Cosmology and Particle Physics

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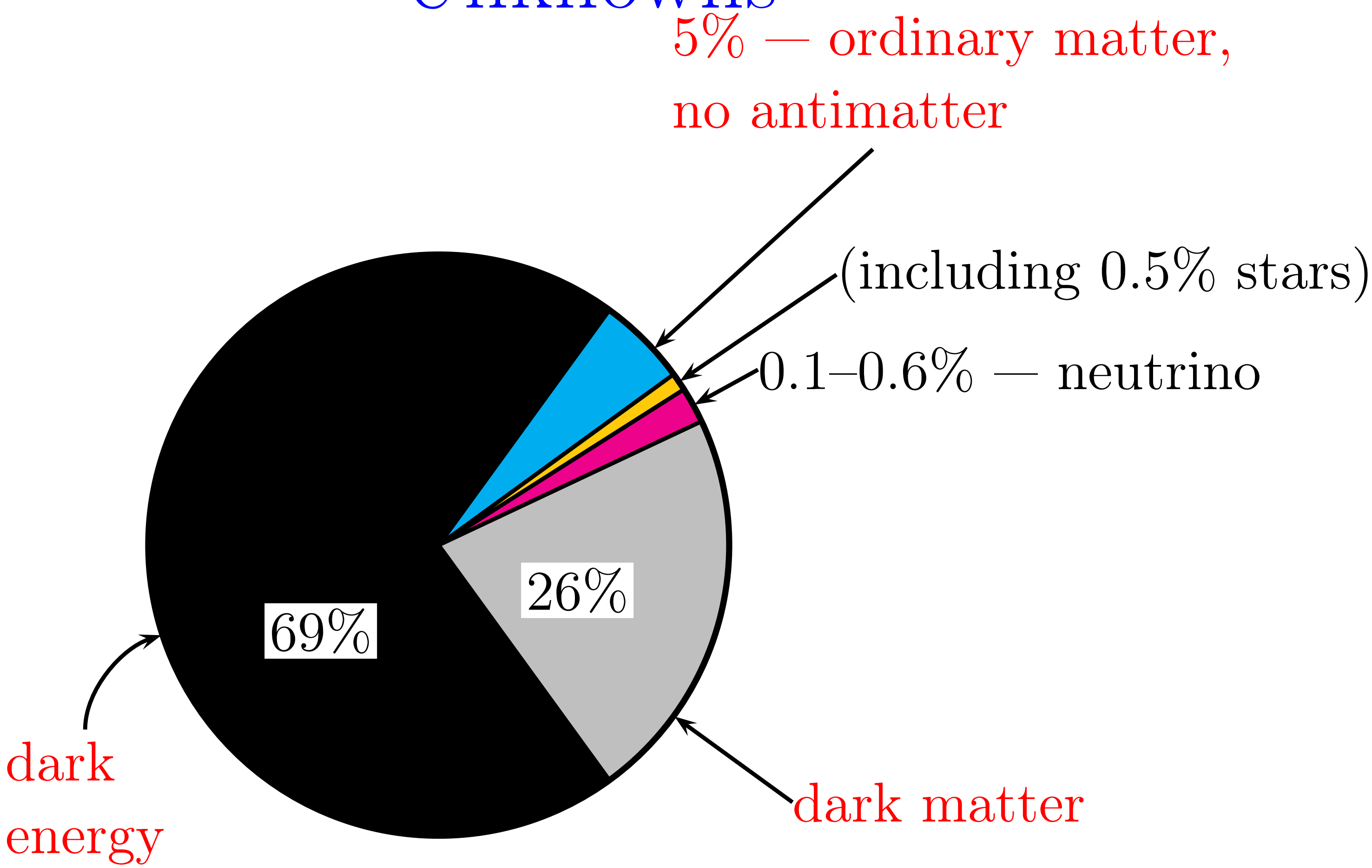
Outline of Lecture 2

- Dark matter: evidence
- Cold and warm dark matter
- Candidates

Unknowns


5% — ordinary matter,
no antimatter
(including 0.5% stars)
0.1–0.6% — neutrino
26%
69%
dark matter
dark
energy

Dark matter

- Astrophysical evidence: measurements of gravitational potentials in galaxies and clusters of galaxies
  - Velocity curves of galaxies

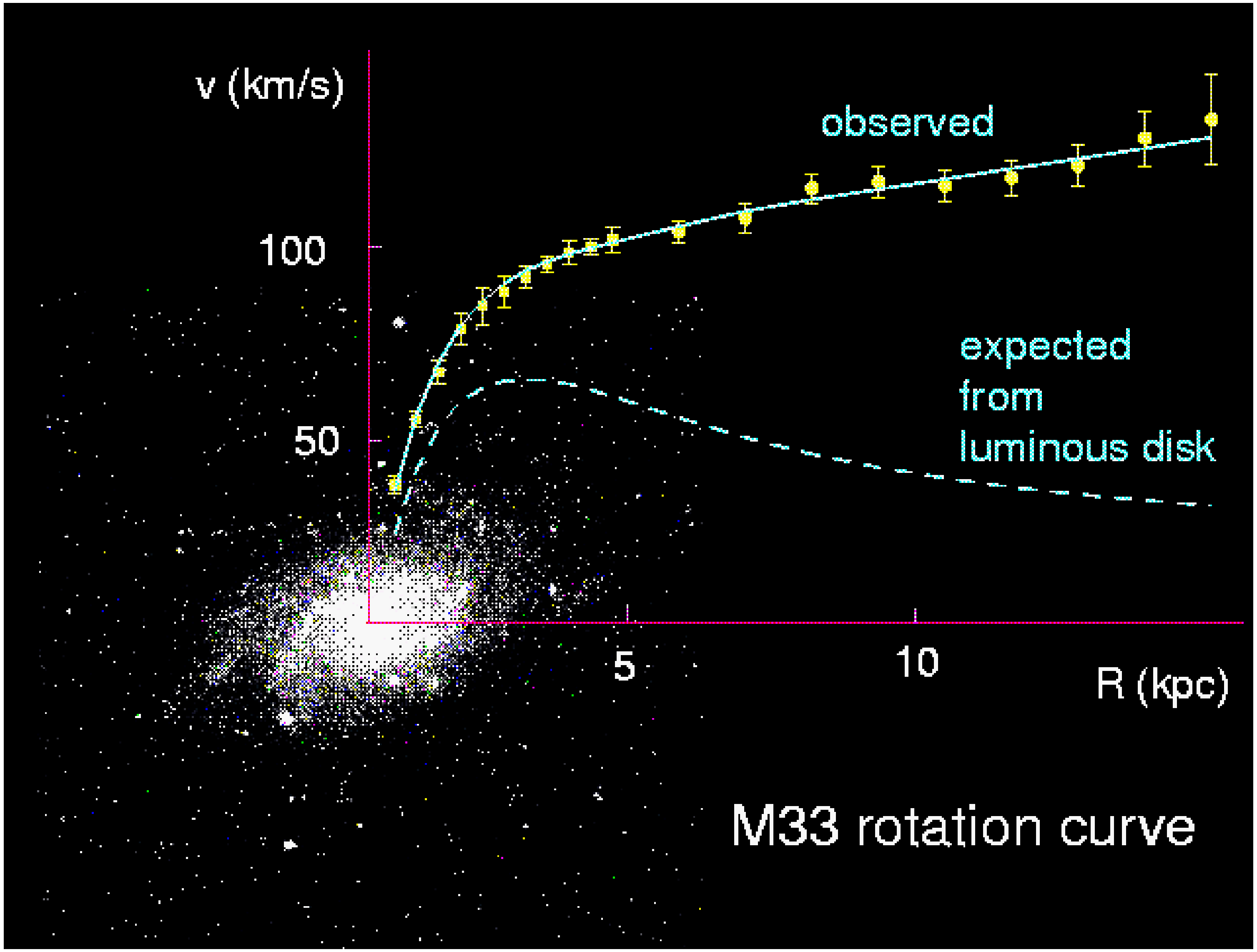
    Fig.
  - Velocities of galaxies in clusters

    Original Zwicky's argument, 1930's

$$v^2 = G\frac{M(r)}{r}$$

  - Temperature of gas in X-ray clusters of galaxies
  - Gravitational lensing of clusters
  - Etc.

Rotation curves



Outcome

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} = 0.2 - 0.3$$

Assuming mass-to-light ratio everywhere the same as in clusters
NB: only 10 % of galaxies gather in clusters

Nucleosynthesis, CMB:

$$\Omega_B = 0.05$$

The rest is non-baryonic, $\Omega_{DM} \approx 0.26$.

Physical parameter: mass-to-entropy ratio. Stays constant in time. Its value

$$\left(\frac{\rho_{DM}}{s}\right)_0 = \frac{\Omega_{DM}\rho_c}{s_0} = \frac{0.26 \cdot 5 \cdot 10^{-6}\ \text{GeV cm}^{-3}}{3000\ \text{cm}^{-3}} \simeq 4 \cdot 10^{-10}\ \text{GeV}$$

Both Ω_{DM} and Ω_B are determined with good precision from CMB anisotropies.

Baryons and DM: standard ruler — BAO

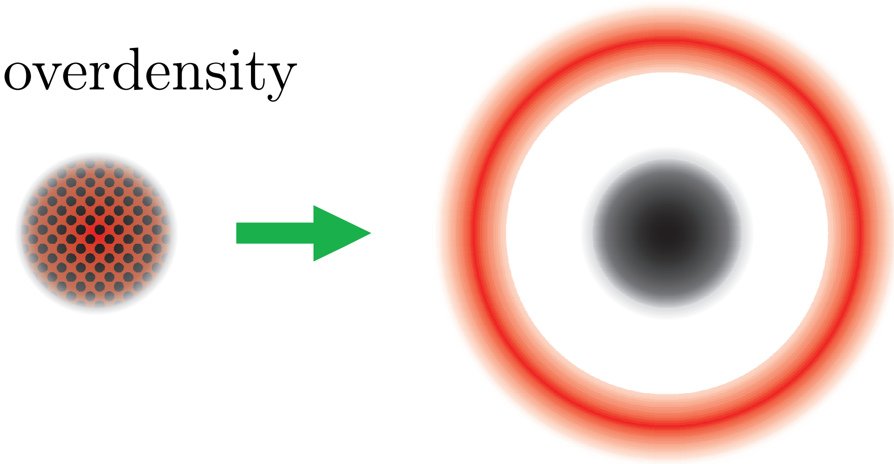
Sound speed in baryon-photon plasma before recombination $\sim 1/\sqrt{3}$; zero after recombination.
Baryon perturbations freeze in at recombination.
Standard ruler: sound horizon at recombination $r_s(t_r) = \int_0^{t_r} v_s \frac{dt}{a(t)}$

Then increases due to expansion of Universe: $r_s(t) = r_s(t_r) \frac{a(t)}{a(t_r)}$

Present size 150 Mpc = 450 mln. light years

"Baryon acoustic oscillations", BAO A.D. Sakharov' 1965

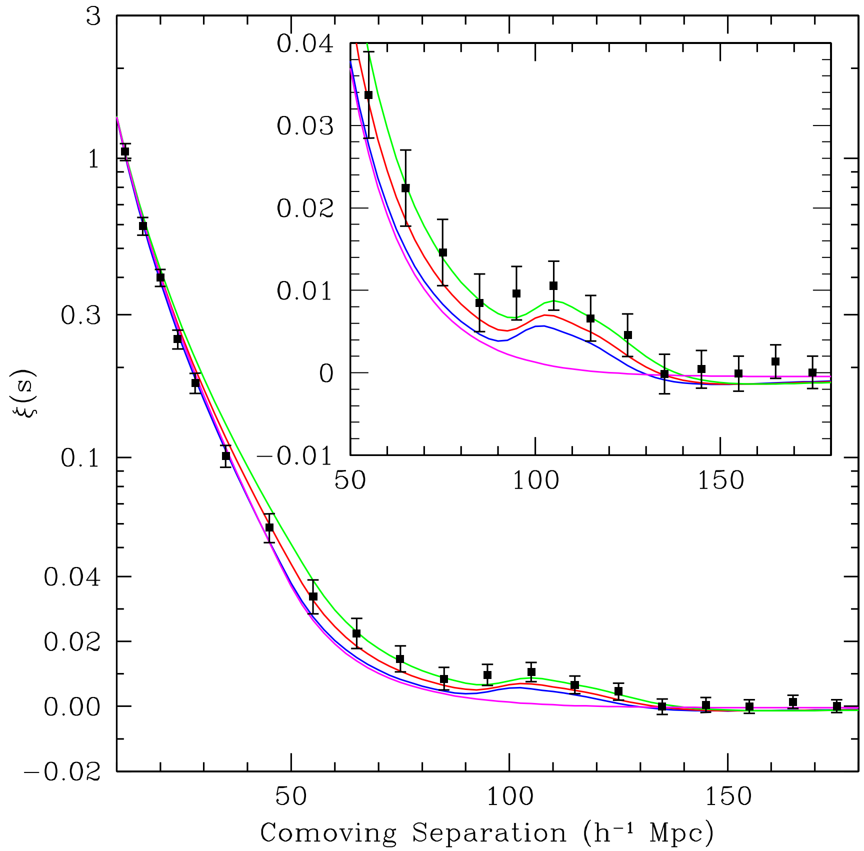
baryons
and dark matter overdensity

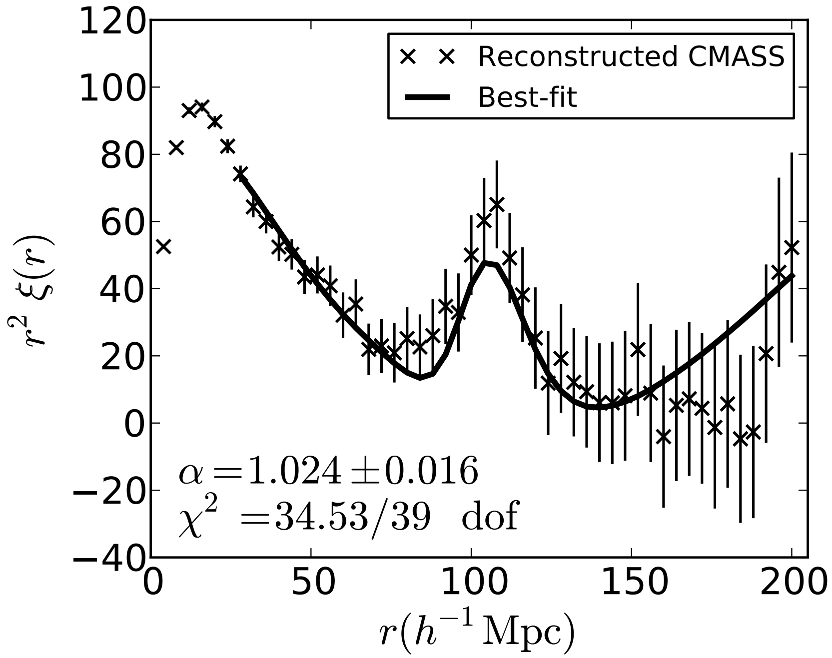


very early times recombination epoch and later

BAO show up in distribution of galaxies

Galaxy correlation function





Very sensitive to expansion history.
NB: Way to infer time-(in)dependence of dark energy.

Dark matter: growth of structure

CMB: baryon density perturbations at recombination $T = 3000$ K, $z = 1100$:

$$\delta_B \equiv \left(\frac{\delta\rho_B}{\rho_B}\right)_{z=1100} \simeq \left(\frac{\delta T}{T}\right)_{CMB} \lesssim 10^{-4}$$

In matter dominated Universe, matter perturbations grow as

$$\frac{\delta\rho}{\rho}(t) \propto a(t)$$

Perturbations in baryonic matter grow after recombination only
If not for dark matter,

$$\left(\frac{\delta\rho}{\rho}\right)_{today} \lesssim 1100 \times 10^{-4} \sim 0.1$$

No galaxies, no stars...
Perturbations in dark matter start to grow much earlier
(already at radiation-dominated stage)

NB: Need dark matter particles non-relativistic early on.

Neutrinos are not considerable part of dark matter
(way to set cosmological bound on neutrino mass,
$m_\nu \lesssim 0.1$ eV for every type of neutrino)

UNKNOWN DARK MATTER PARTICLES ARE
CRUCIAL FOR OUR EXISTENCE

Scandalous situation for ~ 40 years.

Cold and warm dark matter

- Cold dark matter: non-relativistic during all relevant cosmological epochs. Velocities of DM particles negligible for all purposes.
- Warm dark matter: relativistic until fairly late in the Universe. This suppresses formation of small structures.

  Example: sterile neutrinos

Thermal scenario

- WDM particles decouple when relativistic, $T_f \gg m$.
- Remain relativistic until $T \sim m$. Do not feel gravitational potential before that.
- Perturbations of wavelengths shorter than horizon size at that time get smeared out $\Longrightarrow$ small size objects do not form ("free streaming")

Why can WDM be useful?

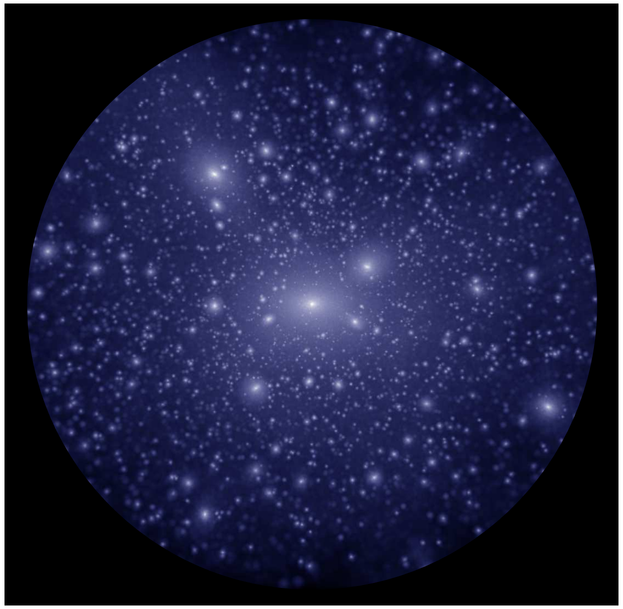
Clouds over CDM

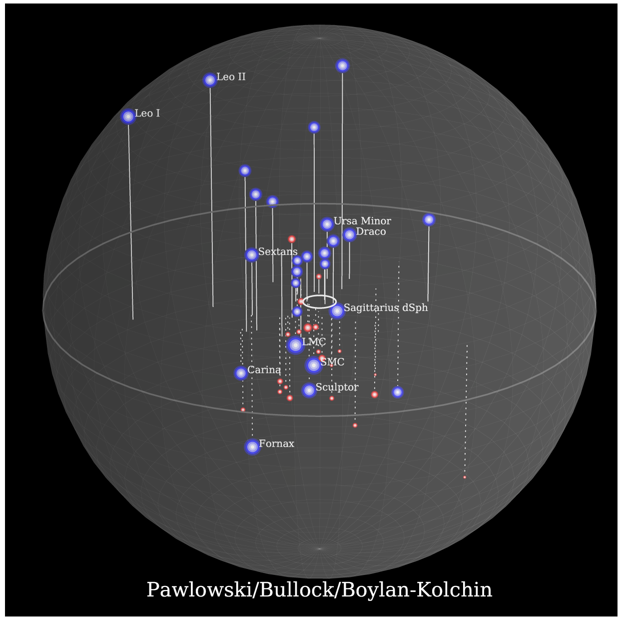
- Traditionally: "missing satellite problem"
  Numerical simulations of structure formation with CDM used to show too many dwarf galaxies:
  A few hundred small dark matter halos, satellites of a galaxy like ours — but only about a dozen observed until recently. Fig.
  - No longer so serious problem:
    - Many dwarf satellites of Milky Way discovered recently: SDSS, DES, Subaru: about 50 satellites by now; expected about 100 with full sky coverage.
    - Effects of baryons (star bursts, etc.): only heavy halos ($M > 10^9 M_\odot$) host visible dwarf galaxies.
- "Too big to fail": large, dense (and hence bright) satellite galaxies ($M \gtrsim 10^{10} M_\odot$) are also much less abundant compared to CDM predictions.
- "Cusp problem": CDM predicts cusps in galactic centers that are not observed.

Not serious worry yet, but what if small scales are suppressed?

CDM simulations

Observations





Bullock, Boylan-Kolchin' 17

250 kpc around Milky Way

- Horizon size at $T \sim m$

$$l(T) \simeq H^{-1}(T \sim m)$$

Friedmann equation at radiation domination:

$$H^2 = \frac{8\pi}{3M_{Pl}^2} \# g_* T^4 \quad \Longrightarrow \quad H(T) = \frac{T^2}{M_{Pl}^*}$$

with $M_{Pl}^* = M_{Pl}/(1.66\sqrt{g_*}) \sim 5 \cdot 10^{18}$ GeV at $T \lesssim 1$ MeV

- Horizon size at $T \sim m$

$$l_H(T) = H^{-1}(T \sim m) \sim \frac{M_{Pl}^*}{T^2} = \frac{M_{Pl}^*}{m^2}$$

Present size of this region

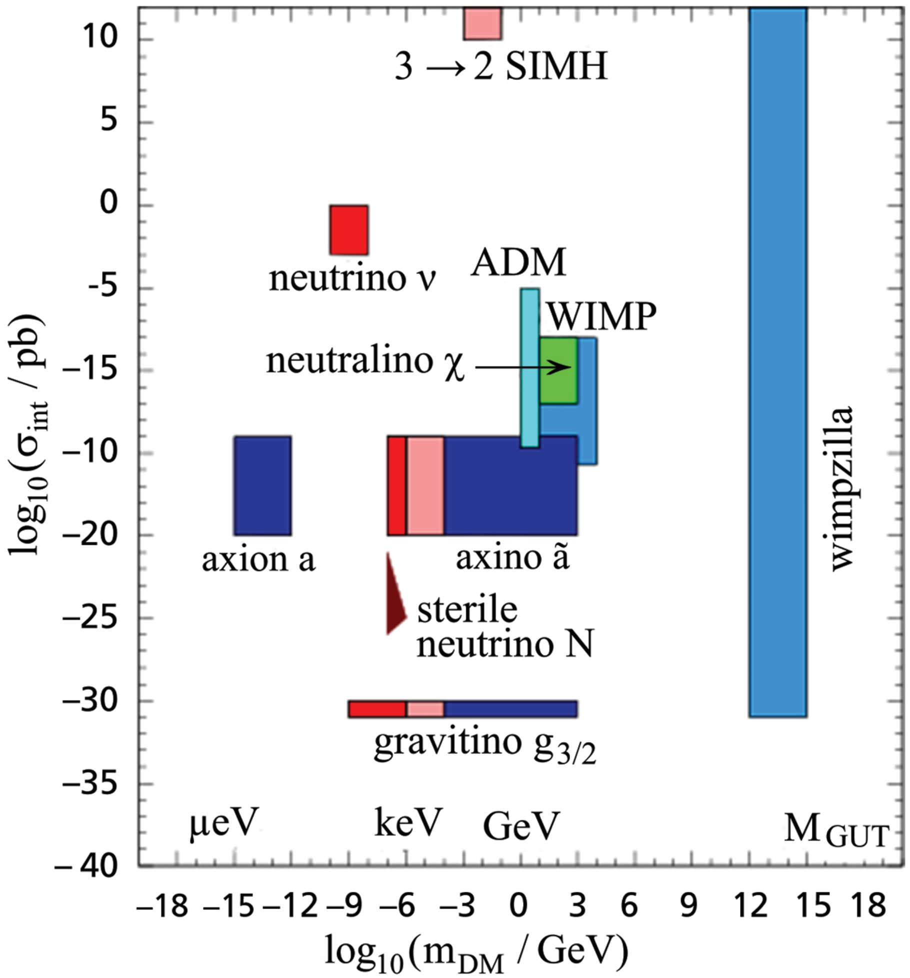
$$l_c = \frac{T}{T_0} l(T) = \frac{M_{Pl}}{mT_0}$$

(modulo g_* factors).

Objects of initial comoving size smaller than l_c are less abundant

- Size of region that collapsed into dwarf galaxy $M \sim 10^8 M_\odot$
  $l_{dwarf} \sim 100$ kpc $\sim 3 \cdot 10^{23}$ cm
  Require
  $$l_c \simeq \frac{M_{Pl}}{mT_0} \sim l_{dwarf}$$
  $\Longrightarrow$ obtain mass of DM particle
  $$m \sim \frac{M_{Pl}}{T_0 l_{dwarf}} \sim 3 \text{ keV}$$
  ($M_{Pl} = 10^{19}$ GeV, $T_0^{-1} = 0.1$ cm).
- Particles of masses in $m =$ a few keV range
  are warm dark matter candidates (assuming they had thermal velocities). Masses $m < 1$ keV ruled out.
- Similar estimates valid for non-thermal relics, if their momenta at decoupling are of order T.

Canidates for Dark Matter particles are numerous



WIMPs

Simple but very suggestive scenario

- Assume there is a new heavy stable particle X
  - Interacts with SM particles via pair annihilation (and crossing processes)

$$X + X \leftrightarrow q\bar{q}\ ,\mathrm{etc}$$

  - Parameters: mass M_X; annihilation cross section at non-relativistic velocity σ
- Assume that maximum temperature in the Universe was high, $T \gtrsim M_X$
- Calculate present mass density

Outcome: mass to entropy ratio

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \langle \sigma v \rangle)}{\langle \sigma v \rangle \sqrt{g_*(T_f)} M_{Pl}} ; \qquad \# = \frac{3\sqrt{5}}{\sqrt{\pi}}$$

- Correct value, mass-to-entropy$= 4 \cdot 10^{-10}$ GeV, for

$$\sigma_0 \equiv \langle \sigma v \rangle = (1 \div 2) \cdot 10^{-36} \text{ cm}^2 = (1 \div 2) \text{ pb}$$

  - Weak scale cross section. WIMP miracle: gravitational physics and EW scale physics combine into

    $$\text{mass-to-entropy} \simeq \frac{1}{M_{Pl}} \left(\frac{\text{TeV}}{\alpha_W} \right)^2 \simeq 10^{-10} \text{ GeV}$$

  - Mass M_X should not be much higher than 1 TeV

Weakly interacting massive particles, WIMPs.

Cold dark matter candidates SUSY: LSP neutralinos, $X = \chi$

But situation is rather tense: annihilation cross section is often too low; WIMPs overproduced.

WIMP search: direct

Difficulties in direct search for WIMPs

- Recoil energy of nucleus A (NB: suppressed for $M_X \ll M_A$)

$$E_{rec} \leq \frac{2M_A v_X^2}{(1+M_A/M_X)^2} = \frac{2M_X^2 v_X^2}{M_A(1+M_X/M_A)^2} \sim 10 \div 100 \text{ keV}$$

$v_X \simeq 200$ km/s = WIMP velocity in our Galaxy.

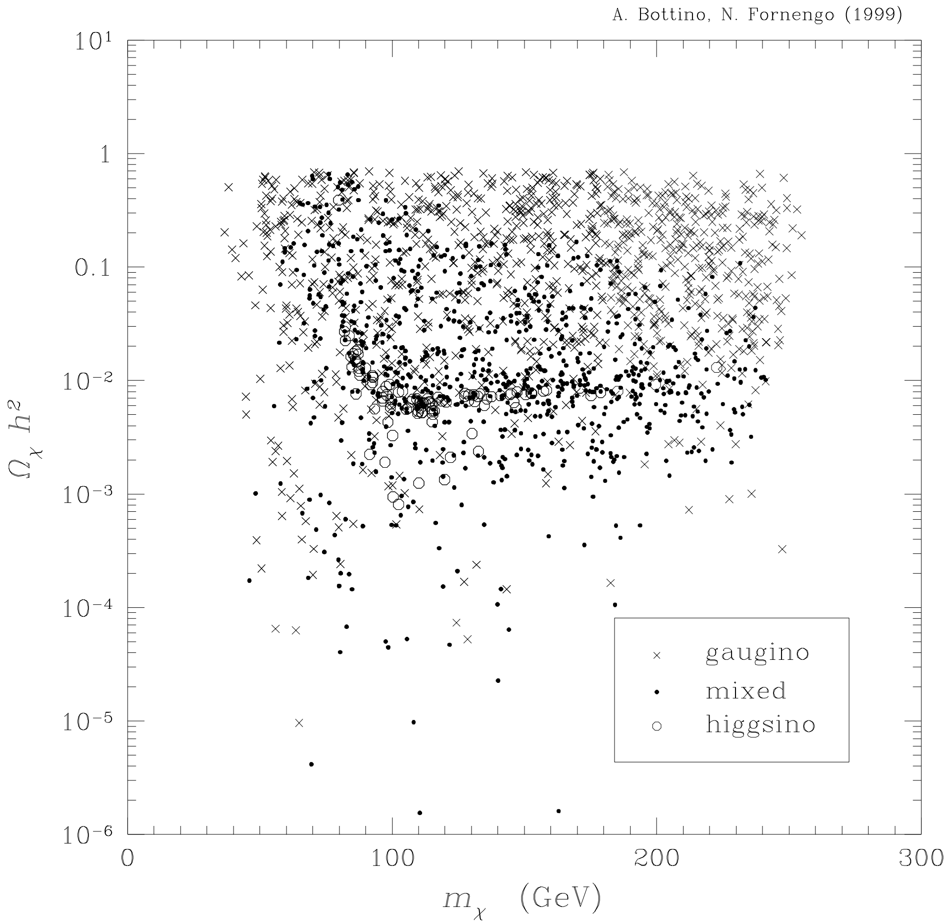
- Rate in detector of mass M_{det} (spin-independent)

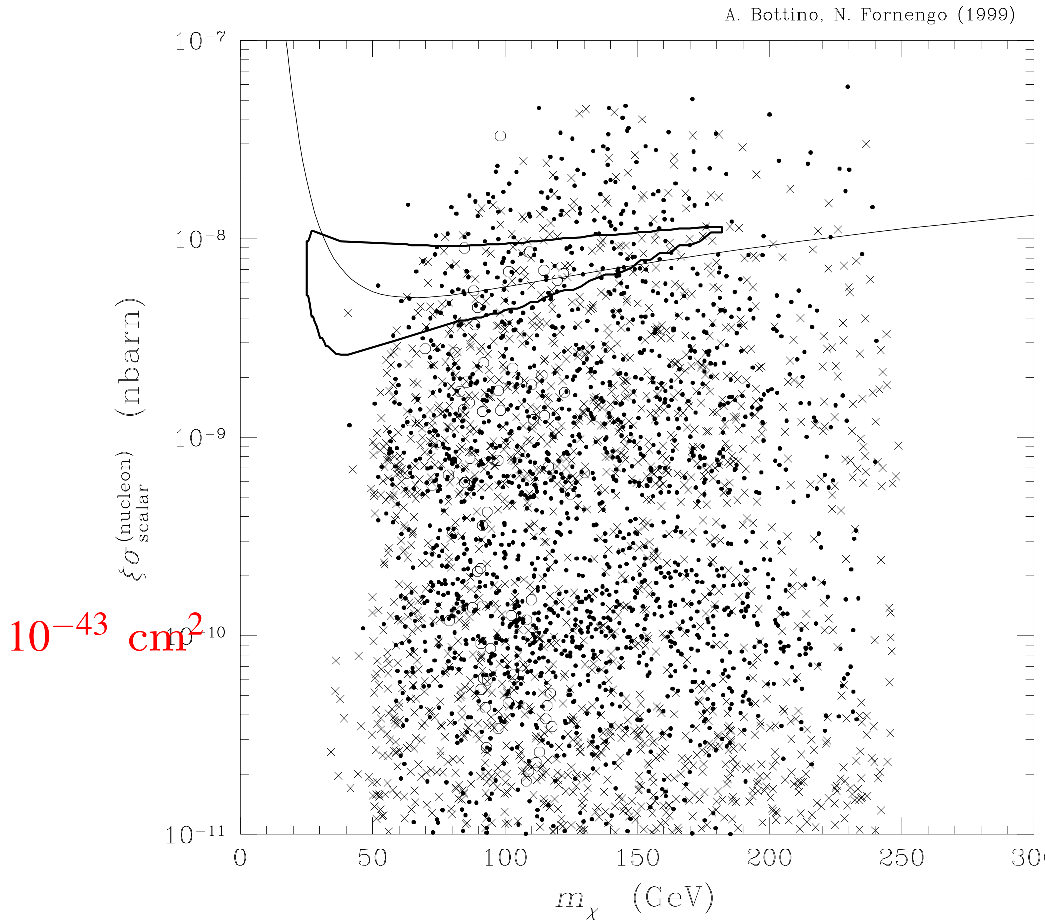
$$\Gamma \simeq v_X \frac{\rho_{loc}}{M_X} (\sigma_N A^2) \frac{6 \cdot 10^{23}}{A} \frac{M_{det}}{\text{g}}$$
$$\simeq 0.2 \frac{\text{events}}{\text{yr}} \cdot \frac{100 \text{ GeV}}{M_X} \frac{A}{100} \frac{M_{det}}{\text{tonn}} \frac{\sigma_N}{10^{-45} \text{ cm}^2}$$

$\rho_{loc} \simeq 0.3$ GeV/cm^3 = local DM mass density.

SUSY WIMPs 20 years ago

Direct detection (spin independent) expectations and limits

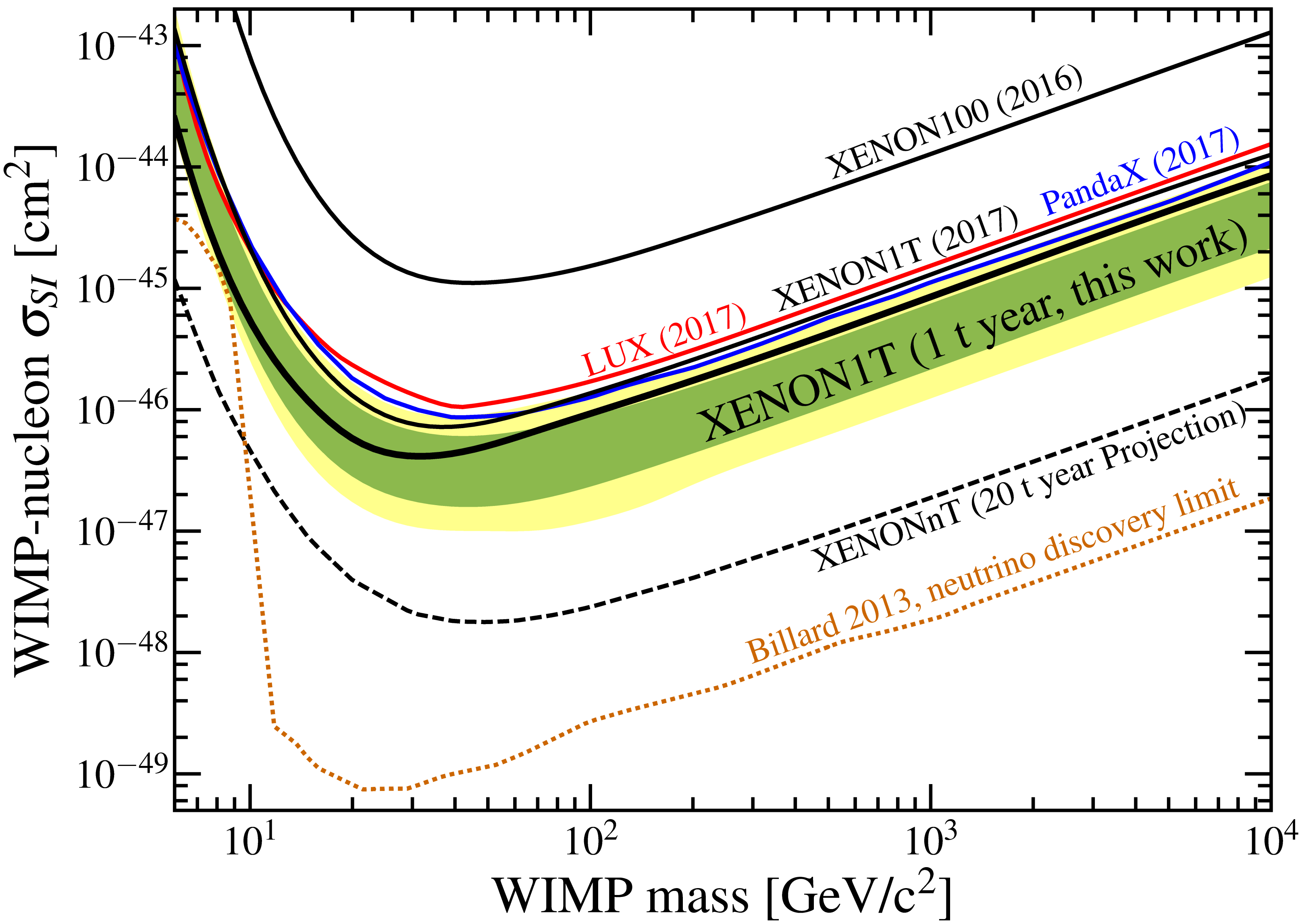




Bottino, Fornengo' 1999

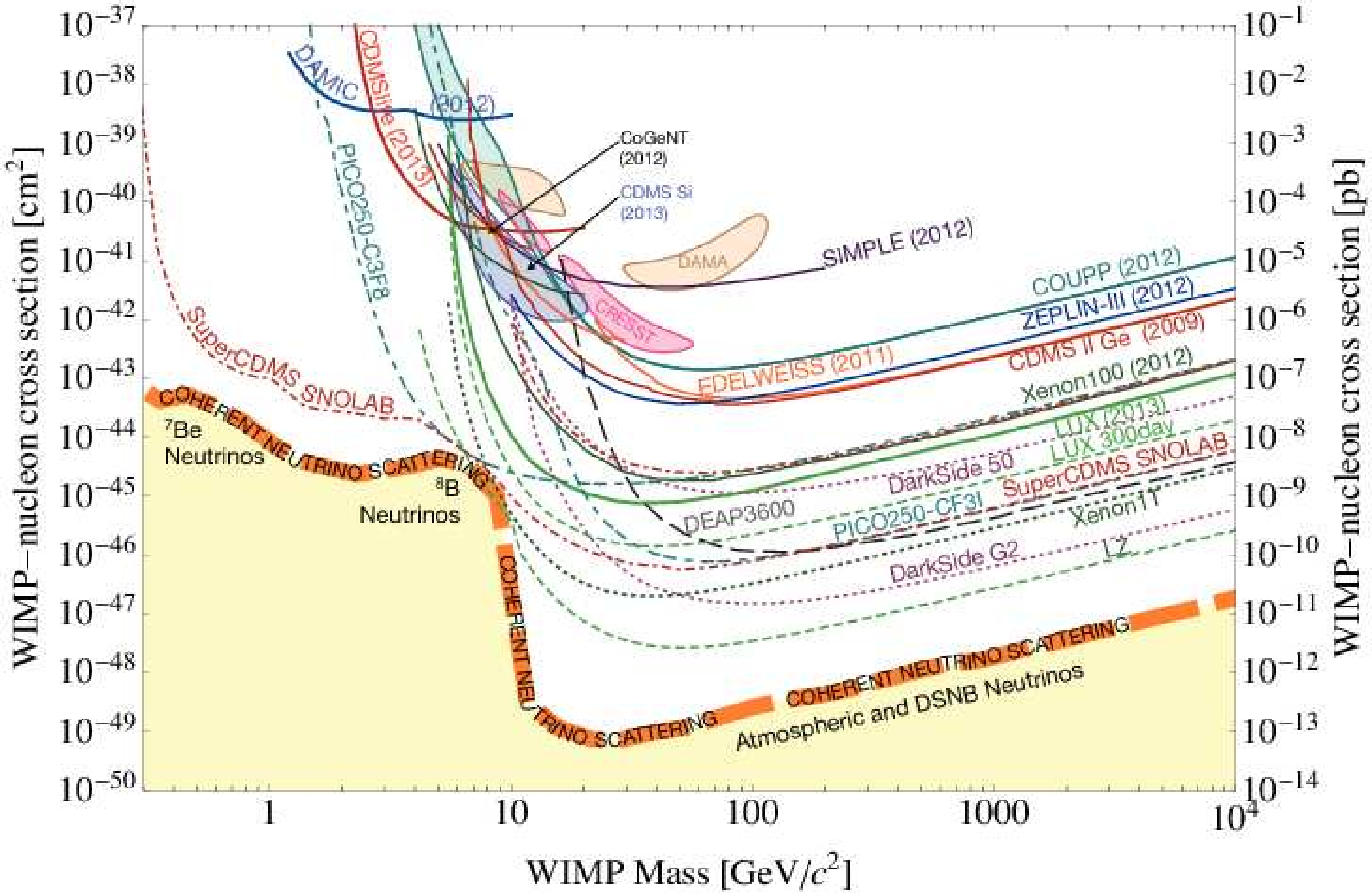
Xenon-1T, PandaX, LUX

Spin-independent, direct detection



Direct detection limits today and tomorrow

Roszkowski, Sessolo, Trojanowski 1707.06277



Many other possibilities

Example: Higgs portal

Just to have a WIMP, introduce scalar singlet S.

Renormalizable interaction with Higgs only. Impose symmetry $S \to -S \implies$ stable S .

$$L_S = \frac{1}{2}(\partial_\mu S)^2 - \left(\frac{\mu_s^2}{2} S^2 + \frac{\lambda_{SH}}{4} S^2 H^\dagger H + \frac{\lambda_S}{4} S^4 \right)$$

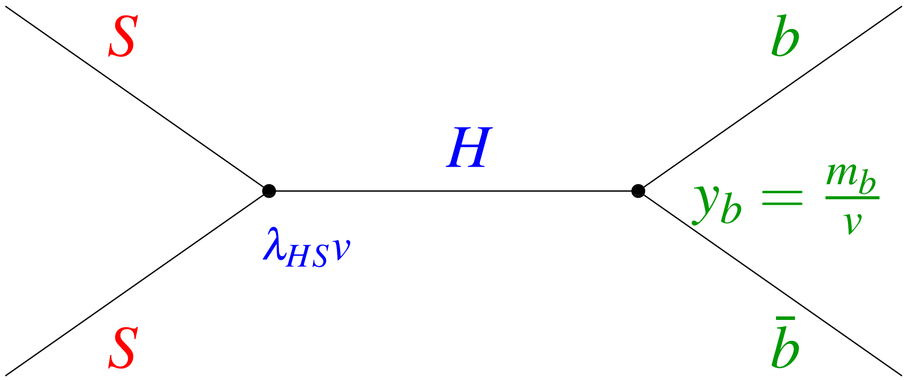
In vacuo $H = v/\sqrt{2} + h/\sqrt{2}$: vertices

$$\frac{\lambda_{SH}}{4} v h S^2 + \frac{\lambda_{SH}}{8} h^2 S^2$$

- Light S, $\quad m_S \ll m_H/2$.

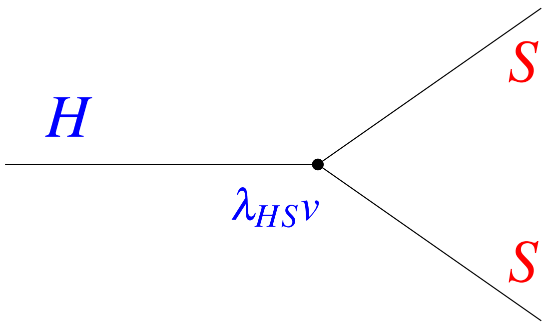
  Fairly popular before the LHC

  Main annihilation channel $SS \to b\bar{b}$.



$\langle \sigma v \rangle = 1$ pb $\Longrightarrow$ quite large λ_{SH} $\Longrightarrow$

Signature: invisible Higgs decay $H \to SS$.



Excersize:
calculate $\langle \sigma v \rangle$
and $\Gamma(H \to SS)$

- Degeneracy: m_s just below $m_H/2$.

  Pole enhanced $\langle \sigma v \rangle \Longrightarrow$ not so large $\lambda_{SH} \Longrightarrow$

  $+$ threshold suppression of invisible Higgs decay $H \to SS \Longrightarrow$

  viable and interesting (but fine tuned).

  Signature: invisible Higgs decay.

- Heavy S: $m_s > m_W$.

  Main annihilation channels $SS \to WW, ZZ, HH$.

  Interesting for direct dark matter detection experiments and LHC, $m_s > 1$ TeV because of existing constraints.

  Signature $pp \to H^* + \text{jet} \to \text{jet} + SS$;

$$\text{jets} + \text{missing } E_T$$

Indirect searches

- DM annihilation in centers of Sun, Earth

$$X + \bar{X} \to \pi^{\pm}\,,\ K^{\pm} + \ldots \to \nu\,,\ \bar{\nu} + \ldots$$

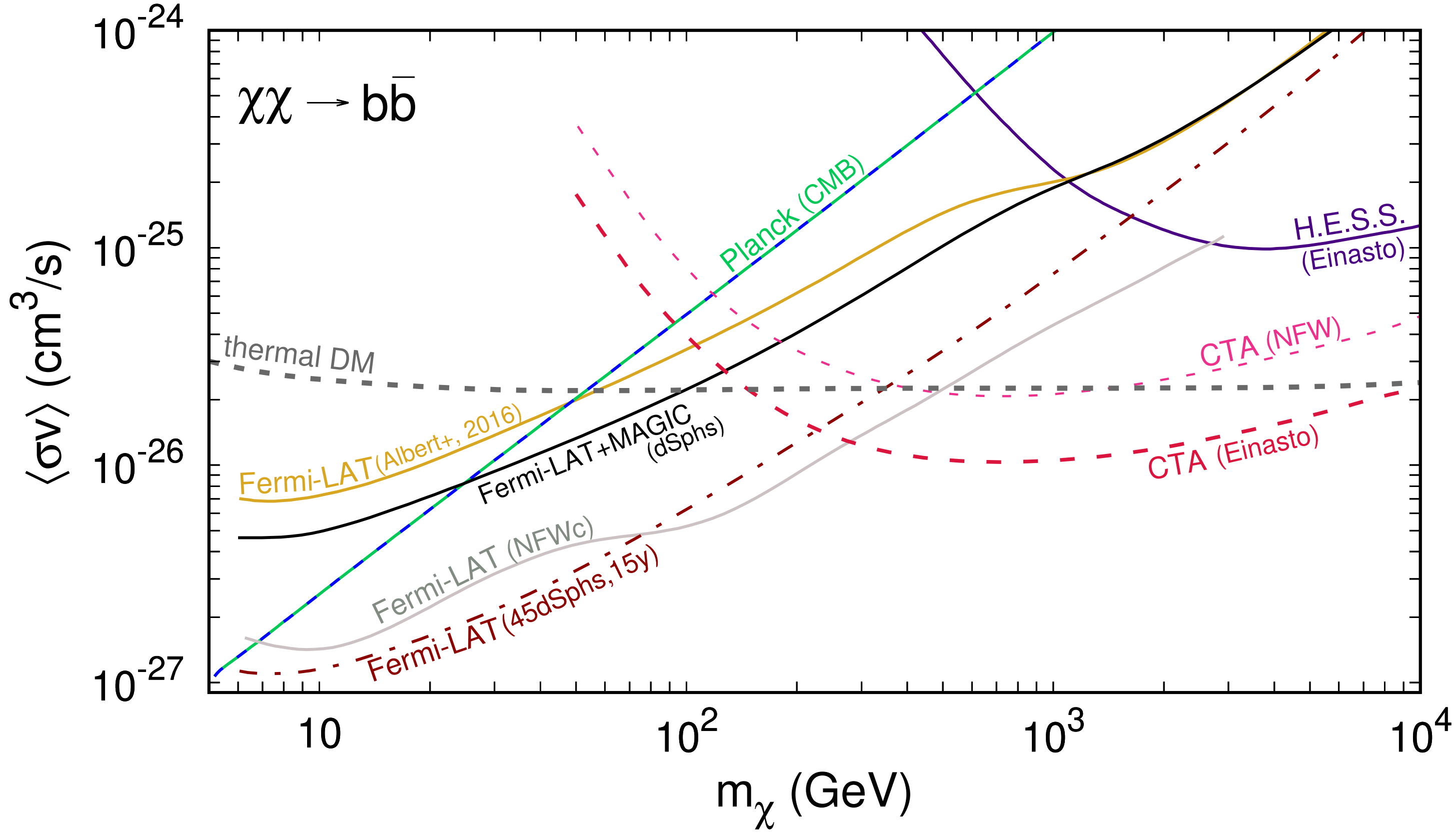
High energy neutrinos $\Longrightarrow$

- Baksan Underground Scintillation Telescope
- Super-K
- IceCube
- Baikal GVD

- DM annihilation in space

e^{+}, $\bar{p}$ in cosmic rays (PAMELA, AMS),
annihilation γ's (Fermi-LAT, MAGIC, HESS, CTA ...).

Limits from annihilation γ's



Current limits, solid
Projected limits, dashed
NFW, Einasto: dark matter profiles in galaxies
Thermal DM: WIMP annihilation cross section, assuming domination of $X \to b\bar{b}$

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

By no means. Other good DM candidates:

axion, sterile neutrino, gravitino

Plus a lot of exotica...

Sterile neutrinos: WDM candidates

- Needed to give masses to ordinary neutrinos
- One sterile neutrino species can be light.
  Seemingly, nothing wrong with m_{ν_s} = a few keV
- Mix with ordinary neutrinos (say, ν_e), mixing angle θ_s.
  In vacuum, and in Universe below $T \sim 200$ MeV

$$P_{\nu_e \to \nu_s} = \sin^2 2\theta_s \cdot \sin^2 \left(\frac{\Delta m^2 \, t}{E} \right)$$

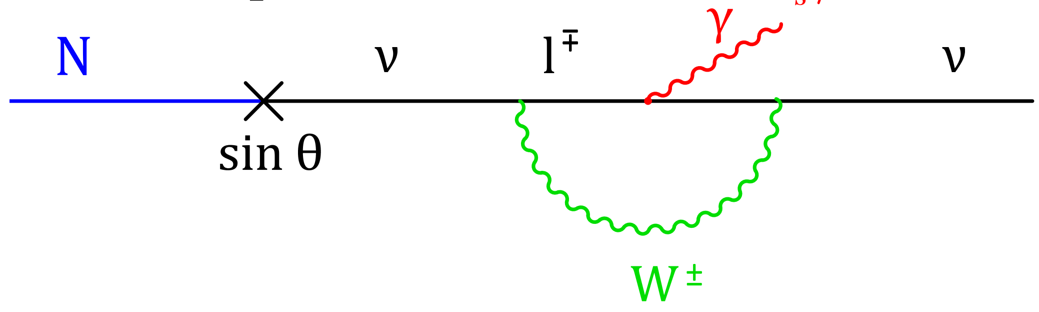
Rapid oscillations, $P_{\nu_e \to \nu_s} = \frac{1}{2} \sin^2 2\theta_s$. Process starts anew after collision of ν_e with another particle in cosmic plasma.

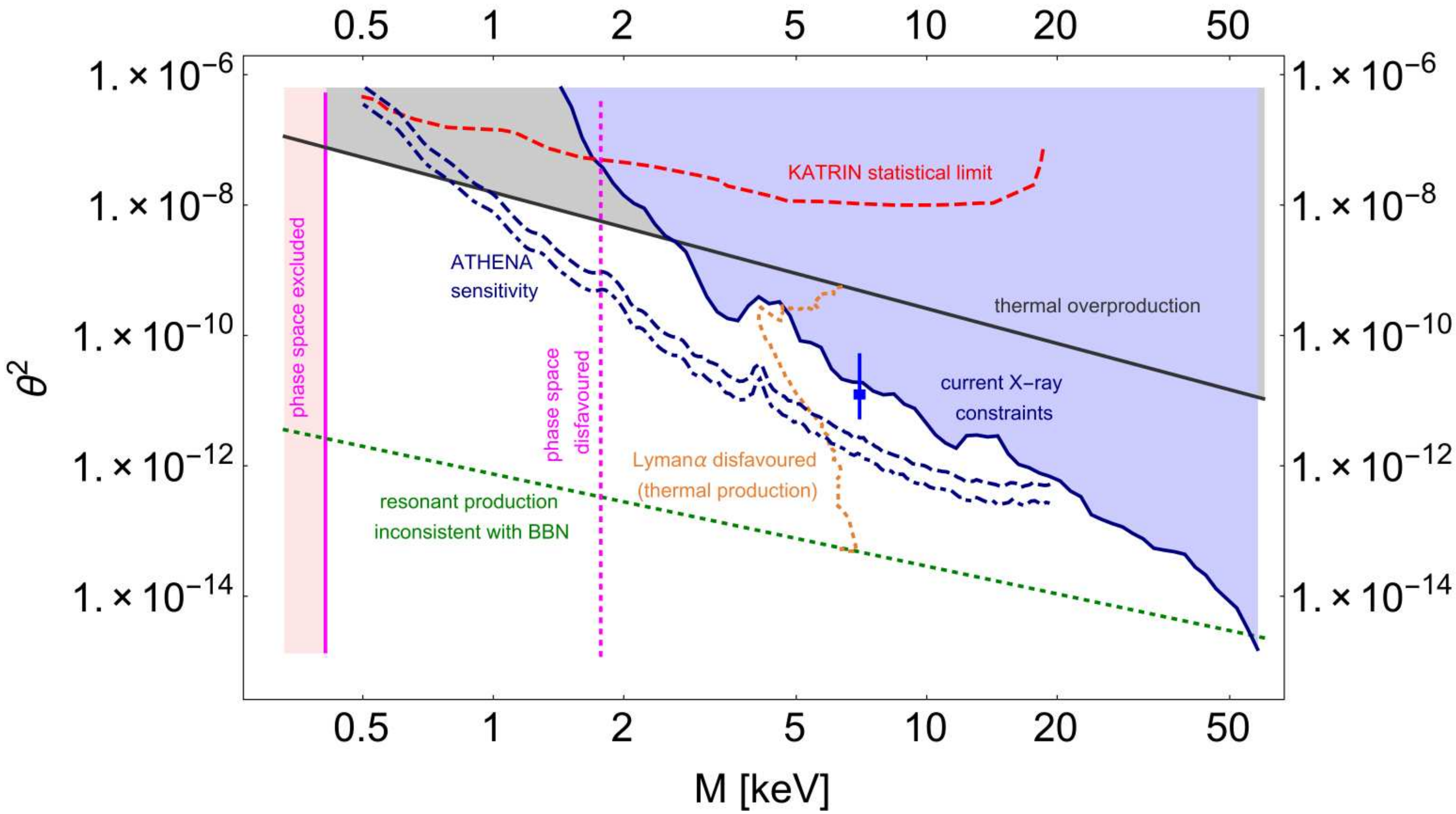
Outcome:

$$\Omega_s \simeq 0.2 \cdot \left(\frac{\sin 2\theta_s}{10^{-4}} \right)^2 \cdot \left(\frac{m_{\nu_s}}{1 \text{ keV}} \right)$$

- Long lifetime: $\tau_{\nu_s} \gg 10^{10}$ yrs for $m_{\nu_s} = 3 - 10$ keV,
  $\sin 2\theta_s = 10^{-4} - 10^{-5}$

$\nu_s \to \nu\gamma \Longrightarrow$ Search for photons with $E = m_{\nu_s}/2$ from sky

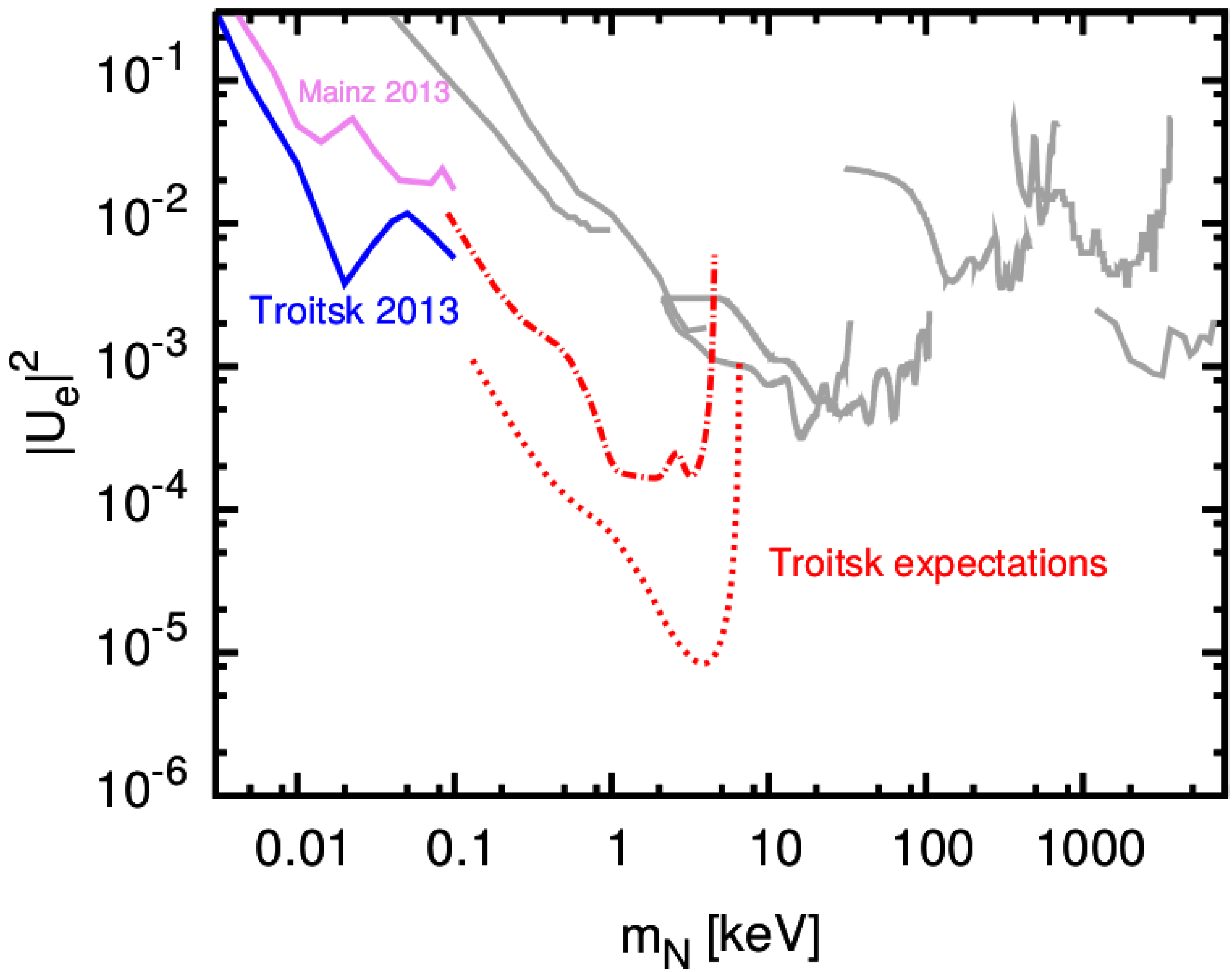




Straightforward version of scenario ruled out

But more contrived are not

Laboratory search: long way to go



Axions

Motivation: solution of strong CP problem

What's the problem?

To make long story short: in general, QCD Lagrangian involves θ-term:

$$\frac{\alpha_s}{16\pi}\,\theta_0\,\varepsilon^{\mu\nu\lambda\rho}G^a_{\mu\nu}G^a_{\lambda\rho}$$

and θ_0 violates CP !

Neutron edm $d_n < 3 \cdot 10^{-26} e \cdot \mathrm{cm} \Longrightarrow$

$$\theta_0 \lesssim 10^{-10}$$

Strong CP problem. Fine tuning? Mechanism that ensures $\theta_0 = 0$

Peccei–Quinn: promote θ to a field.

$$L_\theta = \frac{1}{2} f_{PQ}^2 (\partial_\mu \theta)^2 - V(\theta)\,, \qquad V(\theta) \simeq -m_q \langle \bar{q} q \rangle \cos\theta = \frac{1}{2} m_q \langle \bar{q} q \rangle \theta^2$$

Axion field $\theta(x) = a(x)/f_{PQ}$:

$$m_a^2 \simeq \frac{m_q \langle \bar{q} q \rangle}{f_{PQ}^2} \simeq \frac{m_\pi^2 f_\pi^2}{4 f_{PQ}^2} \quad \Longrightarrow \quad m_a = 6 \cdot 10^{-6}\ \text{eV} \cdot \left(\frac{10^{12}\ \text{GeV}}{f_{PQ}} \right)$$

Thus, Peccei–Quinn solution to strong CP problem predicts axion with mass

$$m_a = 6\ \mu\text{eV} \cdot \left(\frac{10^{12}\ \text{GeV}}{f_{PQ}} \right)$$

and $a\gamma\gamma$ interaction

$$C_{a\gamma\gamma} \frac{\alpha}{2\pi} \frac{a(x)}{f_{PQ}} (\vec{E} \cdot \vec{H})$$

where $C_{a\gamma\gamma} \sim 1$ is model-dependent, and f_{PQ} is the only free parameter. Larger $f_{PQ} \Longrightarrow$ smaller m_a, weaker interactions.

Why is this interesting for cosmology?

- Axion is practically stable:

$$\Gamma(a \to \gamma\gamma) = C_{a\gamma\gamma}^2 \left(\frac{\alpha}{8\pi}\right)^2 \frac{m_a^3}{4\pi f_{PQ}^2} \quad \Longrightarrow \quad \tau_a = 10^{17} \left(\frac{\text{eV}}{m_a}\right)^5 \text{ yrs}$$

- Interacts very weakly $\Longrightarrow$ dark matter candidate
- May never be in thermal equilibrium $\Longrightarrow$ cold dark matter if momenta are negligibly small.

Q. How can one arrange for negligibly small momenta for particles with sub-eV masses?

A. Condensates (not the only option)

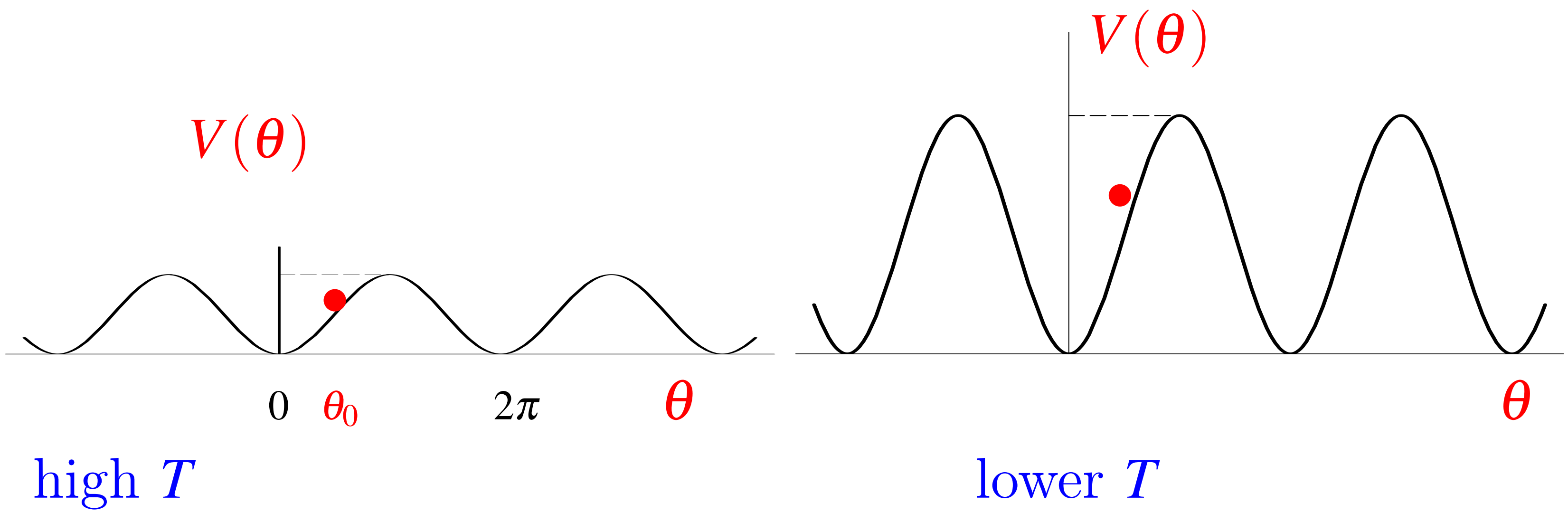
Axion production: misalignment

Recall $V(\theta) \simeq -m_q \langle \bar{q} q \rangle \cos\theta$

Early Universe, high T: $\langle \bar{q} q \rangle = 0 \implies V(\theta) = 0$.

No preferred value of $\theta \implies$ Initial condinion θ_0 anywhere between $-\pi$ and π.

At QCD epoch ($T \sim 200$ MeV) potential $V(\theta)$ builds up. θ starts to roll down. Homogeneous scalar field $=$ collection of quanta with zero spatial momenta.



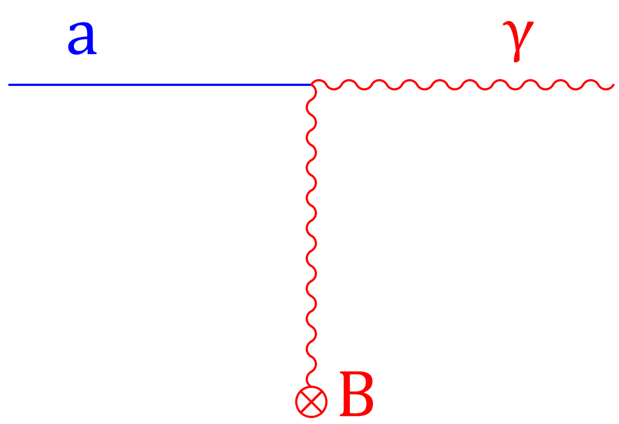
There are other non-thermal production mechanisms.

Outcome: axions of mass $m_a = 1 - 30\ \mu\mathrm{eV}$ are good candidates for cold dark matter.

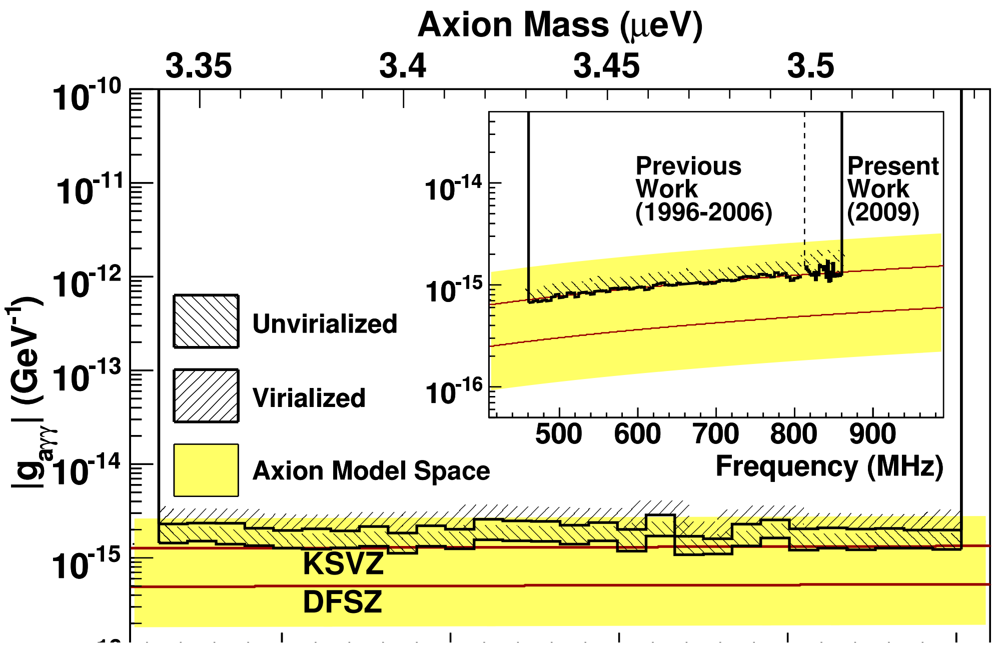
Search

$$a\gamma\gamma \text{ interaction} \qquad C_{a\gamma\gamma}\frac{\alpha}{2\pi}\frac{a(x)}{f_{PQ}}(\vec{E}\cdot\vec{H})$$

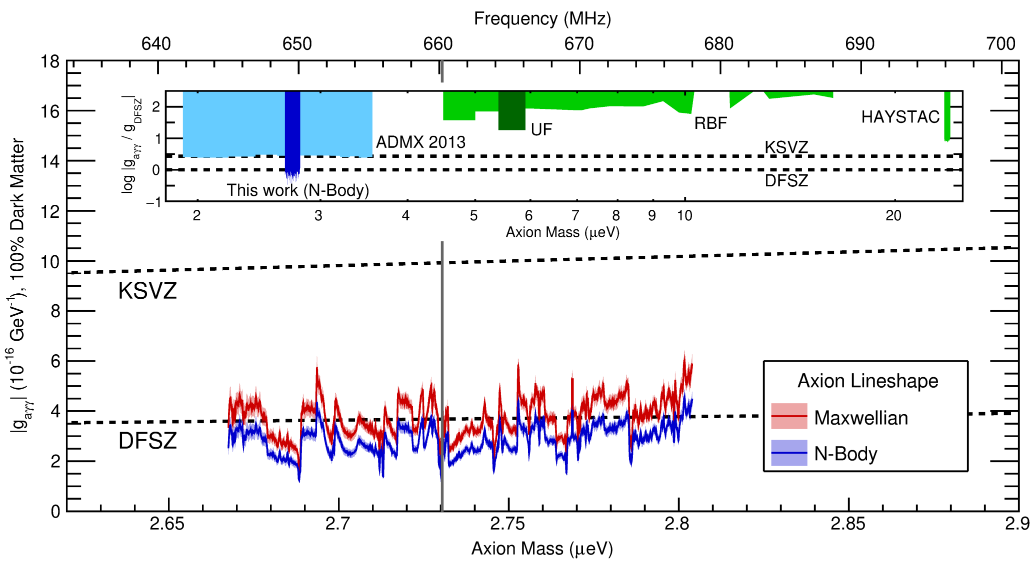
Conversion of DM axion into photon in magnetic field in a resonant cavity. $10^{-6}\ \mathrm{eV}/2\pi = 240\ \mathrm{MHz}$.



Need high Q resonator to collect photons, narrow bandwidth, go small steps in m_a. Long story.



ADMX, PRL ’2010

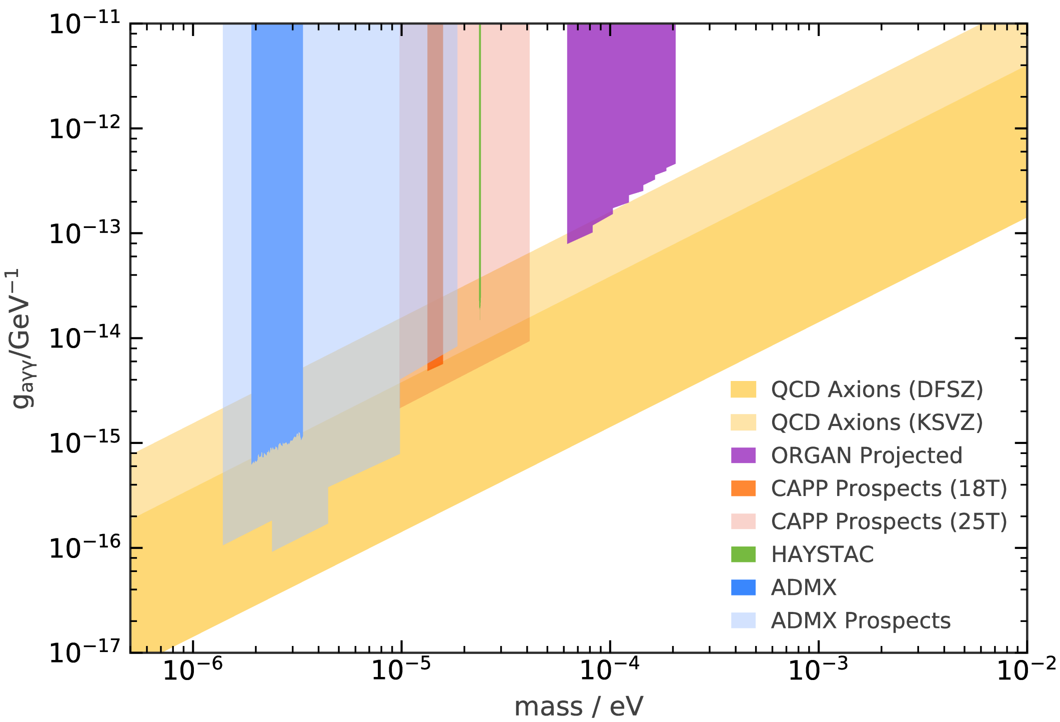


ADMX, PRL ’2018

New efforts in axion searches:

- CAPP, axion-photon conversion in magnetic field, $m_a = (3 \cdot 10^{-6} - 10^{-4})$ eV;
- MADMAX, axion-photon conversion at boundaries of dielectric discs in magnetic field $m_a \gtrsim 4 \cdot 10^{-5}$ eV
- CASPEr, time-varying EDM of nuclei in oscillating axion background $\Longrightarrow$ spin precession, $m_a \lesssim 10^{-9}$ eV

All aim at dark matter QCD axions



Axion-like particles, ALPs

Axions: $m_a f_a = (m_\pi f_\pi)/2 = 6 \cdot 10^{-3}\ \mathrm{GeV}^2$

ALPs: No relationship between m_a and f_a.

Possible origin: pseudo-Nambu–Goldstone bosons of approximate global symmetry

Coupling to photons

$$C_{a\gamma\gamma}\frac{\alpha}{2\pi}a(x)(\vec{E}\cdot\vec{H})$$

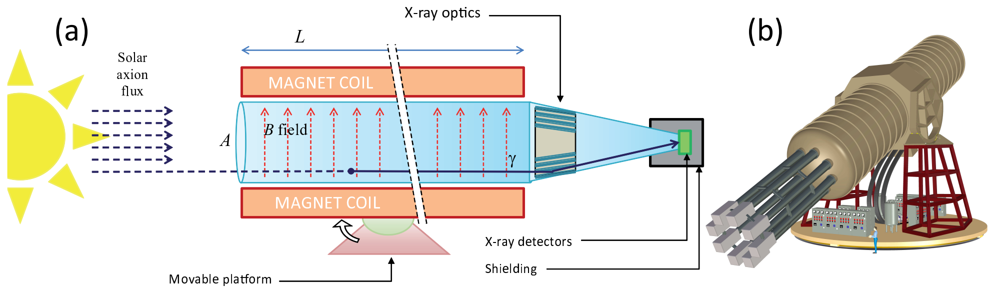
Coupling to SM fermions f through Higgs:

$$C_{aff}\,aH\bar{f}f \implies C_{aff}\langle H\rangle a\bar{f}f$$

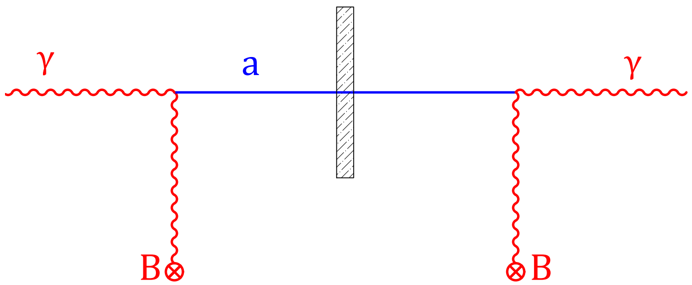
Large $f_a \implies$ small $C_{a\gamma\gamma}, C_{aff} \propto f_a^{-1}$.

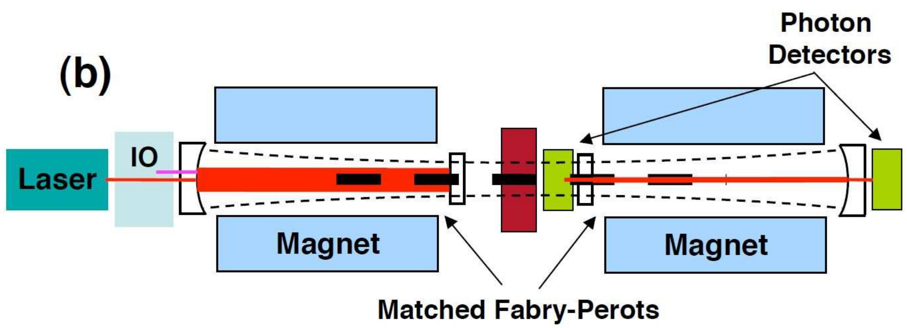
ALP searches, present and future

- Haloscopes – ALPs from dark matter halo: ADMX, CAPP, MADMAX, CASPEr
- Helioscopes – ALPs from the Sun: CAST, IAXO, TASTE



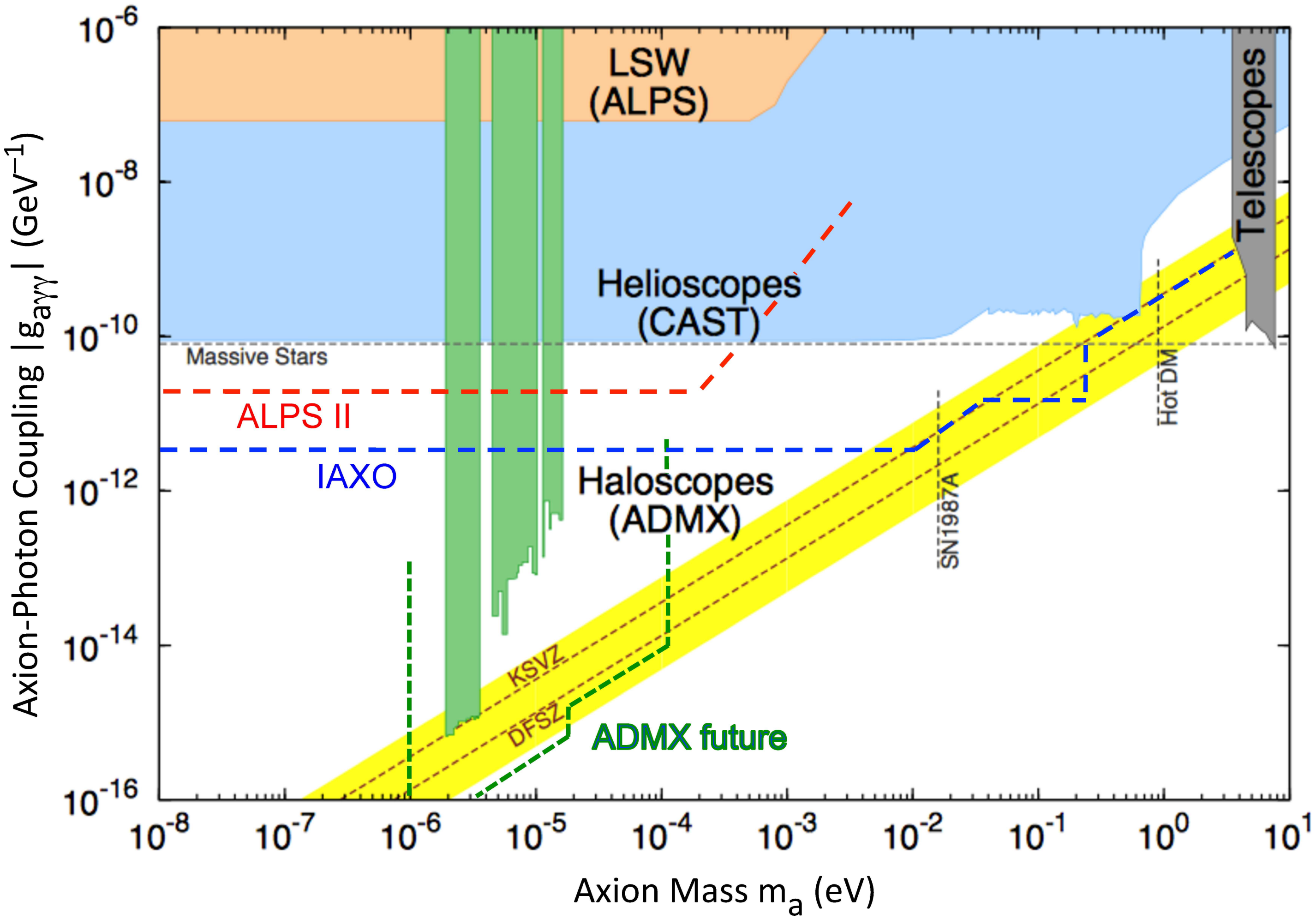
- Light shining through wall, ALPS I, ALPS II





- Beam-dump searches: SHiP

Still a lot of parameter space to explore



Dark matter summary

- No strong preference of one candidate over others.

  Wide program of searches with very diverse techniques.

- Astrophysics will hopefully give more hints

  CDM vs WDM

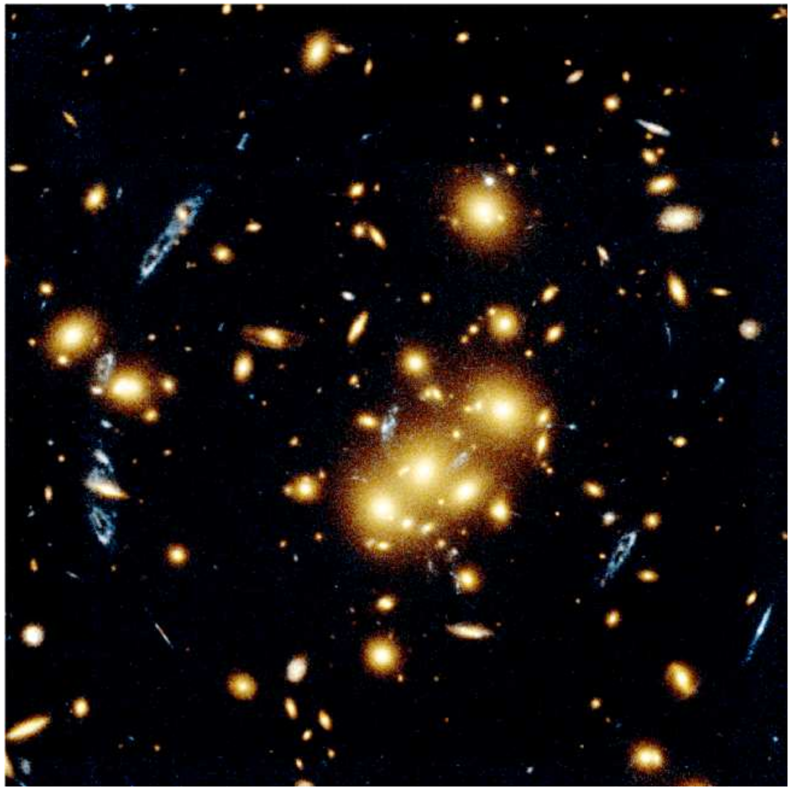
  Bose stars, axion clusters,.....

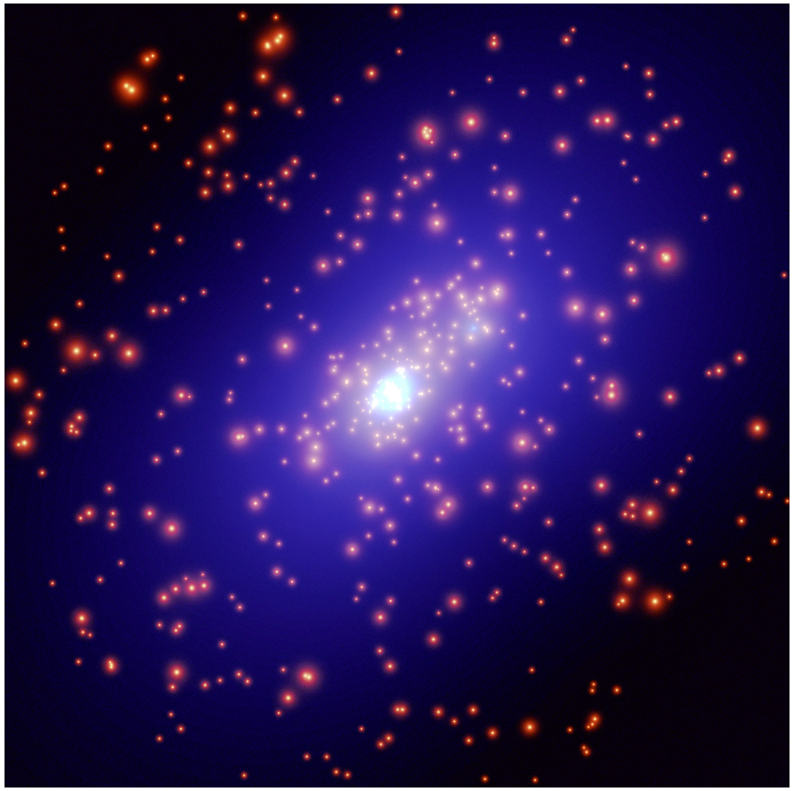
- Primordial black holes is yet another option (not yet ruled out!)

Interesting times ahead

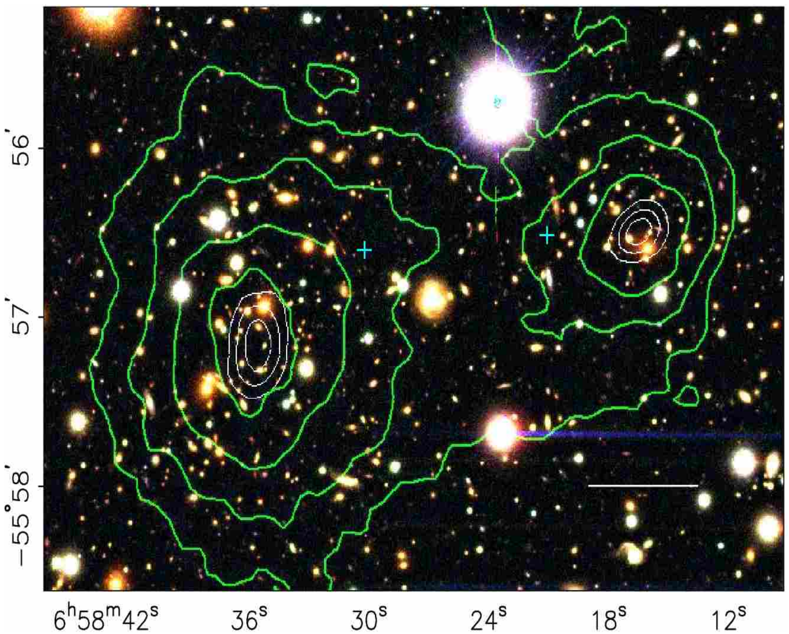
Backup

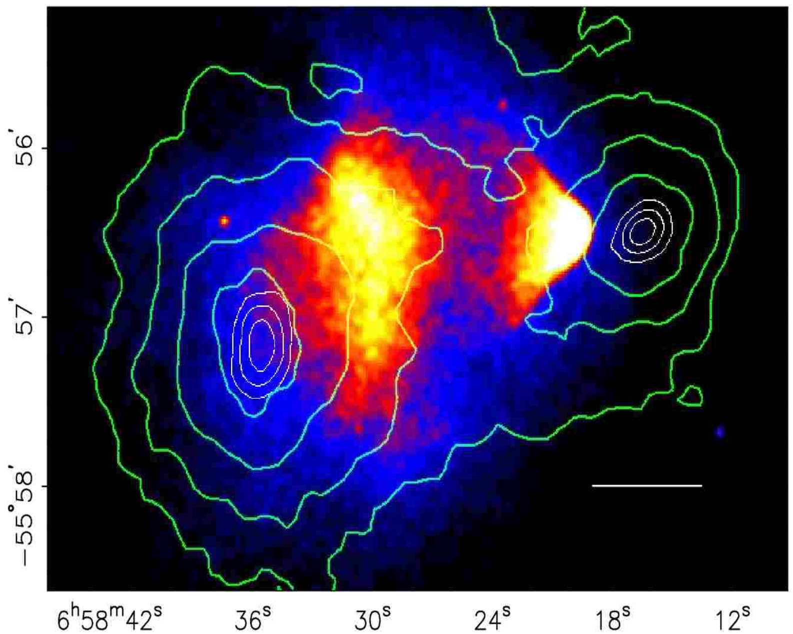
Gravitational lensing





Bullet cluster





Ω_{DM} from CMB angular spectrum.

Before recombination: density perturbation due to baryons and dark matter (Fourier):

$$\frac{\delta\rho}{\rho}(\vec{k},t) \equiv \delta(\vec{k},t) = \delta_B(\vec{k},t) + \delta_{DM}(\vec{k},t)$$

$\vec{k}$ = comoving momentum, constant in time; $\vec{p} = \vec{k}/a(t)$ = physical momentum, gets redshifted.

δ_B: sound wave in baryon-electron-photon plasma,

$$\delta_B(\vec{k},t) = A(\vec{k}) \cos\left(\int_0^t v_s \frac{k}{a(t)}\, dt\right)$$

v_s = sound speed ($\approx 1/\sqrt{3}$).

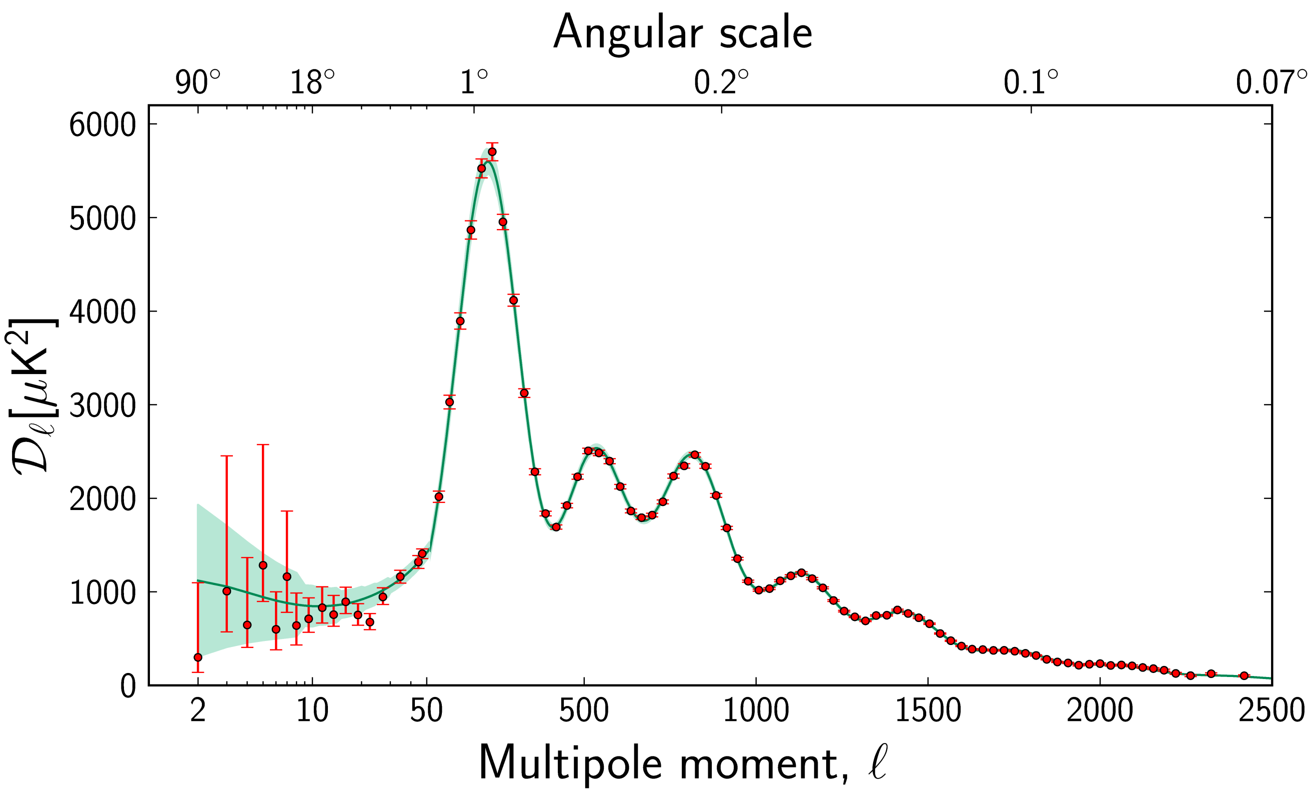
δ_{DM} nearly time-independent.

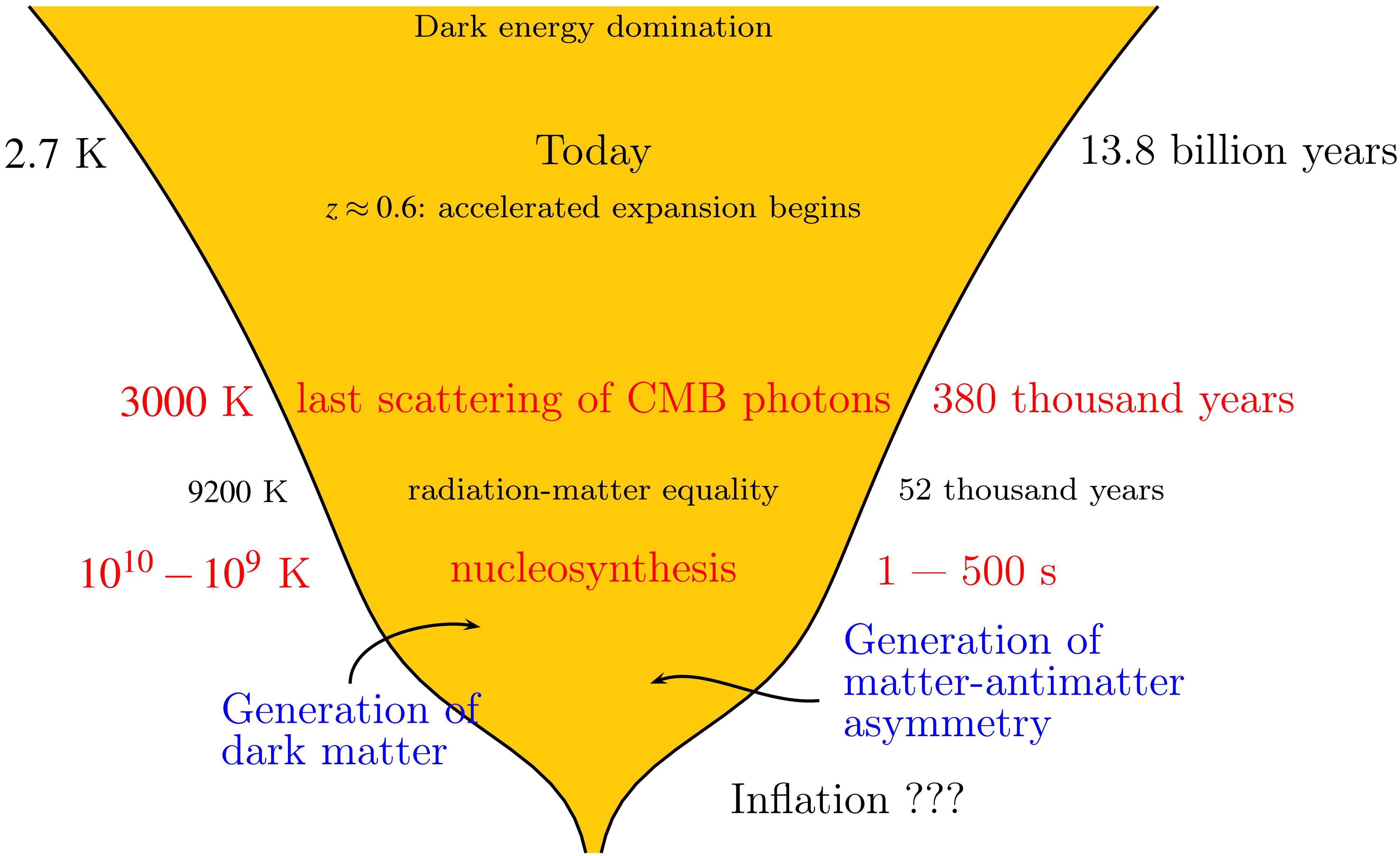
At recombination

$$\delta(\vec{k},t_r) = A(\vec{k})\cos\left(\int_0^{t_r} v_s \frac{k}{a(t)}\,dt\right) + \delta_{DM}(\vec{k},t_r)$$

Part that oscillates in k (due to baryon-photon plasma) + smooth part (due to dark matter)

Translates into oscillations + smooth part of $\delta T/T$ as function of multipole number l. Strong sensitivity to both Ω_B and Ω_{DM}.




Dark energy domination
2.7 K
Today
13.8 billion years
$z \approx 0.6$: accelerated expansion begins
3000 K
last scattering of CMB photons
380 thousand years
9200 K
radiation-matter equality
52 thousand years
$10^{10} - 10^{9}$ K
nucleosynthesis
1 — 500 s
Generation of matter-antimatter asymmetry
Generation of dark matter
Inflation ???

Simplified calculation of WIMP mass density

- Expansion at radiation domination
  - Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

$(M_{Pl} = G^{-1/2} = 10^{19}$ GeV$)$

  - Radiation energy density: Stefan–Boltzmann

$$\rho = \frac{\pi^2}{30} g_* T^4$$

g_*: number of relativistic degrees of freedom (about 100 in SM at $T \sim 100$ GeV). Hence

$$H(T) = \frac{T^2}{M_{Pl}^*}\,, \qquad M_{Pl}^* = \frac{M_{Pl}}{1.66\sqrt{g_*}}$$

- Number density of X-particles in chemical equilibrium at $T < M_X$: Maxwell–Boltzmann with chem. potential $\mu = 0$

$$n_X = g_X \int \frac{d^3p}{(2\pi)^3} \mathrm{e}^{-\frac{\sqrt{M_X^2+p^2}}{T}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{M_X}{T}}$$

- Mean free time wrt annihilation: travel distance $\tau_{ann} v$, meet one X particle to annihilate with in volume $\sigma \tau_{ann} v \Longrightarrow$

$$\sigma \tau_{ann} v n_X = 1 \qquad \Longrightarrow \qquad \tau_{ann} = \frac{1}{n_X \langle \sigma v \rangle}$$

- Freeze-out: $\tau_{ann}^{-1}(T_f) \sim H(T_f) \Longrightarrow \; n_X(T_f)\langle \sigma v \rangle \sim T_f^2/M_{Pl}^* \Longrightarrow$

$$T_f \simeq \frac{M_X}{\ln(M_X M_{Pl}^* \langle \sigma v \rangle)}$$

NB: large log $\Longleftrightarrow T_f \sim M_X/30$

Define $\langle \sigma v \rangle \equiv \sigma_0$ (constant for s-wave annihilation)

- Number density at freeze-out

$$n_X(T_f) = \frac{T_f^2}{\sigma_0 M_{Pl}^*}$$

- Number-to-entropy ratio at freeze-out and later on

$$\frac{n_X(T_f)}{s(T_f)} = \# \frac{n_X(T_f)}{g_* T_f^3} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g_* M_{Pl}^*}$$

where $\# = 45/(2\pi^2)$.

- Mass-to-enropy ratio

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Most relevant parameter: annihilation cross section $\sigma_0 \equiv \langle \sigma v \rangle$ at freeze-out