The scaling limit of the XXZ spin chain and integrable structures in CFT

Moscow International School of Physics

Ivan Gusev
Alexey Litvinov

March 4, 2024

Skoltech
Skolkovo Institute of Science and Technology

Presentation Plan

- Introduction
- Research Problem
- Goals and objectives
- Methods
- Results

Introduction

Integrable models

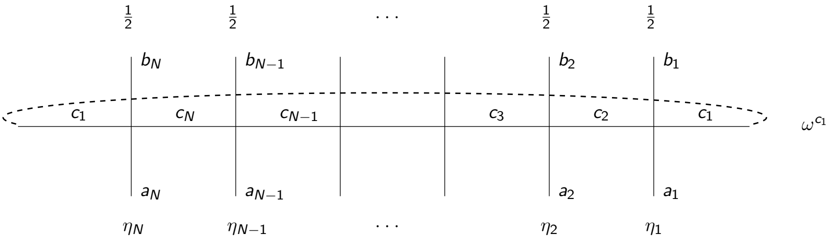
1. Integrable CFT with $\mathcal{Z}_r$ symmetry and BLZ approach
2. CFT with $\mathfrak{sl}(n)$ symmetry and auxiliary non-interacting bosonic field

Skoltech
Skolkovo Institute of Science and Technology

Introduction

Spin-$\frac{1}{2}$ chain with $\mathcal{Z}_r$ symmetry

Transfer matrix $(\mathbb{T}(\zeta))^{b_N b_{N-1} \dots b_2 b_1}_{a_N a_{N-1} \dots a_2 a_1}$



Introduction

Commuting family

- Hamiltonian: $\ell = 1, \ldots, r$

$$\mathbb{H}^{(\ell)} = 2\mathrm{i}\zeta\partial_\zeta \log(\mathbb{T}(-q^{-1}\zeta))|_{\zeta=\eta_\ell} - 2\mathrm{i}\sum_{J=1}^{N}(1 - q^2\eta_J/\eta_\ell)^{-1}$$

- Operators $\mathbb{T}$, $\mathbb{H}$, $\mathbb{Q}$, $\mathbb{S}^z$, etc.
  1. act in the "quantum space" $\mathscr{V}_N = \mathbb{C}^2_N \otimes \mathbb{C}^2_{N-1} \otimes \cdots \otimes \mathbb{C}^2_1$
  2. belong to commuting family

Research Problem

The key problem of this project is the solution of non-linear Bethe ansatz equations (BAE) in the scaling limit:

$$\prod_{J=1}^{N} \frac{\eta_J + q\zeta_m}{\eta_J + q^{-1}\zeta_m} = -\mathrm{e}^{2\pi \mathrm{i} \mathrm{k}} q^{2S^z} \prod_{j=1}^{M} \frac{\zeta_j - q^2\zeta_m}{\zeta_j - q^{-2}\zeta_m}$$

$$m = 1, 2, \ldots, M, \quad S^z = \frac{1}{2}N - M \geq 0$$

The parameter $q = \mathrm{e}^{i\gamma}$ is called anisotropy, and the set of complex numbers $\{\eta_J\}_{J=1}^{N}$ with the periodic condition $\eta_{J+r} = \eta_J$ are the inhomogeneities.

Goals and objectives

- Construct the Hamiltonian spectrum of an inhomogeneous six-vertex model with $\mathcal{Z}_r$-symmetry
- Describe ground and low-energy excited states

Methods

1. Constructed and diagonalized large sparse $\mathbb{H}$ and $\mathbb{Q}$ matrices via Wolfram Mathematica and Python
2. Solved invariant Bethe Ansatz Equations for a small lattice size $N \sim 20$
3. Using the Bethe ansatz equations, extended the RG trajectory of eigenstates up to $N \gg 1$ without explicit construction/diagonalization $\mathbb{H}$ and $\mathbb{Q}$
4. Perturb BAE and find a new solution near the invariant one

Results

Spin-$\frac{1}{2}$ chain with $\mathcal{Z}_1$ symmetry

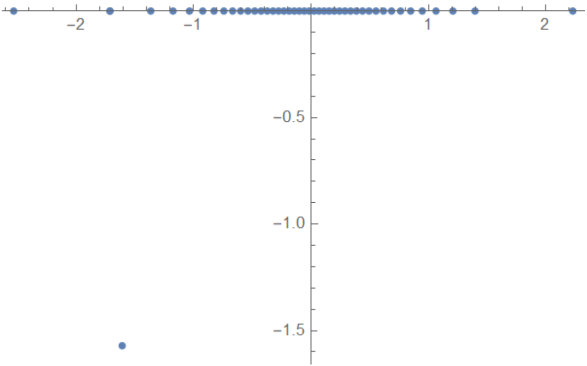


Figure 1: Distribution of roots of the XXZ chain at $N = 90$, $k = \frac{1}{10}$, $S^z = 0$, $L = 2$, $\bar{L} = 0$, $w = 1$, $\beta^2 = 2/5$

Skoltech
Skolkovo Institute of Science and Technology

Results

Spin-$\frac{1}{2}$ chain with $\mathcal{Z}_1$ symmetry

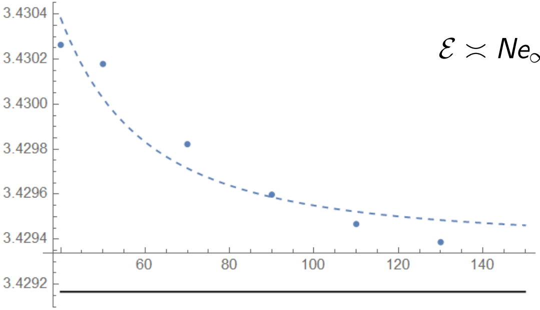


Figure 2: Reduced energy values $\frac{N}{2\pi v_F}(\mathcal{E}-\mathcal{E}_0)$ for the state $L=2$

$$\mathcal{E} \asymp N e_\infty + \frac{2\pi v_F}{N}(\Delta+\bar{\Delta}+L+\bar{L}-\frac{1}{12}) + o\left(\frac{1}{N}\right)$$

Skoltech
Skolkovo Institute of Science and Technology

Results

Spin-$\frac{1}{2}$ chain with $\mathcal{Z}_2$ symmetry

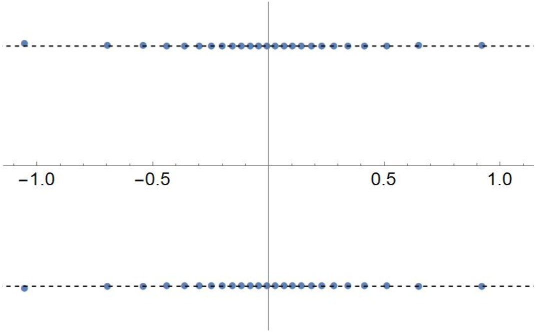


Figure 3: The parameters taken for the figure $S^z = 0$, $\gamma = \frac{143}{200}$, $\eta_1 = \eta_2^{-1} = \mathrm{e}^{\frac{13\mathrm{i}}{10}}$, $\mathrm{k} = \frac{1}{10}$

Results

Spin-$\frac{1}{2}$ chain with $\mathcal{Z}_3$ symmetry

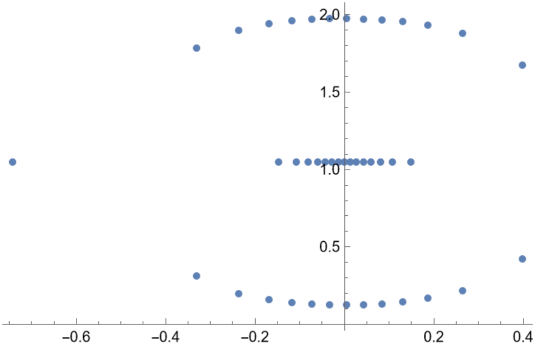


Figure 4: The parameters taken for the figure $S^z = 0$, $\gamma = \frac{\pi}{5}$, $\eta_1 = \mathrm{e}^{-\frac{2\pi \mathrm{i}}{3}}$, $\eta_2 = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}\epsilon}$, $\eta_3 = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}(1-\epsilon)}$, $\epsilon = \frac{1}{10}$, $\mathrm{k} = \frac{1}{20}$

Skoltech
Skolkovo Institute of Science and Technology

Thank you for attention!