Cosmology and particle physics
Lecture #2
Observables in the Hot Big Bang model

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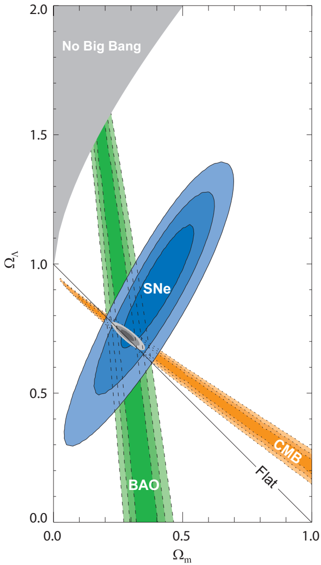
Outline

Astrophysical and cosmological data are in agreement



$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3}\, G\rho^{\text{energy}}_{\text{density}}$$

$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho^{\text{ordinary}}_{\text{matter}} + \rho^{\text{dark}}_{\text{matter}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}$$

$$\frac{3H_0^2}{8\pi G} = \rho^{\text{energy}}_{\text{density}}(t_0) \equiv \rho_c \approx 0.53\times 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}$$

radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5\times 10^{-4}$

Baryons (H, He): $\Omega_\mathrm{B} \equiv \frac{\rho_\mathrm{B}}{\rho_c} = 0.05$

Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

Dark matter: $\Omega_{\mathrm{DM}} \equiv \frac{\rho_{\mathrm{DM}}}{\rho_c} = 0.27$

Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$

Determination of $a(t)$ reveals the composition of the present Universe

$\Delta s^2 = c^2\,\Delta t^2 - a^2(t)\,\Delta\vec{x}^2 \;\to\; ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu$

Light propagation changes. . .

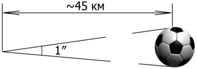
How do we check it?

by measuring distance L to an object!

- Measuring angular size θ of an object of known size d

single-type galaxies

$$\theta = \frac{d}{L}$$



- Measuring angular size $\theta(t)$ corresponding to physical size $d(t)$ with known evolution

– BAO in galaxy distribution
– lensing of CMB anisotropy

$$\theta(t) = \frac{d(t)}{L}$$

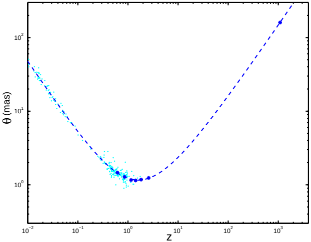
- Measuring brightness J of an object of known luminosity F

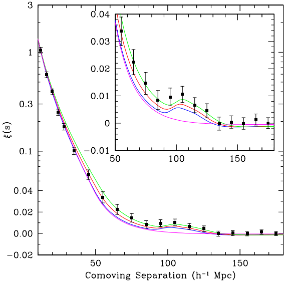
“standard candles”

$$J = \frac{F}{4\pi L^2}$$

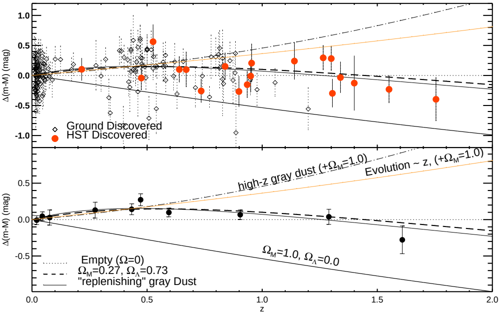
In the expanding Universe all these laws get modified

Results of distance measurements





$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c = 0.8, \Omega_M = 0.2)}$$



Photons in the expanding Universe

$$S = -\frac{1}{4}\int d^4x \sqrt{-g} g^{\mu\nu} g^{\lambda\rho} F_{\mu\lambda} F_{\nu\rho}$$

$dt = a d\eta$ conformally flat metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \longrightarrow ds^2 = a^2(\eta)[d\eta^2 - \delta_{ij}dx^i dx^j]$$

$$S = -\frac{1}{4}\int d^4x\, \eta^{\mu\nu}\eta^{\lambda\rho} F_{\mu\lambda} F_{\nu\rho}\,, \qquad A_\mu^{(\alpha)} = e_\mu^{(\alpha)} \mathrm{e}^{ik\eta - i\mathbf{kx}}\,, \quad k = |\mathbf{k}|$$

$$\Delta x = 2\pi/k\,, \quad \Delta\eta = 2\pi/k$$

$$\lambda(t) = a(t)\Delta x = 2\pi\frac{a(t)}{k}\,, \quad T = a(t)\Delta\eta = 2\pi\frac{a(t)}{k}$$

Redshift and the Hubble law $\lambda_0 = \lambda_i \frac{a_0}{a(t_i)} \equiv \lambda_i(1+z(t_i))$

$$\mathbf{p}(t) = \frac{\mathbf{k}}{a(t)}\,, \quad \omega(t) = \frac{k}{a(t)}$$

for not very distant objects $1\,\text{pc} \approx 3\,\text{ly}$

$$a(t_i) = a_0 - \dot{a}(t_0)(t_0 - t_i) \quad \longrightarrow \quad a(t_i) = a_0[1 - H_0(t_0 - t_i)]$$

$$z(t_i) = H_0(t_0 - t_i) = H_0 r\,, \qquad z \ll 1$$

$$H_0 = h \cdot 100\,\frac{\text{km}}{\text{s}\cdot\text{Mpc}}\,, \qquad h \approx 0.68$$

similar reddening for other relativistic particles (small H, $\dot{H}$, etc.)

$\mathbf{p} = \frac{\mathbf{k}}{a(t)}$ is true for massive particles as well

Gas of free particles in the expanding Universe

homogeneous gas $dN = f(\mathbf{p}, t) d^3\mathbf{X} d^3\mathbf{p}$

in comoving coordinates:

$$d^3\mathbf{x} = \text{const}\,, \quad d^3\mathbf{k} = \text{const}\,, \quad f(k) = \text{const}$$

$$f(k) d^3\mathbf{x} d^3\mathbf{k} = \text{const}$$

comoving volume equals physical volume

$$d^3\mathbf{x} d^3\mathbf{k} = d^3(a\mathbf{x}) d^3\left(\frac{\mathbf{k}}{a}\right) = d^3\mathbf{X} d^3\mathbf{p}$$

$$f(\mathbf{p}, t) = f(\mathbf{k}) = f[a(t) \cdot \mathbf{p}]\,.$$

$$t = t_i \;:\quad f_i(\mathbf{p}) \;\longrightarrow\; f(\mathbf{p}, t) = f_i\left(\frac{a(t)}{a(t_i)}\mathbf{p}\right)$$

Massless bosons (photons) fermions

$$f_i(\mathbf{p}) = f_{\rm Pl}\left(\frac{|\mathbf{p}|}{T_i}\right) = \frac{1}{(2\pi)^3}\frac{1}{e^{|\mathbf{p}|/T_i}-1}$$

$$f(\mathbf{p},t) = f_{\rm Pl}\left(\frac{a(t)|\mathbf{p}|}{a_i T_i}\right) = f_{\rm Pl}\left(\frac{|\mathbf{p}|}{T_{eff}(t)}\right)$$

$$T_{eff}(t) = \frac{a_i}{a(t)} T_i$$

decoupling at $T \gg m$: neutrinos, hot(warm) dark matter

decoupling at $T \ll m$: $f(\mathbf{p}) = \frac{1}{(2\pi)^3}\exp\left(-\frac{m-\mu_i}{T_i}\right)\exp\left(-\frac{a^2(t)\mathbf{p}^2}{2ma_i^2 T_i}\right)$

$$f(\mathbf{p},t) = \frac{1}{(2\pi)^3}\exp\left(-\frac{m-\mu_{eff}}{T_{eff}}\right)\exp\left(-\frac{\mathbf{p}^2}{2mT_{eff}}\right)$$

$$T_{eff}(t) = \left(\frac{a_i}{a(t)}\right)^2 T_i\,, \qquad \frac{m-\mu_{eff}(t)}{T_{eff}} = \frac{m-\mu_i}{T_i}$$

Einstein equations

$T_{\mu\nu}$: macroscopic description $\frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}$

$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - g_{\mu\nu}p$ ideal fluid with $\rho(t)$ and $p(t)$

in the comoving frame $u^0 = 1$, $\mathbf{u} = 0$ (almost) always works

$$T_\mu^\nu = diag(\rho,\, -p)$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j\ ,$$

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R\ :\quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$(00):\quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$$

Dark Energy: all evidences are from cosmology

Working hypothesis is cosmological constant $\Lambda \approx \left(2.5\times 10^{-3}\,\text{eV}\right)^4$:
$p = w(t)\,\rho$, $w = \text{const} = -1$, $\rho = \Lambda$

$$S_\Lambda = -\Lambda \int d^4x\,\sqrt{-\det g_{\mu\nu}}$$

both parts contribute

$$S_{\text{grav}} = -\frac{1}{16\pi G}\int d^4x\,\sqrt{-\det g_{\mu\nu}}\,R\,,$$
$$S_{\text{matter}} = \int d^4x\,\sqrt{-\det g_{\mu\nu}}\left(\frac{1}{2}g^{\lambda\rho}\partial_\lambda\phi\,\partial_\rho\phi - V(\phi)\right)$$

natural values

$$\Lambda_{\text{grav}} \sim 1/G^2 \sim \left(10^{19}\,\text{GeV}\right)^4\,,\quad \Lambda_{\text{matter}} \sim V(\phi_{vac}) \sim (100\,\text{GeV})^4, (100\,\text{MeV})^4, \dots$$

Why Λ is small? Why $\Lambda \sim \rho_{\text{matter}}$? Why $\rho_B \sim \rho_{DM} \sim \rho_\Lambda$ today?

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\left(\rho_{\rm M} + \rho_{rad} + \rho_\Lambda + \rho_{\rm curv}\right)$$

$$\frac{8\pi}{3} G\rho_{\rm curv} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{{\rm M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.52 \cdot 10^{-5} \frac{\rm GeV}{\rm cm^3}\,, \qquad \text{for } \; h = 0.7$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_{\rm M}\left(\frac{a_0}{a}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv}\left(\frac{a_0}{a}\right)^2\right]$$

Brightness–redshift dependence in the Universe

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \sinh^2\chi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

coordinate distance $\rho/R \to \chi = \int_{t_i}^{t_0} \frac{dt}{a(t)}$ $\qquad z(t) = \frac{a_0}{a(t)} - 1$

$$\chi(z) = \int_0^z \frac{dz'}{a_0 H_0} \frac{1}{\sqrt{\Omega_M (z'+1)^3 + \Omega_\Lambda + \Omega_{curv}(z'+1)^2}}$$

$$a_0^2 H_0^2 \Omega_{curv} = 1\,, \quad \Omega_M + \Omega_\Lambda + \Omega_{curv} = 1$$

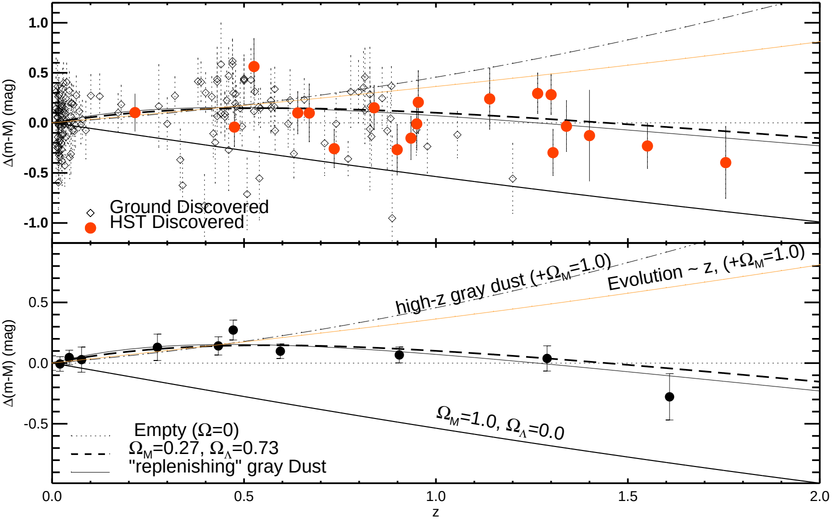
$$S(z) = 4\pi r^2(z)\,, \qquad r(z) = a_0 \sinh\chi(z)$$

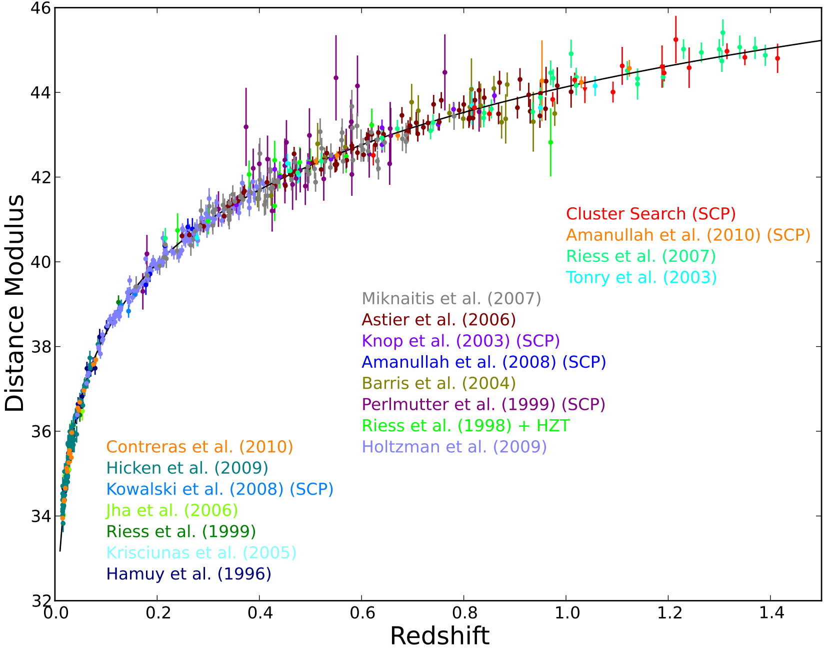
detector: $N_\gamma \propto S^{-1}$, $\omega = \omega_i/(1+z)$, $dt_0 = (1+z)dt_i$
hence the brightness (energy flux measured by a detector) is

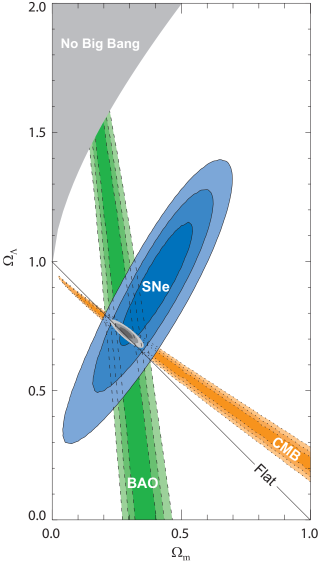
$$J = \frac{L}{(1+z)^2 S(z)} \equiv \frac{L}{4\pi r_{ph}^2}\,, \quad r_{ph} = (1+z)\cdot r(z)$$

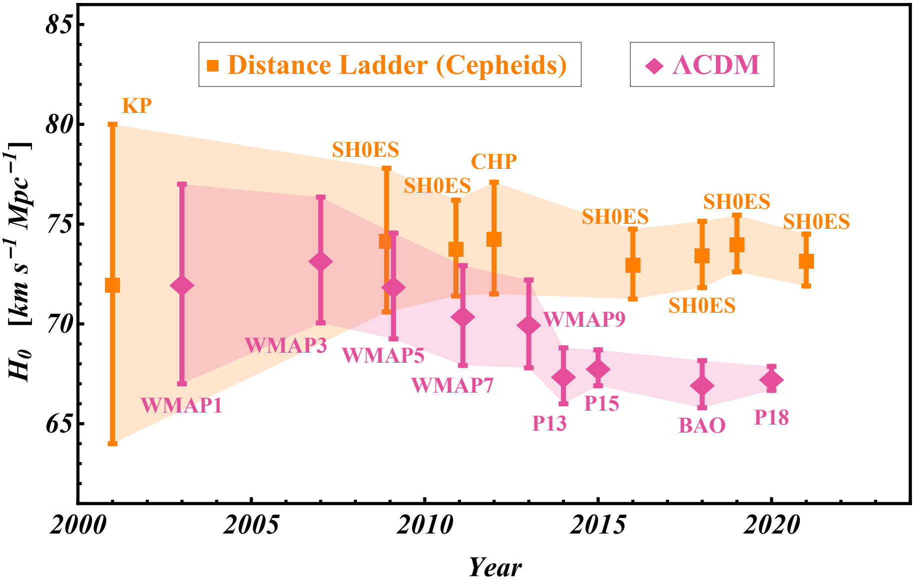
Brightness–redshift dependence: SNe Ia

$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c = 0.8, \Omega_M = 0.2)}$$

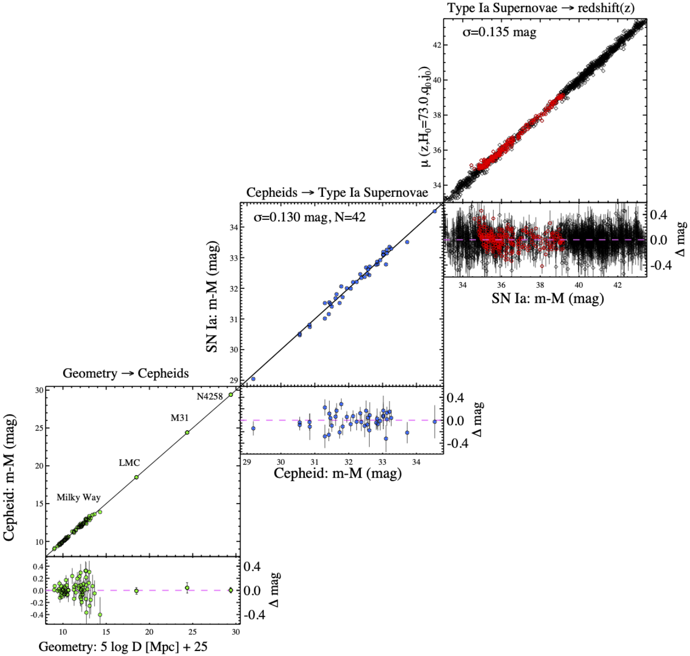



Distance Modulus
Redshift
46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)

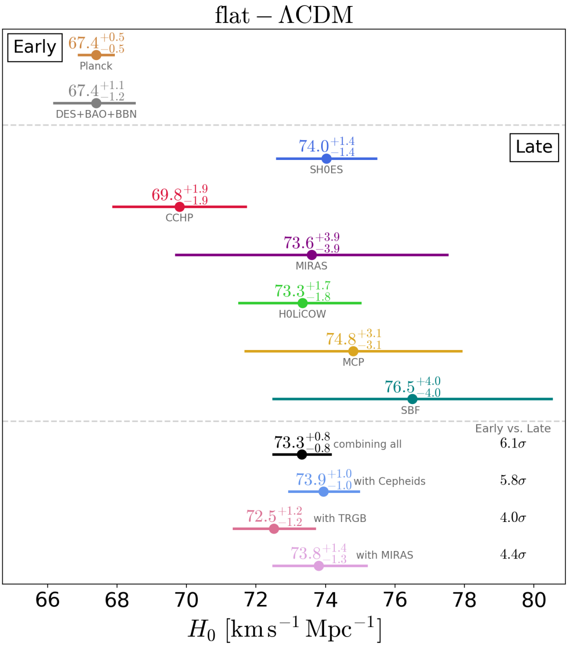

2.0
1.5
1.0
0.5
0.0
Ω_Λ
0.0
0.5
1.0
Ω_m
No Big Bang
SNe
CMB
BAO
Flat


■ Distance Ladder (Cepheids)
◆ ΛCDM
KP
SH0ES
SH0ES
CHP
SH0ES
SH0ES
SH0ES
SH0ES
SH0ES
WMAP1
WMAP3
WMAP5
WMAP7
WMAP9
P13
P15
BAO
P18
H_0 [$km\ s^{-1}\ Mpc^{-1}$]
85
80
75
70
65
2000
2005
2010
2015
2020
Year

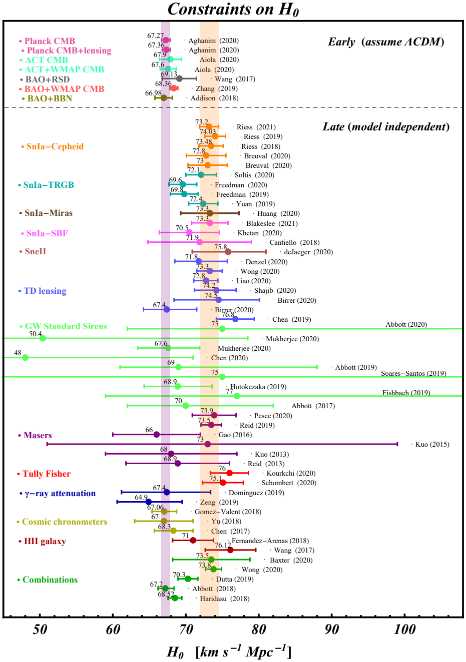
2105.05208


Type Ia Supernovae → redshift(z)
σ=0.135 mag
$\mu\,(z,H_0=73.0,q_0,j_0)$
42
40
38
36
34
0.4
0.0
-0.4
Δ mag
SN Ia: m-M (mag)
Cepheids → Type Ia Supernovae
σ=0.130 mag, N=42
SN Ia: m-M (mag)
34
33
32
31
30
29
Cepheid: m-M (mag)
Geometry → Cepheids
N4258
M31
LMC
Milky Way
Cepheid: m-M (mag)
30
25
20
15
10
0.4
0.2
0.0
-0.2
-0.4
Geometry: 5 log D [Mpc] + 25

2211.04492


flat − ΛCDM
Early
$67.4^{+0.5}_{-0.5}$
Planck
$67.4^{+1.1}_{-1.2}$
DES+BAO+BBN
Late
$74.0^{+1.4}_{-1.4}$
SH0ES
$69.8^{+1.9}_{-1.9}$
CCHP
$73.6^{+3.9}_{-3.9}$
MIRAS
$73.3^{+1.7}_{-1.8}$
H0LiCOW
$74.8^{+3.1}_{-3.1}$
MCP
$76.5^{+4.0}_{-4.0}$
SBF
Early vs. Late
$73.3^{+0.8}_{-0.8}$ combining all
6.1σ
$73.9^{+1.0}_{-1.0}$ with Cepheids
5.8σ
$72.5^{+1.2}_{-1.2}$ with TRGB
4.0σ
$73.8^{+1.4}_{-1.3}$ with MIRAS
4.4σ
66
68
70
72
74
76
78
80
H_0 [km s^{-1} Mpc^{-1}]

1907.10625



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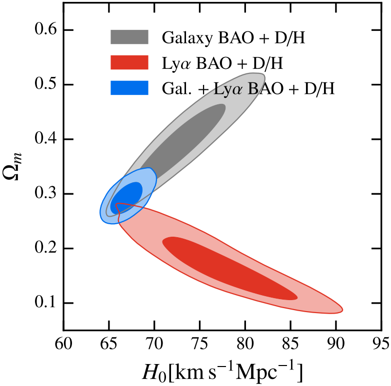

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Last scattering: $\gamma e \to \gamma e$

$$\sigma_{\rm T} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} \approx 0.67\cdot 10^{-24}\,\text{cm}^2\,, \qquad \tau_\gamma = \frac{1}{\sigma_{\rm T}\cdot n_e(T)}$$

last scattering: $\tau_\gamma(T_f) \simeq H^{-1}(T_f) \simeq t_f$

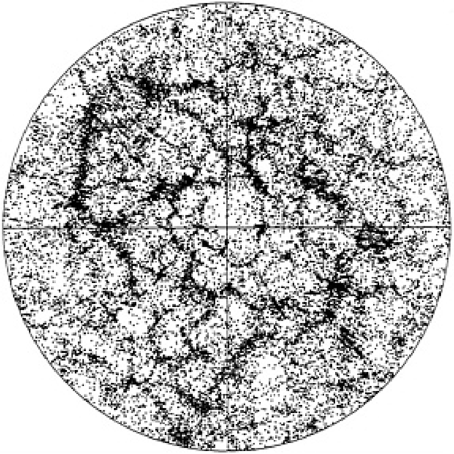
$$T_f = 0.26\ \text{eV}\,, \quad z = 1100\,, \quad t_f = 370\,000\ \text{yr}$$

for general processes one should solve kinetic equations

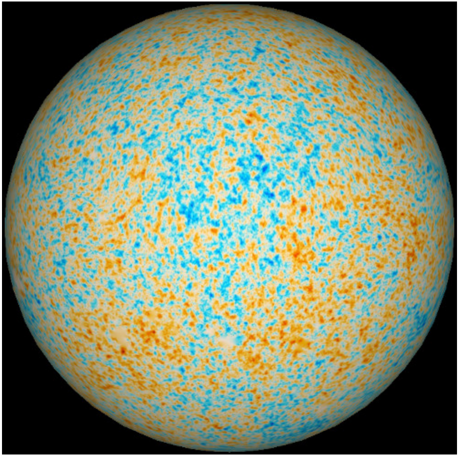
$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \int (production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3\,\int\ldots$

Recombination: $p + e \to H + \gamma$, $T_{rec} \approx 0.25\,\text{eV}$



Large Scale Structure



CMB anisotropy

Sound waves in photon-electron plasma

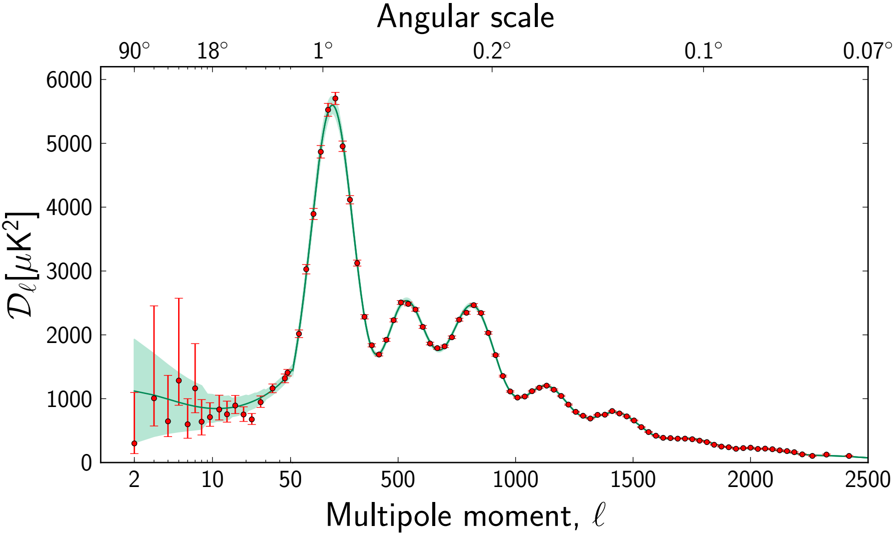
- Subhorizon Inhomogeneities of photons $\delta\rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{RD/MD}/T_{rec}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

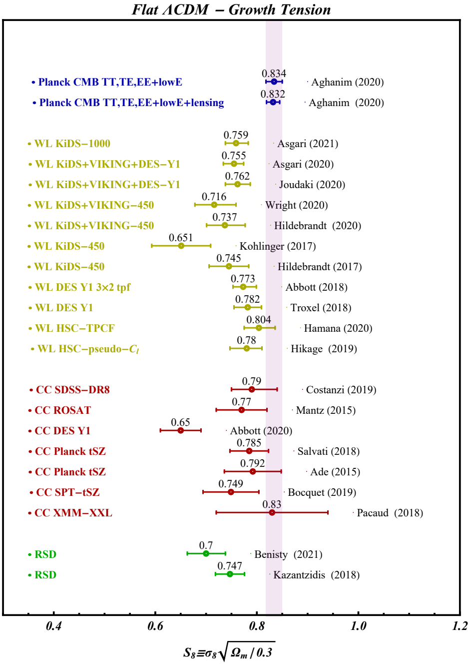
$$\delta\rho_\gamma/\rho_\gamma \propto \cos\left(k\int_0^{t_r}\frac{v_s\,dt}{a(t)}\right) = cos(kl_{sound})$$

-

$$\delta T(\theta,\varphi) = \sum a_{lm} Y_{lm}(\theta,\varphi)\;, \qquad \langle a^*_{lm} a_{lm}\rangle = C_l \equiv 2\pi\mathscr{D}_l/(l(l+1))$$

CMB measurements $l_{rec}, \Omega_{DM}, \Omega_B, \Omega_\Lambda, \Delta_{\mathscr{R}}, n_s, z_{rei}$





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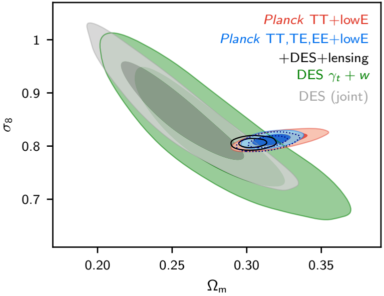

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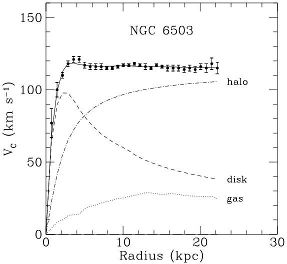
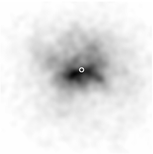
Universe content from astrophysics

Rotational curves





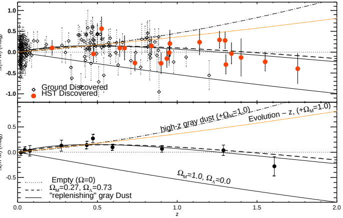
X-rays from centers of galaxy clusters

Gravitational lensing

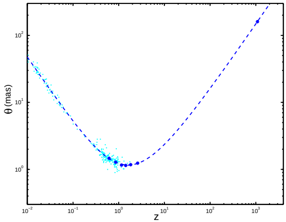
“Bullet” cluster

Universe content from cosmology

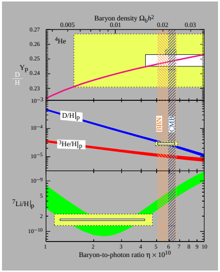
Standard candles

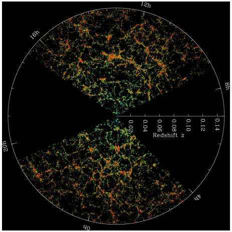


Angular distance

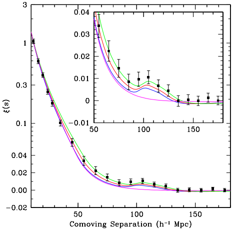


Nucleosynthesis

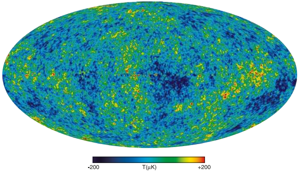




Large Scale Structures



Baryon acoustic oscillations



CMB anisotropy

Friedmann equation for the present Universe

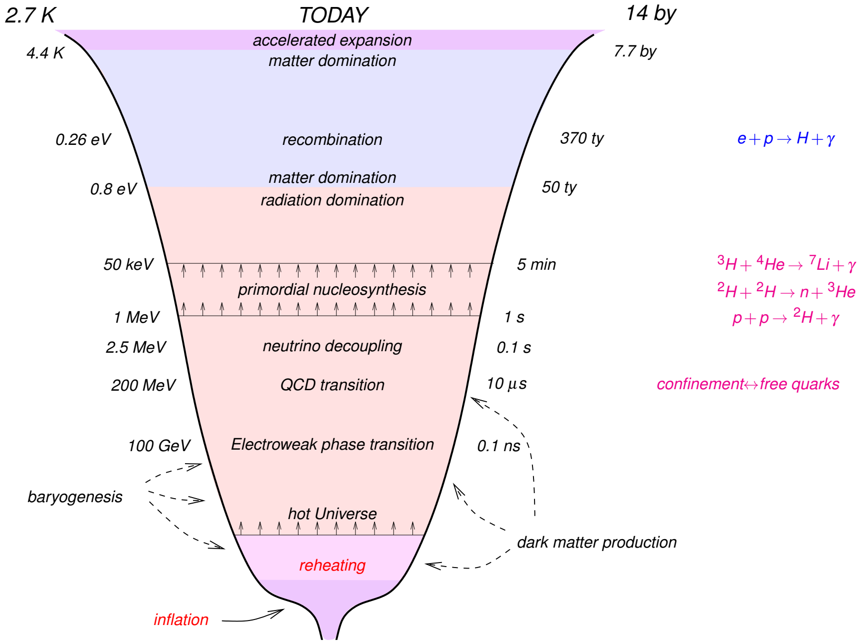
$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\left(\rho_{\rm M} + \rho_{rad} + \rho_\Lambda + \rho_{\rm curv}\right)$$

$$\frac{8\pi}{3}G\rho_{\rm curv} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G}H_0^2$$

$$\rho_c = \rho_{{\rm M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.53\cdot 10^{-5}\,\frac{\rm GeV}{\rm cm^3}\,,$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho_c\left[\Omega_{\rm M}\left(\frac{a_0}{a}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda\right]$$



2.7 K TODAY 14 by

accelerated expansion

4.4 K matter domination 7.7 by

0.26 eV recombination 370 ty $e + p \to H + \gamma$

matter domination

0.8 eV radiation domination 50 ty

50 keV 5 min $^3H + {}^4He \to {}^7Li + \gamma$

primordial nucleosynthesis $^2H + {}^2H \to n + {}^3He$

1 MeV 1 s $p + p \to {}^2H + \gamma$

2.5 MeV neutrino decoupling 0.1 s

200 MeV QCD transition 10 μs confinement↔free quarks

100 GeV Electroweak phase transition 0.1 ns

baryogenesis

hot Universe

dark matter production

reheating

inflation

Examples of cosmological solutions

$$\varkappa = 0 \qquad\qquad \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G\rho$$

dust: $p = 0$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^3}\,, \quad a(t) = const\cdot(t-t_s)^{2/3}\,, \quad \rho(t) = \frac{\text{const}}{(t-t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot a}{a}(t) = \frac{2}{3t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{1}{6\pi G}\frac{1}{t^2}$$

the Universe is too young

$$t_0 = \frac{2}{3H_0} = 0.9\times 10^{10}\ \text{yr} \qquad (h = 0.7)$$

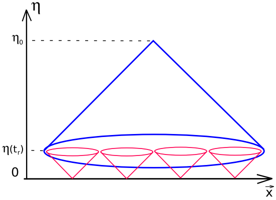
Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates:
$ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals
$\eta(t) = \int d\eta$

$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$



dust

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \qquad l_{H,0} = 2.6 \times 10^{28}\text{cm} \quad (h = 0.7)$$

Examples of cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const\cdot(t-t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t-t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

$$l_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t = \frac{1}{H(t)}\,.$$

In case of thermal equilibrium $T = \text{const}/a$

$$\rho_b = \frac{\pi^2}{30}g_b T^4\,, \quad \rho_f = \frac{7}{8}\frac{\pi^2}{30}g_f T^4$$

$$\rho = \frac{\pi^2}{30}g_* T^4\,, \qquad g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f = g_*(T)$$

Examples of cosmological solutions

vacuum: $T_{\mu\nu} = \rho_{vac}\eta_{\mu\nu}$ $p = -\rho$

$$S_G = -\frac{1}{16\pi G}\int R\sqrt{-g}d^4x\,, \quad S_\Lambda = -\Lambda\int\sqrt{-g}d^4x\,.$$

$$a = \mathrm{const}\cdot \mathrm{e}^{H_{\rm dS}t}\,, \quad H_{\rm dS} = \sqrt{\frac{8\pi}{3}G\rho_{vac}}$$

de Sitter space: space-time of constant curvature

$$ds^2 = dt^2 - \mathrm{e}^{2H_{dS}t}d\mathbf{x}^2$$

$\ddot{a} > 0$, no initial singularity

$ds^2 = dt^2 - e^{2H_{dS}t}d\mathbf{x}^2$

no cosmological horizon: $l_H(t) = e^{H_{dS}t}\int_{-\infty}^{t} dt' e^{-H_{dS}t'} = \infty$

de Sitter (events) horizon ($\mathbf{x} = 0$, t):

from which distance $l(t)$ one can detect light emitted at t?

in conformal coordinates: $ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$

coordinate size: $\eta(t \to \infty) - \eta(t) = \int_t^\infty \frac{dt'}{a(t')}$

physical size: $l_{dS} = a(t)\int_t^\infty \frac{dt'}{a(t')} = \frac{1}{H_{dS}}$

observer will never be informed what happens at distances larger than $l_{dS} = H_{dS}^{-1}$

Our future? with $H_{dS} = 0.8 \times H_0$

Standard cosmological model $ds^2 = dt^2 - a^2(t)dx^2$

$$\left(\frac{\dot a}{a}\right)^2 \equiv H^2 = H_0^2\left[\Omega_\Lambda + (\Omega_{DM} + \Omega_B + \Omega_{\nu,m\neq 0})\left(\frac{a_0}{a}\right)^3 + (\Omega_\gamma + \Omega_{\nu,m=0})\left(\frac{a_0}{a}\right)^4\right]$$

- $T_\gamma = 2.735\,\mathrm{K}, \quad \Longrightarrow \quad \Omega_\gamma \sim 10^{-5}$
- $N_\nu \approx 3, \quad \sum m_\nu < 0.2\,\mathrm{eV} \quad \Longrightarrow \quad \Omega_{\nu,\neq 0},\, \Omega_{\nu,0} \sim 10^{-5}$?
- $\Omega_B = 4.5\% \quad \Longrightarrow \quad \eta_B \equiv n_B/n_\gamma = 6\times 10^{-10}$
- $\Omega_{DM} = 27.5\%$
- $H_0 = 67\,\mathrm{km/s/Mpc} \quad \Longrightarrow \quad \rho_0 = 5\,\mathrm{GeV/m^3}$
- $\Omega_\Lambda = 68\% \quad \Longrightarrow$ flat space
- adiabatic, gaussian matter perturbations

  $$\left\langle\left(\frac{\delta\rho}{\rho}\right)^2\right\rangle \sim A_S \int \frac{dk}{k}\left(\frac{k}{k_*}\right)^{n_S-1}$$

  with $A_S = 3\times 10^{-9}$ and $n_S = 0.97$
- no tensor perturbations, $r \equiv A_T/A_S < 0.05$
- reionization at $z \equiv a_0/a = 10$