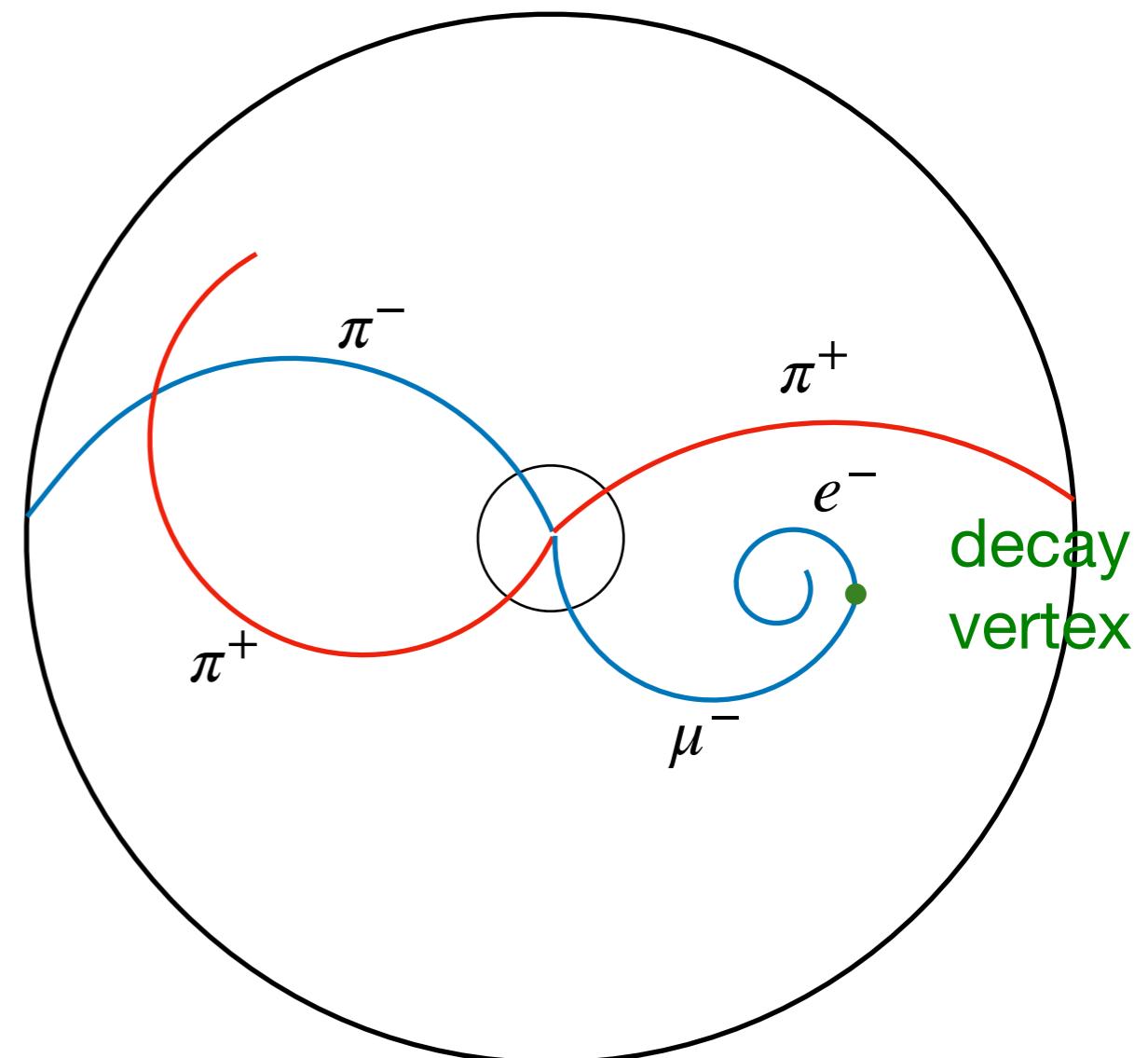
- Michel parameters provide a **model-independent** test of new physics (NP)
- In  $\tau$  lepton decays, due to its much larger mass compared to the muon, the relative contribution of NP processes with an «incorrect» Lorentz structure can be enhanced
- The Michel parameters in  $\tau$  lepton decays are obtained with high precision only in measurements summed over the daughter lepton spin
- The first direct measurement of the Michel parameters, which describe the daughter muon polarization, in  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  decays

95% confidence level experimental limits on coupling constants from PDG

$\tau \rightarrow e \nu_e \nu_\tau$			$\tau \rightarrow \mu \nu_\mu \nu_\tau$		
$ g_{RR}^S  < 0.70$	$ g_{RR}^V  < 0.17$	$ g_{RR}^T  \equiv 0$	$ g_{RR}^S  < 0.72$	$ g_{RR}^V  < 0.18$	$ g_{RR}^T  \equiv 0$
$ g_{LR}^S  < 0.99$	$ g_{LR}^V  < 0.13$	$ g_{LR}^T  < 0.082$	$ g_{LR}^S  < 0.95$	$ g_{LR}^V  < 0.12$	$ g_{LR}^T  < 0.079$
$ g_{RL}^S  < 2.01$	$ g_{RL}^V  < 0.52$	$ g_{RL}^T  < 0.51$	$ g_{RL}^S  < 2.01$	$ g_{RL}^V  < 0.52$	$ g_{RL}^T  < 0.51$
$ g_{LL}^S  < 2.01$	$ g_{LL}^V  < 1.005$	$ g_{LL}^T  \equiv 0$	$ g_{LL}^S  < 2.01$	$ g_{LL}^V  < 1.005$	$ g_{LL}^T  \equiv 0$

# Method of the muon polarization measurement

- The information about muon spin can be restored using its decays due to  $P$ -invariance violation
- The method is based on muon decays-in-flight reconstruction in a tracker as kinks
- Commonly muon flies dozens of meters before decay; thus, events will be rare

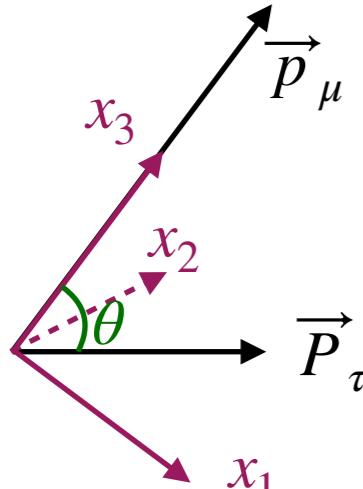


Example of the event

D.A. Bodrov, *Phys. Atom. Nuclei* **84**, 212–215 (2021)

# Differential decay width

- The general expression of differential decay width of  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  in terms of MP ( $F_i(x)$ ) are functions of reduced muon energy parametrized by MP):



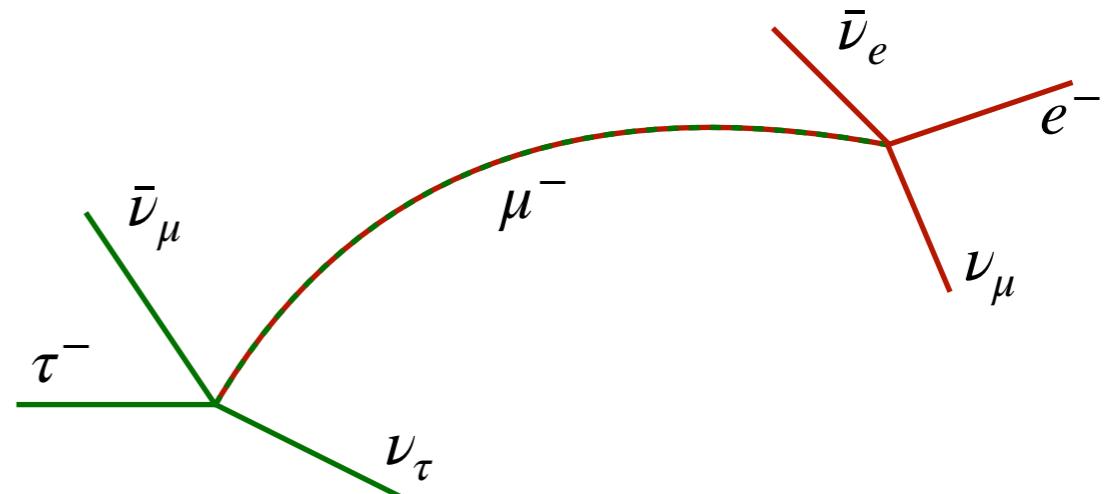
$$\frac{d^2\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)}{dx d\cos\theta} = \frac{m_\tau}{4\pi^3} W_{\mu\tau}^4 G_F^2 \sqrt{x^2 - x_0^2} (F_{IS}(x) \pm P_\tau \cos\theta F_{AS}(x) + F_{T_1}(x)P_\tau \sin\theta \zeta_1 + F_{T_2}(x)P_\tau \sin\theta \zeta_2 + (\pm F_{IP}(x) + F_{AP}(x)P_\tau \cos\theta)\zeta_3)$$

$$W_{\mu\tau} = \frac{m_\mu^2 + m_\tau^2}{2m_\tau}, \quad x = \frac{E_\mu}{W_{\mu\tau}}, \quad x_0 = \frac{m_\mu}{W_{\mu\tau}}$$

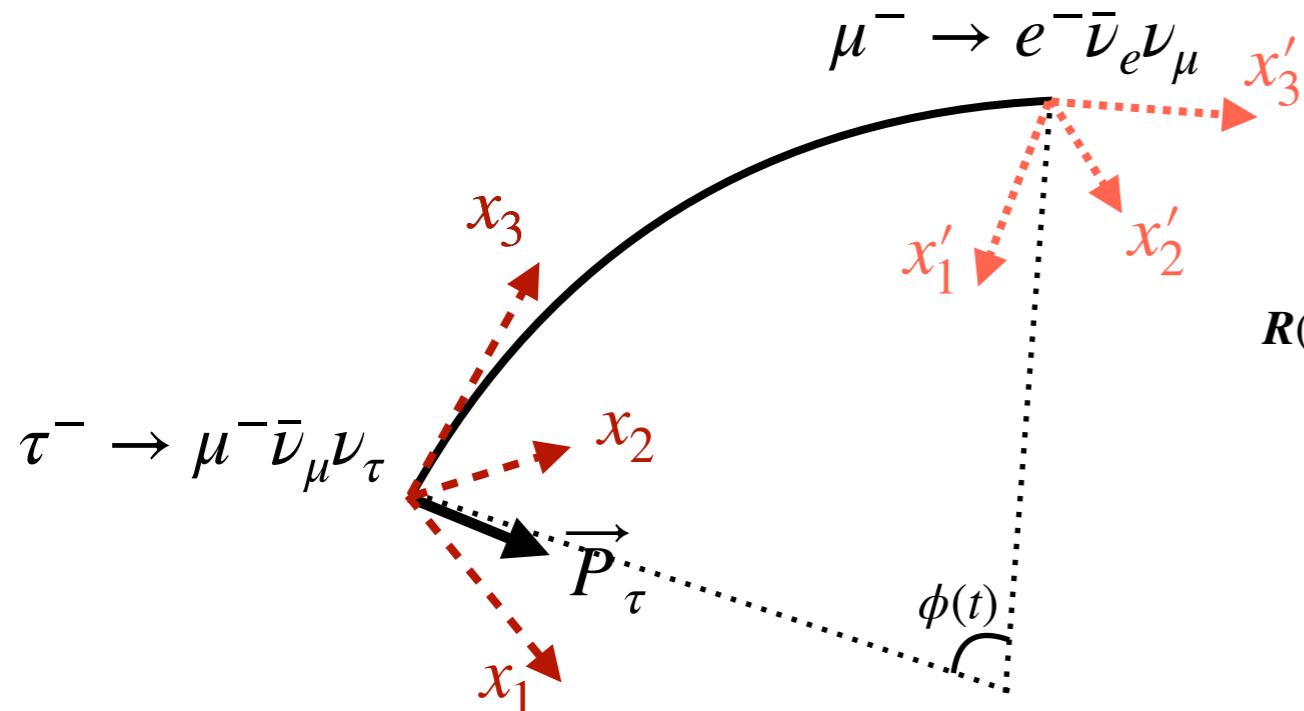
- For  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ , we use SM decay width:

$$\frac{d^2\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)}{dy d\Omega} = \frac{G_F^2 m_\mu^5}{384\pi^4} y^2 [(3 - 2y) \mp (1 - 2y)(\vec{n}_e \cdot \vec{\zeta})] \quad y = \frac{2E_e}{m_\mu}$$

- The muon decay width should be convoluted with  $\tau$  lepton decay width over muon spin  $\zeta$ , taking into account the muon rotation in the detector magnetic field



# SCTF with polarized beams



$$R(\phi) = \begin{pmatrix} c + (1 - c)h_1^2 & -h_3s + (1 - c)h_1h_2 & h_2s + (1 - c)h_1h_3 \\ h_3s + (1 - c)h_1h_2 & c + (1 - c)h_2^2 & -h_1s + (1 - c)h_2h_3 \\ -h_2s + (1 - c)h_1h_3 & h_1s + (1 - c)h_2h_3 & c + (1 - c)h_3^2 \end{pmatrix}$$

$$\vec{n}'_e = (\sin \theta_e \cos \psi_e, \sin \theta_e \sin \psi_e, \cos \theta_e)$$

$$\frac{d^5\Gamma}{dx d\cos\theta dy d\cos\theta_e d\psi_e} = \mathcal{B}(\mu \rightarrow e\nu\nu) \frac{\Gamma_{\tau \rightarrow \mu\nu\nu}}{1 - 3x_0^2} \frac{3}{\pi} y^2 \sqrt{x^2 - x_0^2} [(3 - 2y)G_0 \\ \pm (2y - 1)(G_1 \sin \theta_e \cos \psi_e + G_2 \sin \theta_e \sin \psi_e + G_3 \cos \theta_e)]$$

$$G_0 = F_{IS}(x) \pm F_{AS}(x) P_\tau \cos \theta$$

$$\vec{G} = \left( F_{T_1}(x) P_\tau \sin \theta, F_{T_2}(x) P_\tau \sin \theta, \pm F_{IP}(x) + F_{AP}(x) P_\tau \cos \theta \right)$$

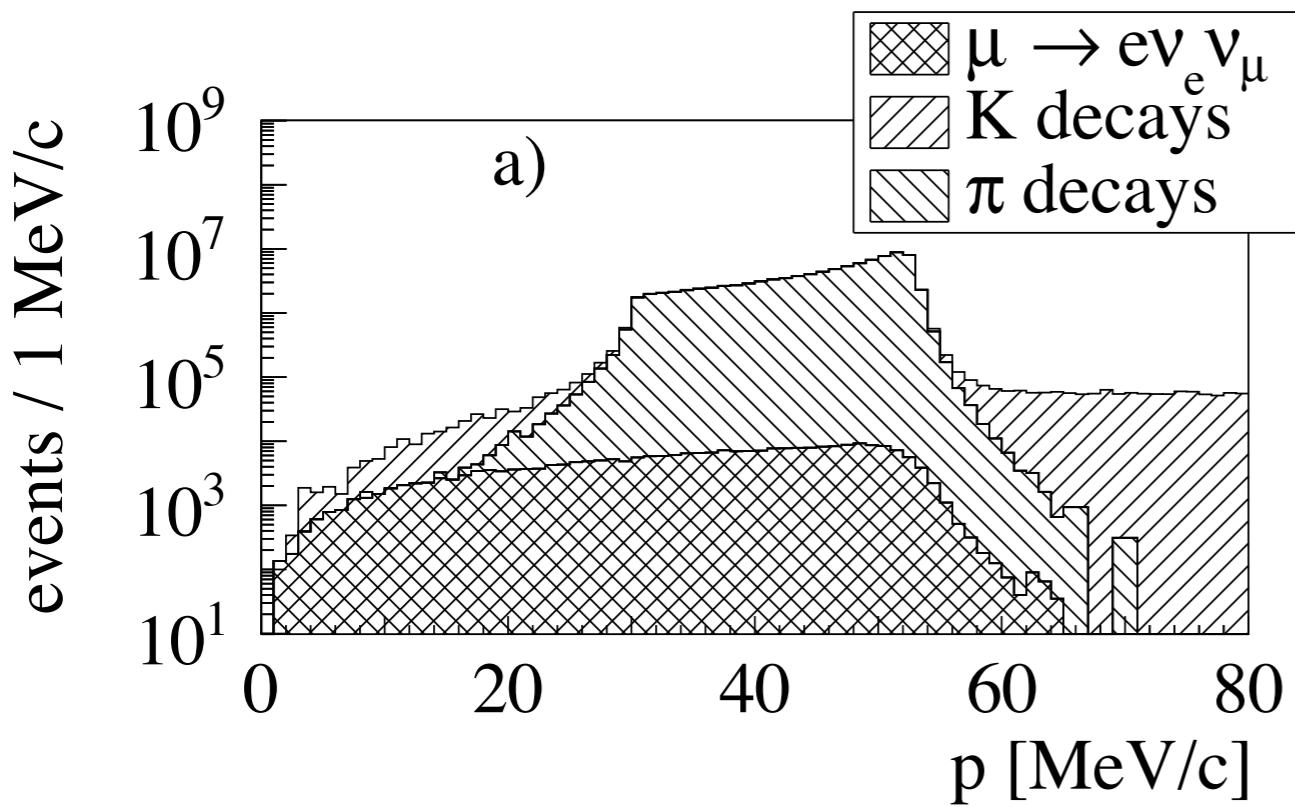
# Precision estimation at SCTF

- Highly polarized beam with  $\xi_{\text{beam}} = 0.8$  ( $P_\tau \approx \xi_{\text{beam}} = 0.8$ )
- We assume the  $\tau^+\tau^-$ -production only at the threshold (the expected integrated luminosity is  $10 \text{ ab}^{-1}$  or  $2.1 \cdot 10^{10} \tau^+\tau^-$ -pairs)
- The  $\tau^+\tau^-$  selection efficiency  $\eta_{\text{tag}} \approx 30\%$  is based on BESIII results
- Simple reconstruction algorithm to find a kink: muons decay inside drift chamber on the first turn of the track and in  $\geq 10 \text{ cm}$  from outer walls. These requirements ensure the reconstruction of the kink by the track reconstruction algorithm with a typical efficiency  $\eta_{\text{kink}} \approx 90\%$
- The probability of muon to decay with these requirements is  $\omega_{\text{dec}} \approx 3.2 \cdot 10^{-4}$
- Perfect resolution and no background (in reality particle scattering and kaon and pion decays imitate signal)

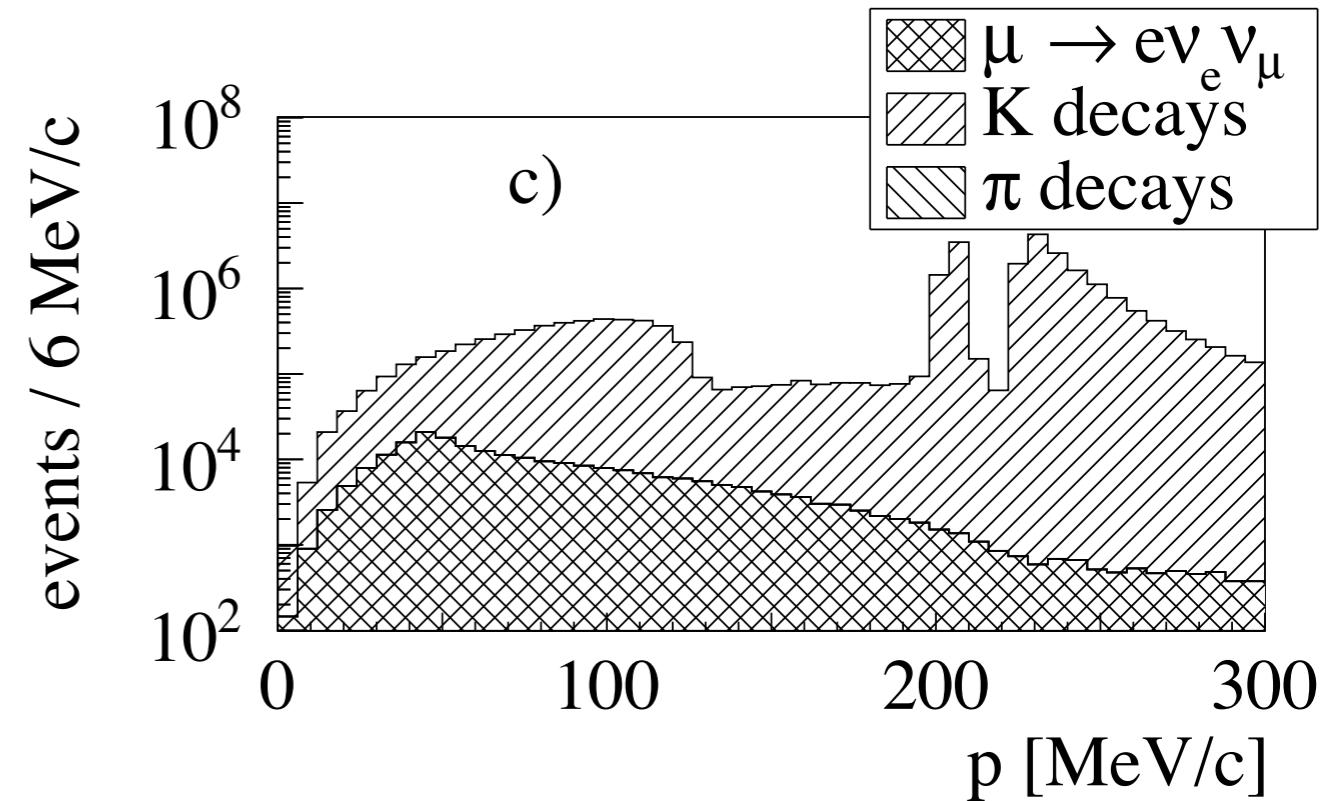
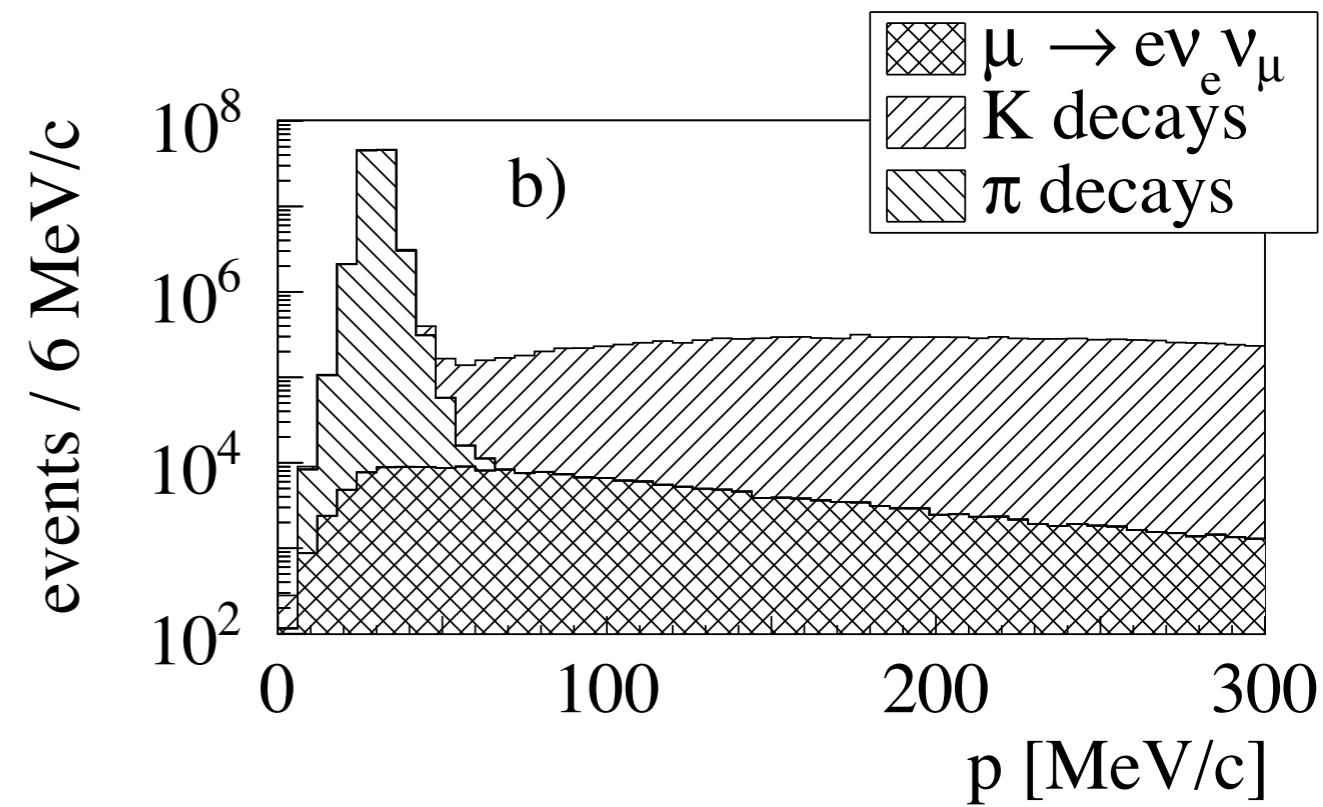
# Background suppression

- The expected background contamination from non- $\tau^+\tau^-$  processes (mainly  $q\bar{q}$ ) was estimated by BESIII to be  $\sim 6\%$ , and we ignore it
- Events that can imitate signal  $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu$ :  $\pi^- \rightarrow \mu^-\bar{\nu}_\mu$ ,  $K^- \rightarrow \mu^-\bar{\nu}_\mu$ ,  $K^- \rightarrow \pi^0\mu^-\bar{\nu}_\mu$ ,  $K^- \rightarrow \pi^0e^-\bar{\nu}_e$ ,  $K^- \rightarrow \pi^-\pi^0$ ,  $K^- \rightarrow \pi^-\pi^0\pi^0$ ,  $K^- \rightarrow \pi^-\pi^+\pi^-$ , electron scattering, muon scattering, hadron scattering
- Initially, background exceeds the signal (muon decays are very rare)
- Kinematics can help: pions and kaons decay mainly to two monochromatic particles producing a narrow line in the mother particle rest frame
- Scattering is elastic and conserves the momentum magnitude of the particle
- The background can be suppressed to a negligible level with  $\eta_{\text{sel}} \approx 80\%$  for signal

# Background suppression (2)



- Daughter particle momentum in the rest frame of decayed one with different mass hypotheses
- MC simulation. a) electron mass hypothesis for the daughter particle and muon mass hypothesis for the mother particle ( $\mu \rightarrow e$ ); b)  $\pi \rightarrow \mu$ ; c)  $K \rightarrow \pi$



# MP uncertainties estimation

- The expected number of signal events is  $N \approx 5 \cdot 10^5$

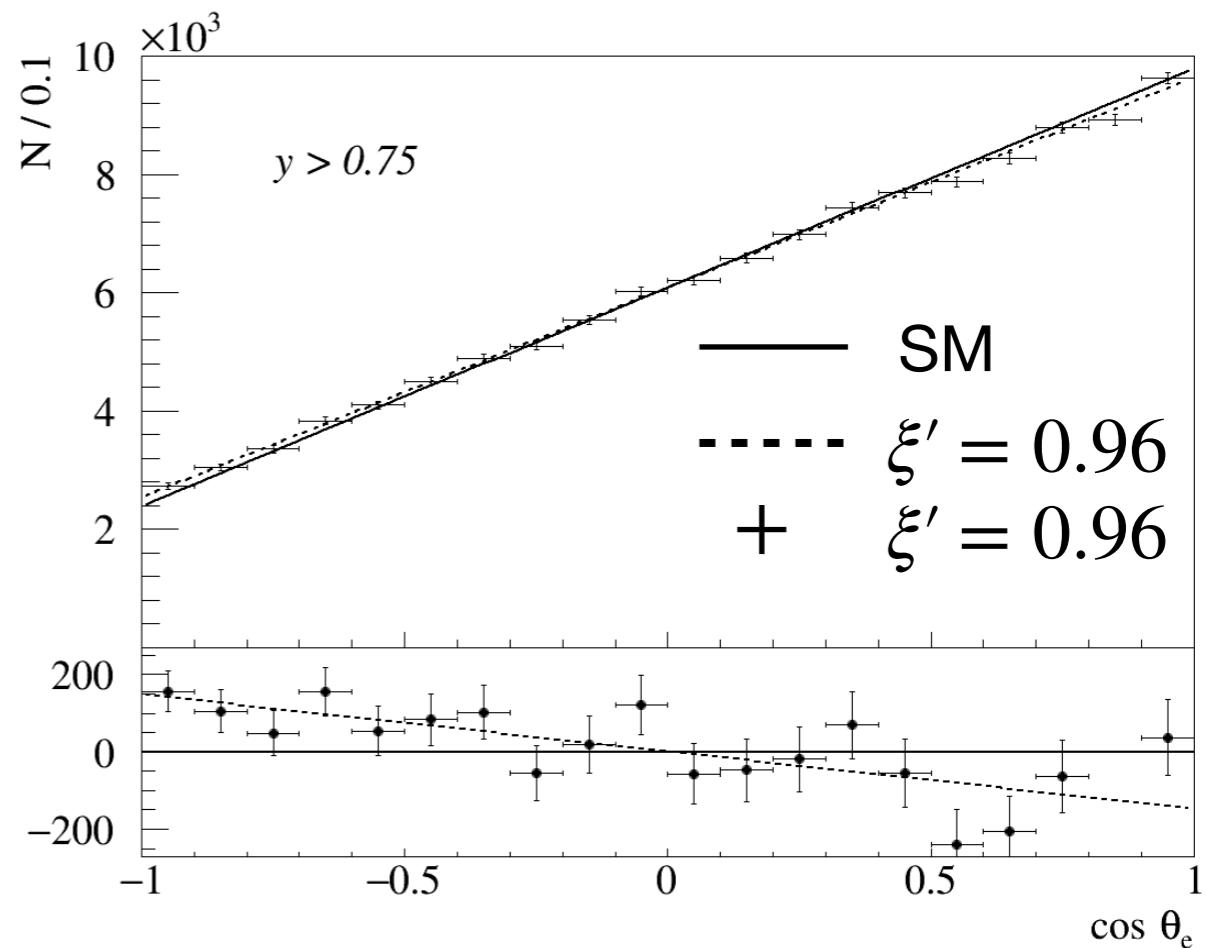
MP (SM)	$\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$	Ideal expectation	Tau spin	Daughter spin
$\rho(0.75)$	$0.763 \pm 0.020$		-	-
$\xi(1)$	$1.030 \pm 0.059$		+	-
$\eta(0)$	$0.094 \pm 0.073$		-	-
$\xi \cdot \delta(0.75)$	$0.778 \pm 0.037$		+	-
$\xi'(1)$	$-2.2 \pm 2.4$	? $\pm 0.006$	-	+
$\xi''(1)$	$6.2 \pm 6.8$	? $\pm 0.03$	+	+
$\eta''(0)$		? $\pm 0.02$	+	+
$\alpha'/A(0)$		? $\pm 0.014$	+	+
$\beta'/A(0)$		? $\pm 0.007$	+	+

- In the ideal case uncertainties are comparable with one in measurements in muon decays!

# Potential application

PDG	SCTF	PDG	SCTF	PDG	SCTF
$ g_{RR}^S $	$< 0.72$	$< 0.18$	$ g_{RR}^V $	$< 0.18$	$< 0.09$
$ g_{LR}^S $	$< 0.95$	$< 0.18$	$ g_{LR}^V $	$< 0.12$	$< 0.05$
$ g_{RL}^S $	$< 2.01$	$< 0.19$	$ g_{RL}^V $	$< 0.52$	$< 0.05$
$ g_{LL}^S $	$< 2.01$		$ g_{LL}^V $	$< 1.005$	

- Upper limits improvement (here we assume that  $\rho$ ,  $\eta$ ,  $\xi$ , and  $\xi\delta$  MP will be measured with  $10^{-3}$  uncertainty)
- As an example, new physics leading to  $\xi' < 0.96$  can be discovered with  $5\sigma$



# Systematics discussion

- The major sources are uncertainties in the efficiency of the signal process reconstruction (depending on the kinetics of muon decay) and the remaining background calculation
- The first one strongly depends on the direction of the daughter electron emission in the muon rest frame. It can be estimated from the data: by inverting the veto on  $K^- \rightarrow \mu^- \bar{\nu}_\mu$  and  $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$  decays, we can select a huge sample of the pseudoscalars decays (uniform decays)
- The background can be extracted from MC simulation (all processes are well known and well described): KKMC and TAUOLA generators are confirmed experimentally in much more precise measurements of other Michel parameters
- The estimation of the detector effects are tied to the experimental realization. Nevertheless, it is always possible to control these effects with large sample of kaons and pions kinks from  $D$ -decays

# Requirements for SCTF

- Polarized beam is crucial for the described method to have the best precision for all Michel parameters
- A big drift chamber, as a number of decayed muons and electron reconstruction efficiency depend on the outer radius of the tracker
- Muon tracks reconstruction using only hits in vertex detector to increase muon reconstruction efficiency
- There must be an algorithm of reconstruction of decayed-in-flight charged particles in a drift chamber together with their daughters and decay vertex (e.g. Kalman filter with kink option)
- High decay vertex resolution and good  $dE/dx$  separation to suppress background

# Conclusion

- This study provides a complete method of the first direct measurement of all Michel parameters that describe daughter muon polarization in  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  decays
- The study shows that SCTF with polarized beam is an optimal experiment for precise measurement, which is quite easy to perform
- Experiment setup with unpolarized beam complicates the procedure and degrades precision for all discussed Michel parameters except for  $\xi'$
- The statistical and systematic precision of the proposed measurement strongly depends on the Hardware and Software requirements for the SCTF detector
- We conclude that systematic errors can be controlled at least at the same level as statistical ones

# Thank you for attention!

For more details, please refer to  
[D. Bodrov and P. Pakhlov, arXiv:2203.12743 \[hep-ph\]](#)

# Backup

# Michel parameters (2)

$$\begin{aligned}
\rho &= \frac{3}{4} - \frac{3}{4} \left\{ (|g_{RL}^V|^2 + |g_{LR}^V|^2) + 2(|g_{LR}^T|^2 + |g_{RL}^T|^2) + Re(g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*}) \right\} \\
\eta &= \frac{1}{2} Re \left\{ g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) + g_{RR}^V g_{LL}^{S*} + g_{LL}^V g_{RR}^{S*} \right\} \\
\eta'' &= \frac{1}{2} Re \left\{ 3g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) + 3g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) - g_{RR}^V g_{LL}^{S*} - g_{LL}^V g_{RR}^{S*} \right\} \\
\xi &= 4Re(g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*}) + |g_{LL}^V|^2 - |g_{RR}^V|^2 + 3(|g_{LR}^V|^2 - |g_{RL}^V|^2) + \\
&\quad + 5(|g_{LR}^T|^2 - |g_{RL}^T|^2) + \frac{1}{4} (|g_{LL}^S|^2 - |g_{RR}^S|^2 + |g_{RL}^S|^2 - |g_{LR}^S|^2) \\
\xi\delta &= \frac{3}{16} (|g_{LL}^S|^2 - |g_{RR}^S|^2 + |g_{RL}^S|^2 - |g_{LR}^S|^2) + \frac{3}{4} (|g_{LL}^V|^2 - |g_{RR}^V|^2 - |g_{LR}^T|^2 + \\
&\quad + |g_{RL}^T|^2 + Re(g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*})) \\
\xi' &= - \left\{ 3 (|g_{RL}^T|^2 - |g_{LR}^T|^2) + (|g_{RR}^V|^2 + |g_{RL}^V|^2 - |g_{LR}^V|^2 - |g_{LL}^V|^2) + \right. \\
&\quad \left. + \frac{1}{4} (|g_{RR}^S|^2 + |g_{RL}^S|^2 - |g_{LR}^S|^2 - |g_{LL}^S|^2) \right\} \\
\xi'' &= 1 - \frac{1}{2} (|g_{RL}^S|^2 + |g_{LR}^S|^2) + 2(|g_{RL}^V|^2 + |g_{LR}^V|^2 + |g_{RL}^T|^2 + |g_{LR}^T|^2) + \\
&\quad + 4Re(g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*}) \\
\alpha &= 8Re \left\{ g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) \right\} \\
\alpha' &= 8Im \left\{ g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) - g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) \right\} \\
\beta &= - 4Re \left\{ g_{RR}^V g_{LL}^{S*} + g_{LL}^V g_{RR}^{S*} \right\} \\
\beta' &= 4Im \left\{ g_{RR}^V g_{LL}^{S*} - g_{LL}^V g_{RR}^{S*} \right\}.
\end{aligned}$$

# Michel parameters (3)

$$F_{IS}(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x),$$

$$F_{AS}(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2} \left[ 1 - x + \frac{2}{3}\delta \left( 4x - 3 - \frac{x_0^2}{2} \right) \right],$$

$$F_{IP}(x) = \frac{1}{54}\sqrt{x^2 - x_0^2} \left[ -9\xi' \left( 2x - 3 + \frac{x_0^2}{2} \right) + 4\xi \left( \delta - \frac{3}{4} \right) \left( 4x - 3 - \frac{x_0^2}{2} \right) \right],$$

$$F_{AP}(x) = \frac{1}{6} \left[ \xi'' \left( 2x^2 - x - x_0^2 \right) + 4 \left( \rho - \frac{3}{4} \right) \left( 4x^2 - 3x - x_0^2 \right) + 2\eta'' x_0(1-x) \right],$$

$$F_{T_1}(x) = -\frac{1}{12} \left[ 2 \left( \xi'' + 12 \left( \rho - \frac{3}{4} \right) \right) (1-x)x_0 + 3\eta(x^2 - x_0^2) + \eta''(3x^2 - 4x + x_0^2) \right],$$

$$F_{T_2}(x) = \frac{1}{3}\sqrt{x^2 - x_0^2} \left( 3\frac{\alpha'}{A}(1-x) + \frac{\beta'}{A}(2 - x_0^2) \right).$$

# Russian SCTF project

Logashenko Ivan

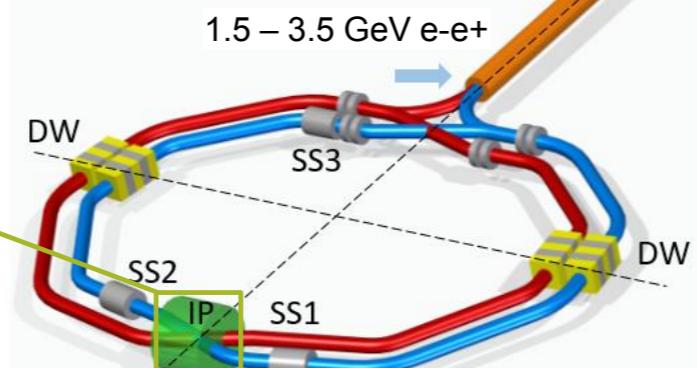
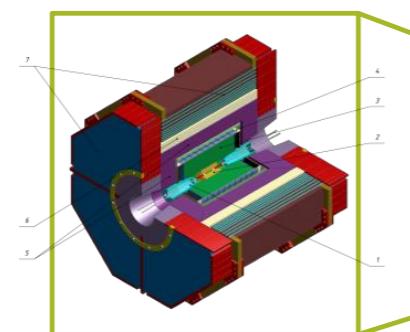
Overview of the SCTF project

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## The overview

Perimeter	632.94 m		
$2\theta$	60 mrad		
$\beta_x^*/\beta_y^*$	100 mm / 1 mm		
$F_{RF}$	350 MHz		
$E_{beam}$ (GeV)	1.5	2.5	3.5
$I$ (A)	2	2	2
$N_{bunch}$	292	328	262
$L_{peak} \times 10^{35}$ ( $\text{cm}^{-2}\text{s}^{-1}$ )	0.8	1.0	1.0

Parameters as of 2021



Super c-tau factory (SCTF) =  
 $e^+e^-$  collider with c.m. energy from 3 to 7 GeV  
+ detector



### Novosibirsk Super Charm Tau Factory

- e+ DR – positron damping ring
- DW – damping wiggler
- SS – Siberian Snake
- CV – electron-positron converter
- Pol e-/e- - polarized/un-polarized electron source