Dual formulation for the massless spin 2 field

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Introduction

The spin 2 massless representation admits alternative field-theoretical descriptions by the tensor with hook Young diagram:

`T. Curtight (1985); C. M. Hull (2001), etc.`

- exist only in $d \geq 5$;
- obstruct inclusion of interactions.

We propose another representation for the spin 2 by the third rank tensor with the hook Young diagram exists in $d \geq 3$.

Spin 1 analogy:

- $dF = 0$ is a topological field theory;
- $\delta F = dA$ is a gauge symmetry;
- general solution $F = dA$ is a pure gauge;
- substituting $F = dA$ into $d * F = 0$, we arrive at equations for A.

Linearised Nordström gravity

Linearised Nordström equation:

$$\left(\partial_\mu \partial_\nu - \eta_{\mu\nu}\Box\right) h^{\mu\nu} = -\frac{2d}{d-2}\Lambda\,, \tag{1}$$

where Λ is the cosmological constant.

Gauge symmetry transformations:

$$\delta_H h^{\mu\nu} = \partial_\lambda H^{\mu\nu\lambda} - \frac{1}{d-1}\eta_{\alpha\beta}\left(\eta^{\mu\nu}\partial_\lambda H^{\alpha\beta\lambda} + \partial^\nu H^{\alpha\beta\mu} + \partial^\mu H^{\alpha\beta\nu}\right), \tag{2}$$

with the gauge parameter $H^{\mu\nu\lambda}$ being the arbitrary tensor with Hook symmetry,

$$H^{(\mu\nu)\lambda} = H^{\mu\nu\lambda}\,, \qquad H^{(\mu\nu\lambda)} = 0\,,$$

This tensor is described by the Young diagram

$$\begin{array}{|c|c|}\hline \mu & \nu \\ \hline \lambda & \\ \cline{1-1} \end{array}\,.$$

Gauge transformations (2) are reducible.

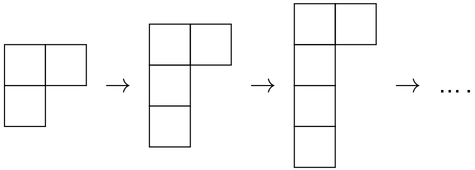
The complete sequence of gauge transformations:

$$\delta_H h^{\mu\nu} = \partial_\lambda H^{\mu\nu\lambda} - \frac{1}{d-1}\eta_{\alpha\beta}(\eta^{\mu\nu}\partial_\lambda H^{\alpha\beta\lambda} + \partial^\nu H^{\alpha\beta\mu} + \partial^\mu H^{\alpha\beta\nu})\,, \tag{3}$$

$$\begin{aligned}\delta_{H_1} H^{\mu\nu\lambda} = \partial_\rho\big(H^{\mu\nu\lambda\rho} - \frac{1}{3}\frac{1}{d-1}\eta_{\alpha\beta}(2\eta^{\mu\nu}H^{\alpha\beta\lambda\rho}\\ - \eta^{\nu\lambda}H^{\alpha\beta\mu\rho} - \eta^{\lambda\mu}H^{\alpha\beta\nu\rho})\big)\,;\end{aligned} \tag{4}$$

$$\delta_{H_k} H^{\mu\nu|\lambda\rho_1\ldots\rho_{k-1}} = \partial_{\rho_k} H^{\mu\nu\lambda\rho_1\ldots\rho_k}\,, \qquad k = 2,\ldots,d-2\,, \tag{5}$$

with the gauge parameters being tensors with hook symmetry:



Dual formulation for massless spin 2

Linearised Einstein gravity equations:

$$L_{\mu\nu} \equiv \frac{1}{2}\big(\partial_\mu\partial^\lambda h_{\nu\lambda} + \partial_\nu\partial^\lambda h_{\mu\lambda} - \Box h_{\mu\nu} - \partial_\mu\partial_\nu h\big) - \frac{1}{2}\eta_{\mu\nu}\big(\partial^\lambda\partial^\rho h_{\lambda\rho} - \Box h\big) - \Lambda\eta_{\mu\nu} = 0\,. \tag{6}$$

General solution:

$$h^{\mu\nu} = \partial_\lambda H^{\mu\nu\lambda} - \frac{1}{d-1}\big(\eta^{\mu\nu}\partial_\lambda H^\lambda + \partial^\nu H^\mu + \partial^\mu H^\nu\big) + \frac{2\Lambda}{(d-2)(d-1)}\big(x_\mu x_\nu + \eta_{\mu\nu}x_\lambda x^\lambda\big)\,. \tag{7}$$

Non-Lagrangian equations:

$$L_{\mu\nu} \equiv \frac{1}{2}\big(\partial_\mu\partial^\lambda\partial^\rho H_{\nu\lambda\rho} + \partial_\nu\partial^\lambda\partial^\rho H_{\mu\lambda\rho} - \Box\partial^\lambda H_{\mu\nu\lambda}\big) - \frac{1}{2}\frac{1}{d-1}\big(\partial_\mu\partial_\nu\partial_\lambda H^\lambda - \eta_{\mu\nu}\Box\partial_\lambda H^\lambda\big) = 0\,. \tag{8}$$

Gauge transformations for (8) coincide with (4)–(5).

DoF number count

The degree of freedom (DoF) number:

$$\mathcal{N} = \sum_n n\Big(t_n - \sum_m (-1)^m \left(l_n^m + r_n^m\right)\Big)\,, \tag{9}$$

where

- t_n is the number of equations of order n;
- l_n^m is the number of gauge identities of total order n and the order of reducibility m;
- r_n^m is the number of gauge symmetries of total order n and the order of reducibility m.

D.S. Kaparulin, S.L. Lyakhovich, A.A. Sharapov, JHEP (2013)

- for linearised Nordström gravity in $d = 4$:

$$t_2 = 1\,, \quad r_1^0 = 20\,, \quad r_2^1 = 15\,, \quad r_3^2 = 4\,,$$

$$\mathcal{N} = 2 - 1 \cdot 20 + 2 \cdot 15 - 3 \cdot 4 = 0\,; \tag{10}$$

for arbitrary d:

$$t_2 = 1\,, \quad r_n^{n-1} = \frac{(n+1)(d+1)!}{(n+2)!(d-n-1)!}\,,$$

$$\mathcal{N} = 2 - \sum_{n=1}^{d-1} (-1)^n \frac{n(n+1)(d+1)!}{(n+2)!(d-n-1)!} = 0\,. \tag{11}$$

- for linearised Einstein gravity in $d = 4$:

$$t_3 = 9\,, \quad l_4^0 = 4\,, \quad r_1^0 = 15\,, \quad r_2^1 = 4\,,$$

$$\mathcal{N} = 3 \cdot 9 - 4 \cdot 4 - 1 \cdot 15 + 2 \cdot 4 = 4\,; \tag{12}$$

for arbitrary d: $\mathcal{N} = d^2 - 3d$.

Conclusion

- The complete reducible gauge symmetry of the Nordström equation is found at linearised level.
- The third-order non-Lagrangian equations are proposed for the tensor with the hook Young diagram, which describe irreducible spin 2 massless representation in any $d \geq 3$.
- Using the Stueckelberg method for reducible gauge symmetries, the Lagrangian formulation is possible which simultaneously involves both metric and the hook tensor. Imposing different gauges one can switch between these two dual formulations.

  `V. Abakumova, S. Lyakhovich, Phys. Lett. B, 2021`

- The higher derivatives in field equations for hooks do not mean instability. Various higher derivative theories with unbounded canonical energy are stable, because there exist the other bounded conserved quantity.

  `V. Abakumova, D. Kaparulin, S. Lyakhovich, Phys. Rev. D, 2019`

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