Probing intermediate-scale Froggatt-Nielsen models at future gravitational wave observatories

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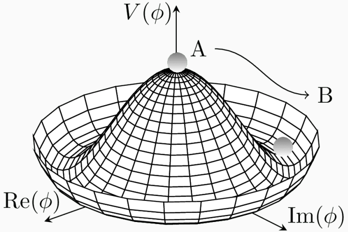
Motivation

Spontaneous symmetry breaking at zero temperature

- Consider a model featuring spontaneous symmetry breaking:

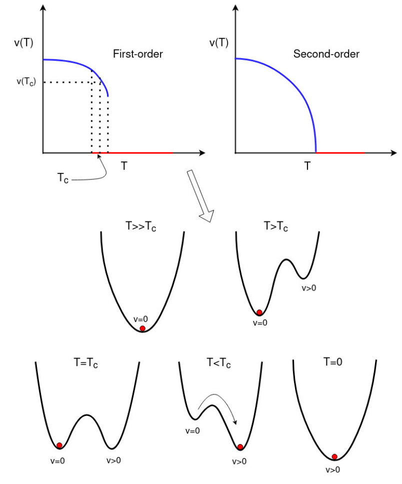
$$G \xrightarrow{v\neq 0} H.$$

- For example, in SM, the Higgs mechanism generates masses for all massive particles.
- The Higgs potential considered is the zero temperature effective potenial, $V_{\text{eff}}(\phi)$.
- $V_{\text{eff}}(\phi) = V_0 + V_1 + \cdots$
- Present temperature of the universe: $T = 2.3 \times 10^{-4}$ eV.
- So $T = 0$ is a good approximation today.
- However, not true in the hot early universe.



Phase transitions in particle physics

- At finite temperature T, $V_{\text{eff}}(\phi, T) \approx V_0(\phi) + V_1(\phi) + V_{1T}(\phi, T)$.
- The *vev* $(v \equiv \langle\phi\rangle)$ lies at the minimum of the effective potential, $V_{\text{eff}}(\phi, T)$.
- Generlaly at high T, $v \to 0$ due to thermal effects [Kirzhnitz, 1975] (symmetry restoration).
- As the universe cools, there is a phase transition from $v = 0$ to $v \neq 0$.
- Order parameter: $v(T)$.

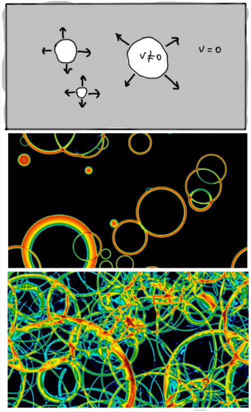


FOPT and GWs

- FOPT : bubbles of true phase nucleate in a sea of false phase.
- Tunnelling rate per unit volume [Callan, Coleman 1977, Linde 1977]

$$\Gamma(T) \approx T^4 e^{-\frac{S_3}{T}}.$$

- Bubbles expand rapidly, collide with each other.
- Strength of FOPT characterized by,
  1. T_n: $\left.\frac{\Gamma}{H^4}\right|_{T_n} \sim \mathcal{O}(1)$,
  2. Ratio of latent heat and radiation density, α,
  3. Rate of PT, $\beta \equiv -\left.\frac{dS}{dt}\right|_{t=t_*} = TH_* \left.\frac{dS}{dT}\right|_{T=T_*}$.
- GWs are produced by: bubble collisions, sound waves in the plasma, and MHD turbulence.



[D. Weir, 2017]

GW spectrum

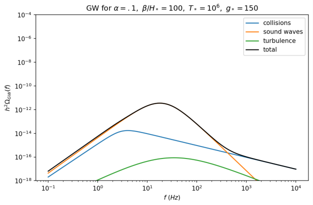
- GWs are produced by 3 processes:

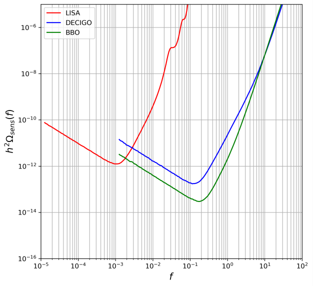
$$h^2\Omega_{\rm GW} \approx h^2\Omega_{\rm col} + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}.$$

- $\Omega_{\rm GW} \;\propto\; v_w \left(\frac{H_*}{\beta}\right)^x \left(\frac{\kappa\alpha}{1+\alpha}\right)^y \left(\frac{100}{g_*}\right)^{1/3} \mathcal{S}(f)$, $x, y \geq 0$.
- Large α, small β/H_* $\implies$ strong FOPT $\implies$ strong GW background.
- $f_{\rm peak} \propto T_n$.
- For detection time τ,

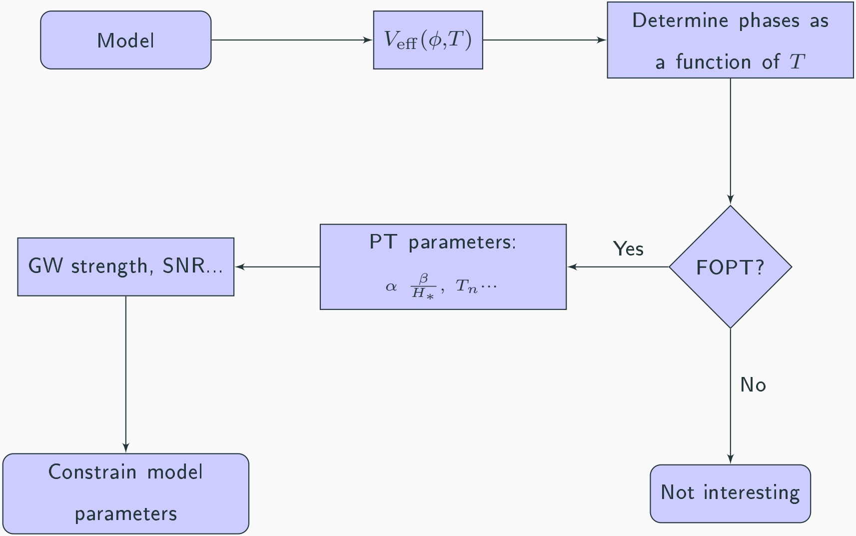
$$\mathrm{SNR} = \sqrt{\tau \int_{f_{\rm min}}^{f_{\rm max}} df \left[\frac{\Omega_{\rm GW}(f)h^2}{\Omega_{\rm sens}(f)h^2}\right]^2}$$

- Signal is detectable if $\mathrm{SNR} > \mathrm{SNR_{thr}}$.





Constraining models with GWs from FOPT



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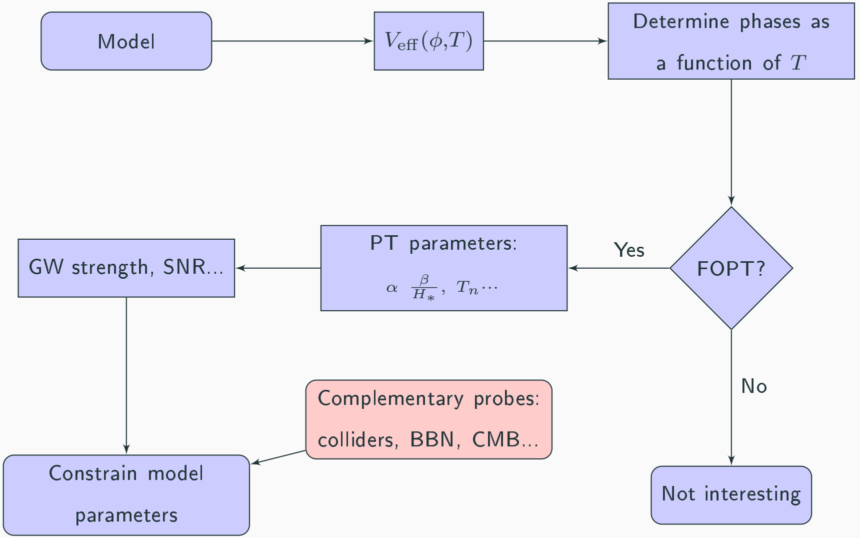


Flavon phase transition

Froggatt-Nielsen Mechanism[1]

- SM flavor puzzle:
  1. Quarks: $m_u : m_c : m_t \approx 1 : 550 : 7.5 \times 10^4$,
     $m_d : m_s : m_b \approx 1 : 20 : 870$.
  2. Leptons: $m_e : m_\mu : m_\tau \approx 1 : 200 : 3.5 \times 10^3$.
- One solution: FN. Assume a $U(1)_{\rm FN}$ flavor symmetry. In the quark sector,

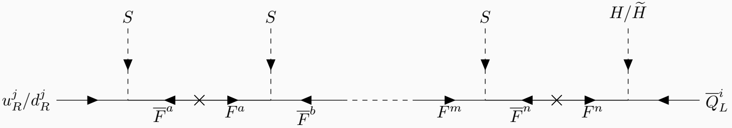
$$\mathcal{L}_{FN} = y^u_{ij} \left(\frac{S}{\Lambda}\right)^{n^u_{ij}} \overline{Q}_{Li} \tilde{H}\, u_{Rj} + y^d_{ij} \left(\frac{S}{\Lambda}\right)^{n^d_{ij}} \overline{Q}_{Li} H\, d_{Rj} + h.c.$$

  where S is called a flavon, and y_{ij} are $\mathcal{O}(1)$.
- $U(1)_{\rm FN}$ is spontaneously broken by, $\langle S \rangle = \epsilon\Lambda$ ($\epsilon < 1$), so that $Y_{ij} = y_{ij}\epsilon^{n_{ij}}$ are the effective Yukawa couplings.
- Bounds from neutral meson mixing and VLQ collider searches: $\Lambda \gtrsim$ few TeV.
- To avoid these bounds, we choose $\Lambda \gtrsim 10^5$ GeV.

[1] Froggatt, Nielsen, 1977

UV Completion

- $\mathcal{L}_{\rm UV} \supset -\overline{F^u}_L \mathcal{M}^u F^u_R - \overline{F^d}_L \mathcal{M}^d F^d_R + \text{h.c.}$



- Model 1: $\epsilon = \lambda \sim 0.23$,
  $Q_{\rm FN}(\overline{Q}_i) = (3, 2, 0)$, $Q_{\rm FN}(u_i) = (5, 2, 0)$, $Q_{\rm FN}(d_i) = (4, 3, 3)$,

$$Y^u_{ij} \sim \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix}, Y^d_{ij} \sim \begin{pmatrix} \epsilon^7 & \epsilon^6 & \epsilon^6 \\ \epsilon^6 & \epsilon^5 & \epsilon^5 \\ \epsilon^4 & \epsilon^3 & \epsilon^3 \end{pmatrix}$$

- $\det Y^u \sim \epsilon^{12}$, $\det Y^d \sim \epsilon^{15}$. Counts the number of VLQs.
- Model 2: $\epsilon = \lambda^2 \sim 0.05$, and
  $Q_{\rm FN}(\overline{Q}_L) = Q_{\rm FN}(u_R) = (2, 1, 0)$, $Q_{\rm FN}(d_R) = (1, 1, 1)$.
- $\det Y^u \sim \epsilon^6$, $\det Y^d \sim \epsilon^6$.

Effective Potential

- Extend SM by a complex scalar (CxSM), with a global $U(1)_{\rm FN}$ symmetry,

$$V_0(S,H) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \lambda_{HS}|S|^2|H|^2.$$

- $S = \frac{1}{\sqrt{2}}s\ e^{i\rho}$, $\langle s\rangle = v_s$. Can choose $\langle H\rangle = 0$.
- 1-loop potential: $V(s,T) = V_0(s) + V_1(s) + V_{1T}(s,T)$,

$$V_0(s) = -\frac{\mu_S^2}{2}s^2 + \frac{\lambda_S}{4}s^4,$$

$$V_{\rm CW}(s) = \sum_i \pm\frac{n_i}{64\pi^2}m_i^4(s)\left[\log\left(\frac{m_i^2(s)}{\mu^2}\right) - c_i\right],$$

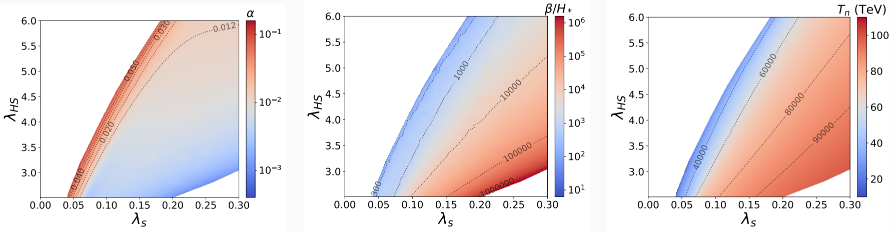
$$V_{1T}(s,T) = \sum_i n_i\frac{T^4}{2\pi^2}\left(\pm\int_0^\infty dy\ y^2\log\left(1\mp e^{-\sqrt{x^2+y^2}}\right)\right),$$

where $x^2 \equiv \frac{m_i^2(s)}{T^2}$.

- $m_i(s)$ are called field dependent masses.

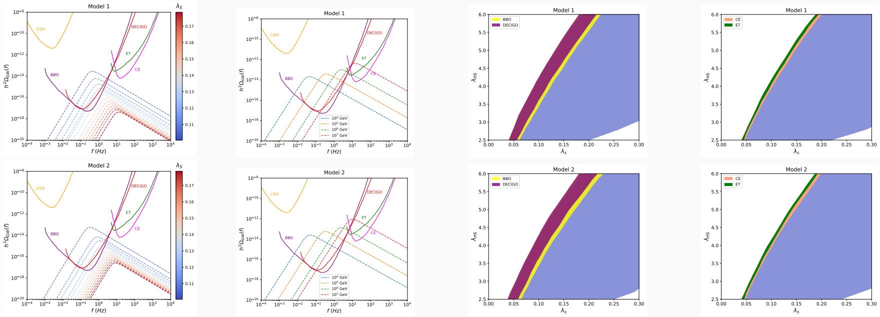
Parameter space

- We construct V_{eff} for the 2 UV completions, and analyse the PT.
- Parameters: $\{y,\ v_s,\ \lambda_S,\ \lambda_{HS}\}$, where y is the Yukawa for $\mathcal{L}_{\text{UV}}$.
- Range: $\lambda_{HS} \in [2.5, 6]$, $\lambda_S \in [0, 0.3]$, with $y = 0.5$.
- To avoid collider bounds, take $v_s = 10^{4-7}$ GeV.



FOPT parameter space for model 1. $y = 0.5$, and $v_s = 10^5$ GeV.

- $h^2\Omega_{\rm GW} \approx h^2\Omega_{\rm col}$
- Possibly observable signals at BBO and DECIGO ($v_s \sim 10^5$ GeV), or CE and ET ($v_s \sim 10^7$ GeV).
- GWs do not disriminate between the 2 models.



Outlook

Conclusions

- GWs are a promising window to the early universe.
- GWs from FOPT can probe symmetry-breaking scales upto 10^7 GeV at upcoming observatories.
- Model discrimination is difficult, but complementary signatures help.
- Interesting problems:
  1. Supercooled PTs lead to exciting possibilities such as PBH production.
  2. Could the NANOGrav signal be due to FOPT?
  3. Identify new methods to discriminate models.
  4. FOPTs combined with topological defects.

Thank You!

Effective potential

- Background field value of the neutral CP-even field, χ^0_{Rr}: R.
- For $SU(2)_R \times U(1)_{B-L}$ breaking PT, $V_{\text{eff}} \equiv V_{\text{eff}}(R, T)$.
- The effective potential is,

$$
\begin{aligned}
V_{\text{eff}}(R,T) &= V_0 + V_{\text{CW}} + V_{\text{c.t.}} + V_{1T} + V_{\text{Daisy}} \\
V_0(R) &= -\frac{\mu_3^2}{2} R^2 + \frac{\rho_1}{4} R^4, \\
V_{\text{CW}}(R) &= \frac{1}{64\pi^2} \sum_i (-1)^{f_i} n_i m_i^4(R) \left[\log\left(\frac{m_i^2(R)}{\mu^2} \right) - c_i \right], \\
V_{1T}(R,T) &= \frac{T^4}{2\pi^2} \sum_i (-1)^{f_i} n_i J_{b/f}\left(\frac{m_i^2}{T^2} \right), \\
V_{\text{Daisy}} &= -\frac{T}{12\pi} \sum_i n_i \left((m_i^2(R) + \Pi_i(T))^{3/2} - (m_i^2(R))^{3/2} \right).
\end{aligned}
$$

- $V_{\text{c.t.}}$ is chosen such that the one-loop *vev* and mass coincide with the tree-level values.

Large T expansion of V_{1T}

$$J_{b/f}(x^2) = \pm \int_0^\infty dy\; y^2 \log\left(1 \mp e^{-\sqrt{x^2+y^2}}\right).$$

For small x^2,

$$\begin{aligned}
J_f(x^2, n) = & - \quad \frac{7\pi^4}{360} + \frac{\pi^2}{24}x^2 + \frac{1}{32}x^4\left(\log x^2 - c_f\right) \\
& - \quad \pi^2 x^2 \sum_{l=2}^{n} \left(-\frac{1}{4\pi^2}x^2\right)^l \frac{(2l-3)!!\zeta(2l-1)}{(2l)!!(l+1)} \left(2^{2l-1} - 1\right), \\
J_b(x^2, n) = & - \quad \frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 - \frac{\pi}{6}\left(x^2\right)^{3/2} - \frac{1}{32}x^4\left(\log x^2 - c_b\right) \\
& + \quad \pi^2 x^2 \sum_{l=2}^{n} \left(-\frac{1}{4\pi^2}x^2\right)^l \frac{(2l-3)!!\zeta(2l-1)}{(2l)!!(l+1)}.
\end{aligned}$$

Small T expansion of V_{1T}

For large x^2, both fermions and bosons have the same expansion,

$$J_{b/f}(x^2,n) = -\exp\left(-(x^2)^{1/2}\right)\left(\frac{\pi}{2}(x^2)^{3/2}\right)^{1/2} \sum_{l=0}^{n} \frac{1}{2^l l!} \frac{\Gamma(5/2+l)}{\Gamma(5/2-l)} (x^2)^{-l/2}.$$

Daisy Resummation

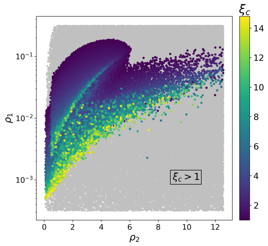
- Particles receive thermal (Debye mass) due to 'hard loops'[2]. Scalars, longitudinal component of gauge bosons and fermions get thermal mass. For $SU(N)$,

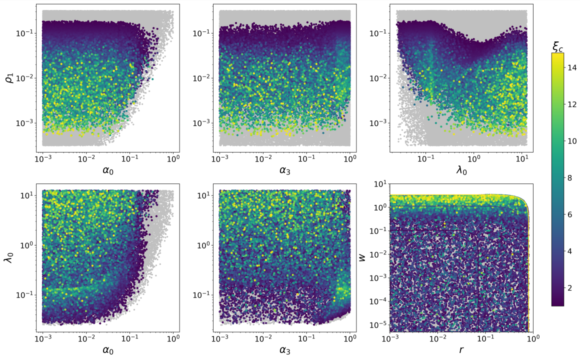
$$\Pi^L = g_N^2 T^2 \left[\frac{N}{3} + \frac{1}{6} \sum_s n_s C(r_s) + \frac{1}{12} \sum_f n_f C(r_f) \right].$$

- For N-loop diagrams with $N-1$ petals, $\Pi_{\text{daisy}} \sim \lambda^N \frac{T^{2N-1}}{\mu^{2N-3}}$.
- Breakdown of perturbativity: At $T_c \sim \frac{\mu}{\sqrt{\lambda}}$, parameter $\alpha \equiv \lambda \frac{T^2}{\mu^2} \sim 1$.
- Resummed contribution from daisy diagrams $\implies m^2 \to m^2 + \Pi(T)$.
- Effect is to shift $V_T \to V_T + V_{\text{daisy}}$.
- Generally, perturbation theory can be applied is $\frac{\phi_c}{T_c} \gtrsim 1$.

[2] Thesis-Breitbach, 2018

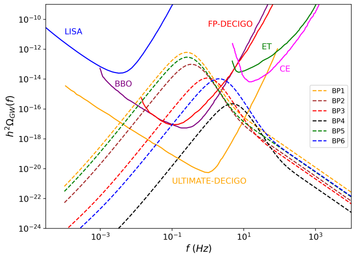
Other projections





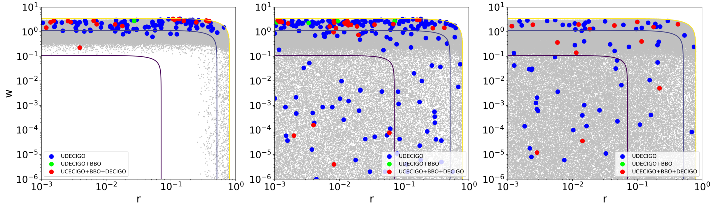
GW Detection

- For detection time τ, $\mathrm{SNR} = \sqrt{\tau \int_{f_{\min}}^{f_{\max}} df \left[\frac{\Omega_{\mathrm{GW}}(f)h^2}{\Omega_{\mathrm{sens}}(f)h^2}\right]^2}$
- Signal is detectable if $\mathrm{SNR} > \mathrm{SNR}_{\mathrm{thr}}$.
- Six BPs were identified with high SNR at BBO, FP-DECIGO and Ultimate DECIGO.



GW spectra for the benchmark points

Detectable points in the $r - w$ plane



GW spectra for the benchmark points

H_3 production channels:

1. H_1-decay, $pp \to H_1 \to hH_3$,
2. decay of boosted h, $pp \to h^* \to hH_3, H_3H_3$,
3. Higgsstrahlung, $pp \to V_R^* \to V_R H_3$,
4. $V_R V_R$ fusion, $pp \to H_3 jj$.