Moscow International School of Physics 2022, Dubna, 24 July – 2 August 2022

T-violation in neutrino spin-flavor oscillation probabilities due to influence of matter and electromagnetic field

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Abstract

T violation in neutrino oscillations in vacuum is a well known fact. Due to the CPT theorem this fact is strictly connected with the CP violating phase in the neutrino mixing matrix. Since neutrino interaction with matter or external electromagnetic field violates Lorentz invariance, influence of external conditions can lead to T violation even in the model with real mixing matrix, e.g. in the two-flavor model. We study neutrino propagation in dense matter and electromagnetic field taking into account both neutrino oscillations and spin rotation. We obtain a sufficient condition of T violation in the general case. In the two-flavor model we derive neutrino spin-flavor transition probabilities and discover a T-violating term, which arises due to simultaneous influence of electromagnetic field and moving matter.

Neutrino evolution equation

For massive Dirac neutrinos two different processes influence the particle evolution: neutrino oscillations and neutrino spin rotation. Both these processes can be described in the framework of quantum field theory within the modification of the Standard model, where fermions with equal electroweak quantum numbers are combined into multiplets [1]. Interaction with matter and electromagnetic field influences both oscillations and neutrino spin rotation. Quasiclassical evolution equation for neutrino interacting with matter and electromagnetic field was derived in [2]. In the most general form the evolution equation can be written as follows:

$$\left(\mathrm{i}\frac{d}{d\tau}-\mathcal{F}\right)\Psi(\tau)=0$$

In the two flavor model $\Psi(\tau)$ is an 8-component object, which in the mass representation consists of two bispinors. This object is transformed as a direct product of Poincarre group and SU(2). Matrix $\mathcal{F}$ includes the neutrino mass matrix and information about interaction with matter and electromagnetic field. The most general form of this matrix for neutrino interacting with matter and field corresponding to the Standard model prediction, can be found in [2]. The evolution matrix can be written as a matrix exponential. Explicit analytical expression for the evolution matrix can be obtained in several special cases [2,3].

Sufficient condition of T violation

Since the evolution matrix determines the neutrino wave function at any moment of time, we write the probabilities of transition between two different spin-flavor states using the the Backer-Campbell-Hausdorff formula as follows

$$W_{\alpha\to\beta}=\frac{1}{2u^0}\mathrm{Tr}\left\{e^{-\mathrm{i}\tau\mathcal{F}}\mathcal{P}_0^{(\alpha)}e^{\mathrm{i}\tau\mathcal{F}}\mathcal{P}_0^{(\beta)}(\gamma^\mu u_\mu+1)\gamma^0\right\}=\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-\mathrm{i}\tau)^n}{n!}\mathrm{Tr}\left\{D_n\mathcal{P}_0^{(\beta)}(\gamma^\mu u_\mu+1)\right\},$$

where $D_0=\mathcal{P}_0^{(\alpha)},\, D_1=[\mathcal{F},\mathcal{P}_0^{(\alpha)}],\ D_2=[\mathcal{F},[\mathcal{F},\mathcal{P}_0^{(\alpha)}]]\ldots$

Here $\mathcal{P}_0^{(\alpha)}$ and $\mathcal{P}_0^{(\beta)}$ are projection operators on the states with definite polarization and flavor, which can be presented as products of a flavor projection operator and a polarization projection operator. There exist the solutions of evolution equation in vacuum, which are characterized by constant values of parameter u^μ such that $u^2=1$ Therefore, quantum numbers u^μ can be interpreted as components of the neutrino 4-velocity.

T violation takes place, when there are nonzero terms of odd power in τ in the power expansion of probability. A nonzero value of the term containing τ^3 is a sufficient condition of T violation, since the term linear in τ is identically equal to zero. For neutrino in vacuum this condition is satisfied only when the mixing matrix is complex. In the general case the interaction with matter and electromagnetic field can lead to the presence of a nontrivial coefficient in front of τ^3 even when the CP-violating phase in the mixing matrix is equal to zero.

Using this condition to study all the possible situations with T violation is a rather complicated task. For this reason we will work in the two-flavor model, where the mixing matrix is real and is determined by one mixing angle. That is, we consider the case, when the T invariance breaking can be caused only by the influence of external conditions.

T violation in matter and electromagnetic field

In the most simple case, when neutrino interacts with matter via neutral currents, and the transition moments are not taken into account, matrix, which defines the evolution equation, takes the form

$$\mathcal{F}=\mathcal{M}+\frac{1}{2}(f^{(\mathrm{N})}u)+\frac{1}{2}R_{\mathrm{N}}\gamma^5\gamma^\sigma s_\sigma^{(\mathrm{N})}\gamma^\mu u_\mu-\mathcal{M}_d\gamma^5\gamma^\mu\,{}^\star F_{\mu\nu}u^\nu.$$

Here $\mathcal{M}$ is the mass matrix, and $\mathcal{M}_d$ is the matrix of diagonal neutrino magnetic moments. We assume multiplying the first two terms by a unity matrix from the algebra of Dirac matrices. Here $\gamma^5=-\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, tensor ${}^\star F_{\mu\nu}=-\frac{1}{2}e^{\mu\nu\rho\lambda}F_{\rho\lambda}$ is dual to the electromagnetic field tensor $F^{\mu\nu}$, and $e^{0123}=1$. The interaction with matter is determined by effective potential $f_\mu^{(\mathrm{N})}$. Here we use the following notations

$$s_\mu^{(\mathrm{N})}=\frac{u_\mu(f^{(\mathrm{N})}u)-f_\mu^{(\mathrm{N})}}{R_{\mathrm{N}}},\qquad R_{\mathrm{N}}=\sqrt{(f^{(\mathrm{N})}u)^2-(f^{(\mathrm{N})})^2}.$$

In this simple case the evolution matrix in the mass representation can be written in the explicit form

$$U(\tau)=\frac{1}{4}\sum_{\zeta=\pm1}\sum_{k=1,2}\exp\left\{-\mathrm{i}\tau\left[m_k+(f^{(\mathrm{N})}u)-\frac{1}{2}\zeta R_k\right]\right\}\times\Big(1-(-1)^k\sigma_3\Big)\big(1-\zeta\gamma^5\gamma_\mu s_k^\mu\big),$$

where the indices k = 1, 2 correspond to the neutrino mass states. Here the following notations are introduced

$$s_k^\mu=(u^\mu(f^{(\mathrm{N})}u)-f^{(\mathrm{N})\mu}-2\mu_k\,{}^\star F^{\mu\nu}u_\nu)/R_k,$$

$$R_k=\Big((f^{(\mathrm{N})}u)^2-(f^{(\mathrm{N})})^2+4\mu_k^2\,u^\mu\,{}^\star F_{\mu\alpha}\,{}^\star F^{\alpha\nu}u_\nu-4\mu_k f_\mu^{(\mathrm{N})}\,{}^\star F^{\mu\nu}u_\nu\Big)^{1/2}.$$

Using the evolution matrix we obtain the transition probabilities between state with flavor α and helicity ζ_α and with flavor β and helicity ζ_β $(\zeta_\alpha,\zeta_\beta=\pm1)$.

As a result, we obtain the spin-flavor transition probabilities in the two flavor model. T violation arises in the probabilities with spin rotation. For example, the transition between left-handed and right-handed neutrinos with electron flavor violates T symmetry. T-violating term is proportional to the value [3]

$$e_{\mu\nu\rho\lambda}u^\mu s_0^\nu s_1^\rho s_2^\lambda=2\frac{\mu_2-\mu_1}{R_1R_2}\frac{1}{|\mathbf{u}|}([\mathbf{u}\times\mathbf{f}]\cdot(u_0\mathbf{B}-[\mathbf{u}\times\mathbf{E}])).$$

That is, for neutrino in matter at rest the spin-flavor transition probabilities are T-invariant. However, even when neutrino interacts with moving matter only via neutral currents, simultaneous effect of matter and electromagnetic field leads to T violation in the two flavor model.

The results can be generalized to the three flavor model. In this case the probabilities with spin rotation include two T-violating terms: one term depends on the Jarlskog invariant, and the other is due to interaction with external conditions and depends on the real part of product of four matrix elements of the mixing matrix.

Antineutrino transitions

The same procedure can be performed for antineutrino propagation. As a result we obtain that within the two flavor model left-handed neutrino propagates in electromagnetic field in the same way as right-handed antineutrinos do. This corresponds to the invariance of the transition probabilities under CP operation performed for neutrino only. This result is rather surprising since electromagnetic interaction is known to be P even. When be compare left-handed neutrino propagation in electromagnetic field and matter composed of particle and antineutrino propagation in matter composed of antiparticles in the presence of electromagnetic field, we find out that the expressions of corresponding spin-flavor transition probabilities differ only in the sign of the T violating term. However, this is not a consequence of the CPT theorem, since the model under consideration is Lorentz-violating.

Conclusion

In the two-flavor model we derive the transition probabilities for neutrino in matter and electromagnetic field and obtain T violating terms. T violation implies a difference between the probabilities of transitions from the state α to the state β and from the state β to the state α ($W_{\alpha\to\beta}-W_{\beta\to\alpha}\neq0$). T violation in two-flavor neutrino oscillations becomes possible due to the combined effect of matter and electromagnetic field. In this case T violation is actually caused by the fact that the neutrino helicity is not conserved. Obviously, it cannot arise for two-flavor neutrino in matter at rest. That is, we reveal an extra source of T violation, which emerges due to collective effects and is not caused by complex entries in the mixing matrix. This source of T violation does not require introducing new symmetries, which imply either new particles to be included into the Standard Model or new properties of the presently known particles

Acknowledgements

A. V. Chukhnova is grateful to the Theoretical Physics and Mathematics Advancement Foundation "BASIS" for support (Grant No. 19-2-6-100-1)

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