Muon anomalous magnetic moment:
the Standard Model prediction

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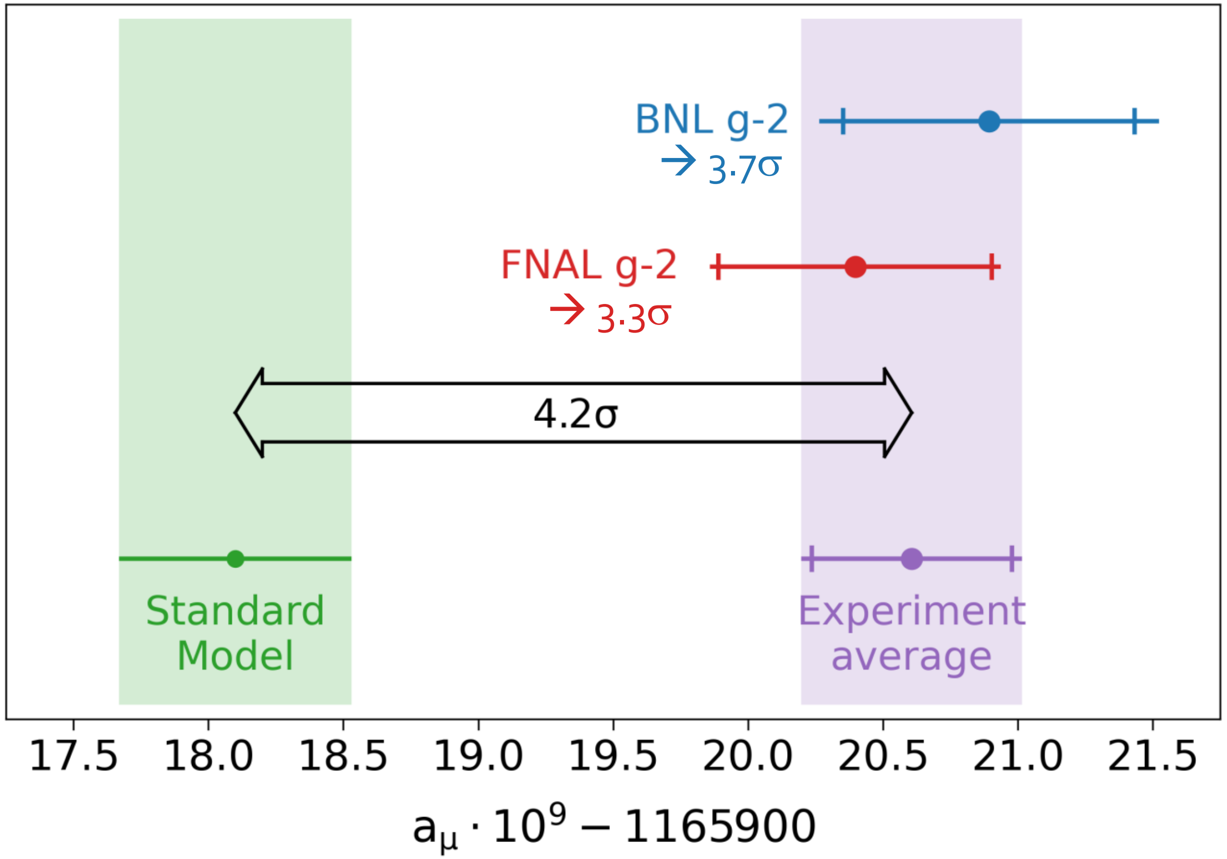
At the beginning of 2023…

$a_\mu(SM) = 0.00116591810(43) \rightarrow 368$ ppb



$a_\mu(Exp) - a_\mu(SM) = 0.00000000251(59) \rightarrow 4.2\sigma$

On a theoretical side…

We are interested not in the value of anomalous magnetic moment, but in its difference from the Standard Model prediction

$$\Delta a_\mu(New\ Physics) = a_\mu(exp) - a_\mu(SM)$$

a_μ in Standard Model

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{Weak}$$

Evaluation of $a_\mu(SM)$ is as important, as the measurement of $a_\mu(exp)$!

Evaluation of a_μ in Standard Model

Magnetic moment

Suppose that there is a point particle f at rest in an external magnetic field $\vec{B}$. If the interaction Hamiltonian H_{mdm} between f and $\vec{B}$ is given by

$$H_{\text{mdm}} = -\vec{\mu} \cdot \vec{B} \,,$$

then $\vec{\mu}$ is called the **magnetic dipole moment** of f.

- If f has a non-zero spin $\vec{s}$, then $\vec{\mu} \propto \vec{s}$
- H_{mdm} is P-even and T-even
- Its cousins:

  EDM $\vec{d}$: $H_{\text{EDM}} = -\vec{d} \cdot \vec{E}$ (P-odd, T-odd)
  (EDM: electric dipole moment)

  anapole $\vec{a}$: $H_{\text{ana}} = -\vec{a} \cdot (\nabla \times \vec{B})$ (P-odd, T-even)

Moments of spin ½ particle

For a spin-1/2 particle f,

$$\langle f(p')|J^{\text{em}}_\mu|f(p)\rangle = \bar{u}_f(p')\Gamma_\mu u_f(p)\ ,$$

$$\Gamma_\mu = F_1(q^2)\gamma_\mu + \frac{i}{2m_f}F_2(q^2)\sigma_{\mu\nu}q^\nu - F_3(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 - F_4(q^2)(\gamma_\mu q^2 - 2m_f q_\mu)\gamma_5$$

There are no other independent form factors of a spin-1/2 particle other than $F_1(q^2), \ldots, F_4(q^2)$ (See e.g., Nowakowski, Paschos, & Rodriguez, physics/0402058)

$F_1(0) = -eQ_f$ (electric charge) **By definition!**

$F_2(0) = -eQ_f a_f$ (a_f : anomalous magnetic moment)

$F_3(0) = d_f$ (EDM)

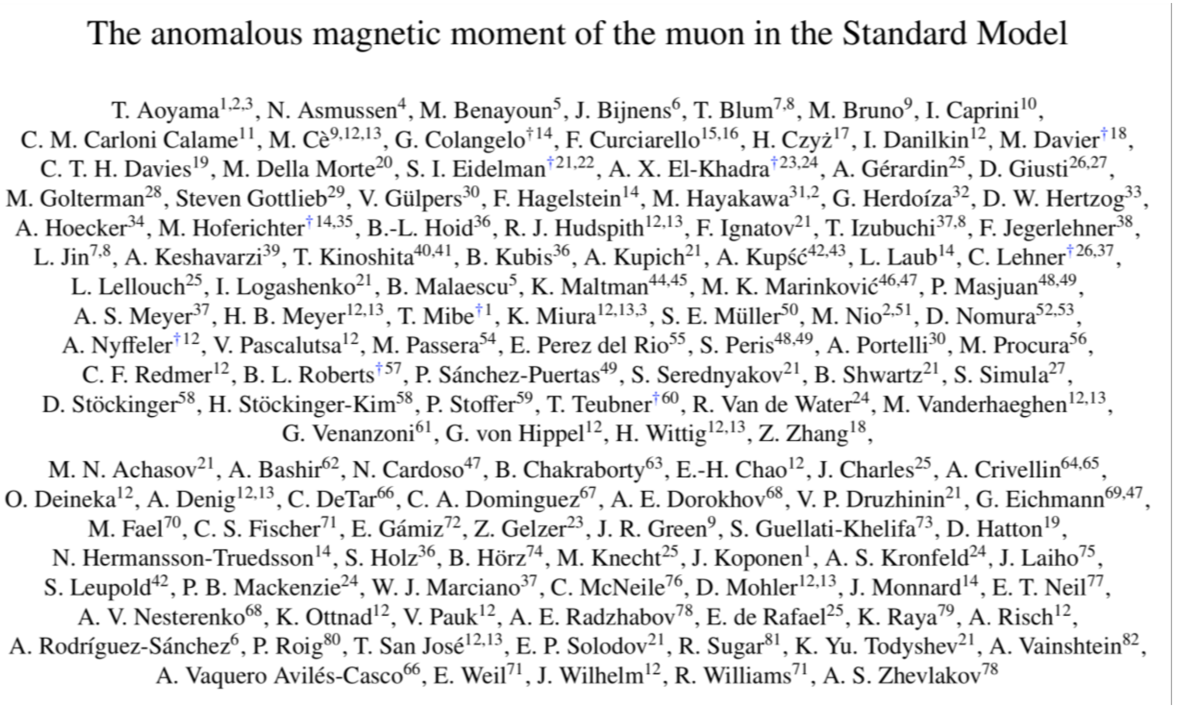
$F_4(0) = \tilde{a}_f$ (anapole moment)

If f is a Majorana particle, then $F_1(q^2) = F_2(q^2) = F_3(q^2) = 0$.

Muon g-2 Theory Initiative

A consortium of > 100 theorists formed in advance of the new FNAL results to compile all the theoretical inputs and provide recommendations

- White Paper posted 10 June 2020 (132 authors, 82 institutions, 21 countries)
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]



The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10],
C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[†14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[†18],
C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[†21,22], A. X. El-Khadra[†23,24], A. Gérardin[25], D. Giusti[26,27],
M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33],
A. Hoecker[34], M. Hoferichter[†14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38],
L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[†26,37],
L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49],
A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[†1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53],
A. Nyffeler[†12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56],
C. F. Redmer[12], B. L. Roberts[†57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27],
D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[†60], R. Van de Water[24], M. Vanderhaeghen[12,13],
G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

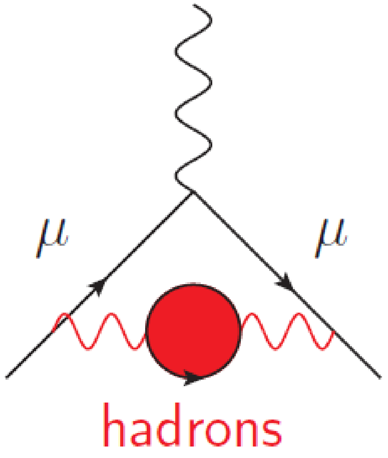
M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65],
O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47],
M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19],
N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75],
S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77],
A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12],
A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82],
A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

a_μ in Standard Model

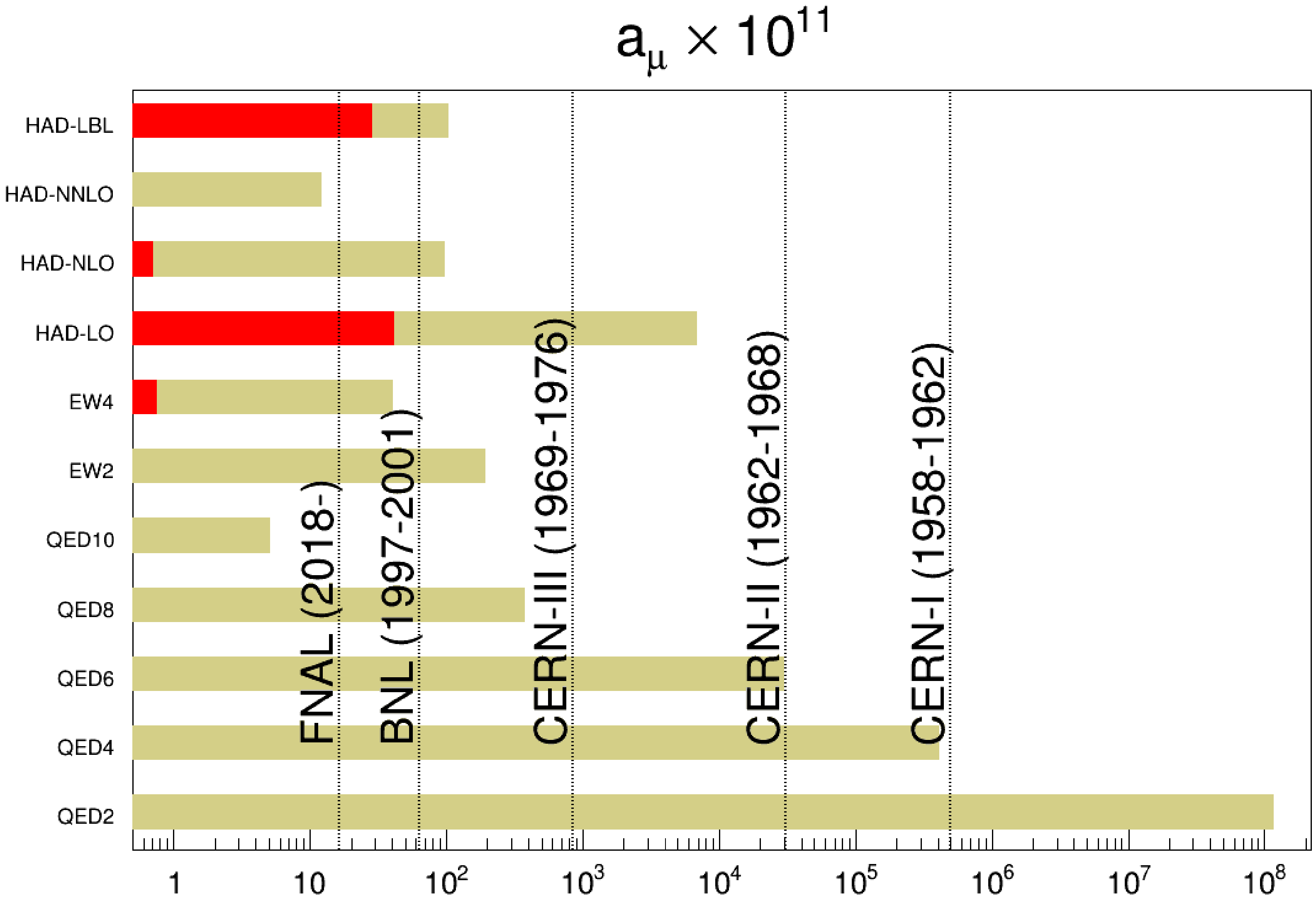
	a_μ $[10^{-11}]$	Δa_μ $[10^{-11}]$
experiment	116 592 061.	41. → 22
QED $\mathcal{O}(\alpha)$	116 140 973.321	0.023
QED $\mathcal{O}(\alpha^2)$	413 217.626	0.007
QED $\mathcal{O}(\alpha^3)$	30 141.902	0.000
QED $\mathcal{O}(\alpha^4)$	381.004	0.017
QED $\mathcal{O}(\alpha^5)$	5.078	0.006
QED total	116 584 718.931	0.030
electroweak	153.6	1.0
had. VP (LO)	6931.	40.
had. VP (NLO)	–98.3	0.7
had. LbL	92.	19.
total	116 591 810.	43.

BNL E821 2006
+ Fermilab 2021

Aoyama et al. 2020



Reach of various measurements a_μ

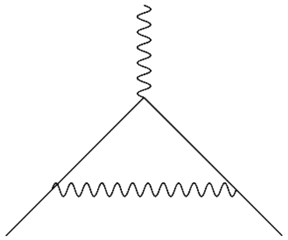


QED contribution

$$a_\mu^{QED} = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$a_\mu^{QED} = \underset{\substack{\uparrow \\ \text{universal} \\ A_1(\mu) = A_1(e)}}{A_1} + \underbrace{A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau)}_{\substack{\text{differ for } e,\, \mu,\, \tau \\ \text{leptons in the loop differ from external leptons}}}$$

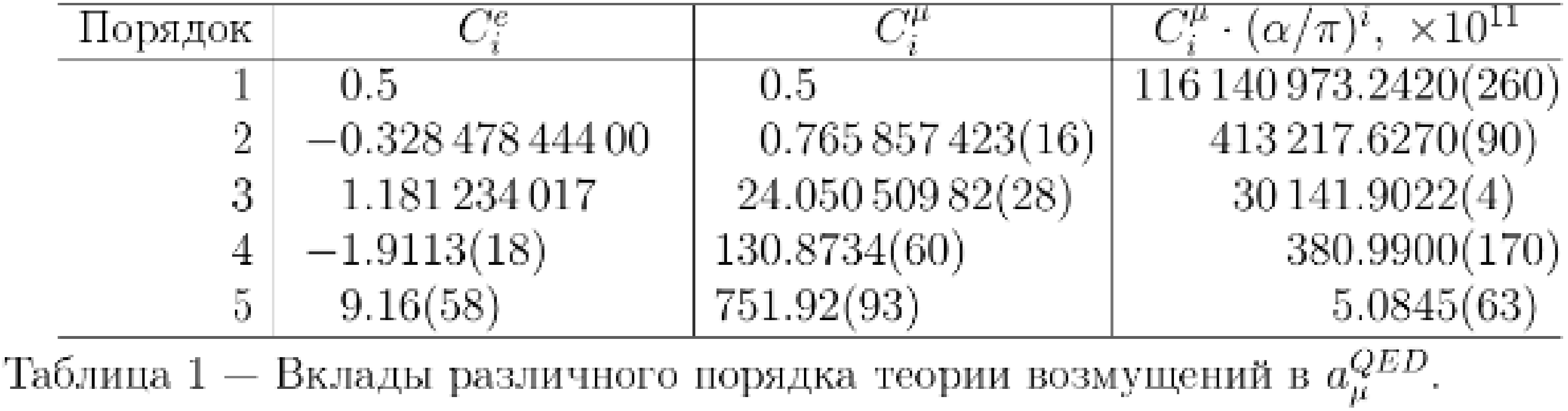
$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + A_i^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$



$$C_1 = A_1^{(2)} = \frac{1}{2}$$

QED contribution

$$a_\mu^{QED} = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$



Порядок	C_i^e	C_i^μ	$C_i^\mu \cdot (\alpha/\pi)^i,\ \times 10^{11}$
1	0.5	0.5	116 140 973.2420(260)
2	−0.328 478 444 00	0.765 857 423(16)	413 217.6270(90)
3	1.181 234 017	24.050 509 82(28)	30 141.9022(4)
4	−1.9113(18)	130.8734(60)	380.9900(170)
5	9.16(58)	751.92(93)	5.0845(63)

Таблица 1 — Вклады различного порядка теории возмущений в a_μ^{QED}.

$$a_\mu^{QED} = 116\,584\,718.842\ (.028)\ (.007)\ (.017)\ (.006)(.100)\ [.106] \times 10^{-11}$$

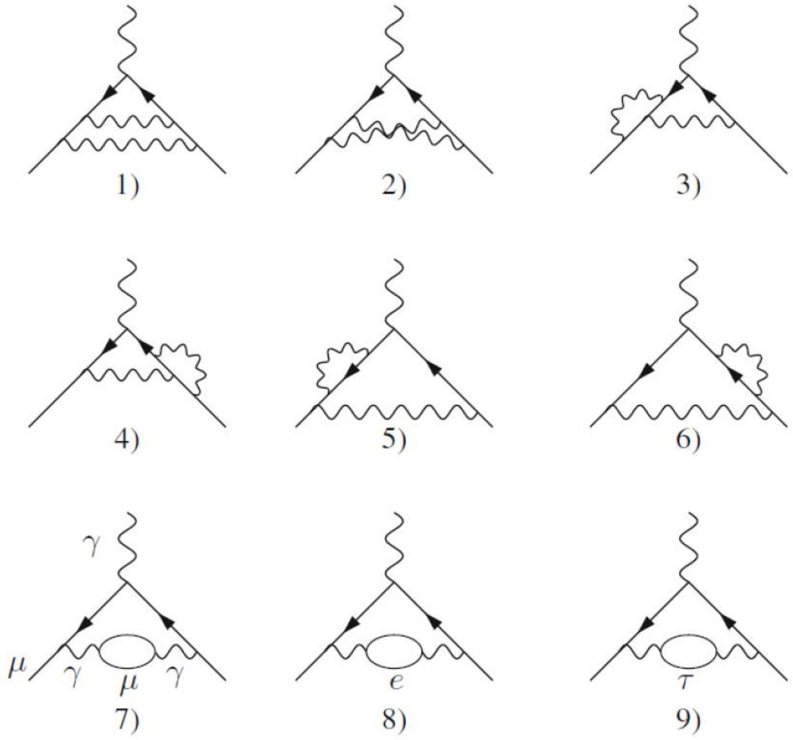
WP2020

α m_τ C_4 C_5 C_6

0.91 ppb

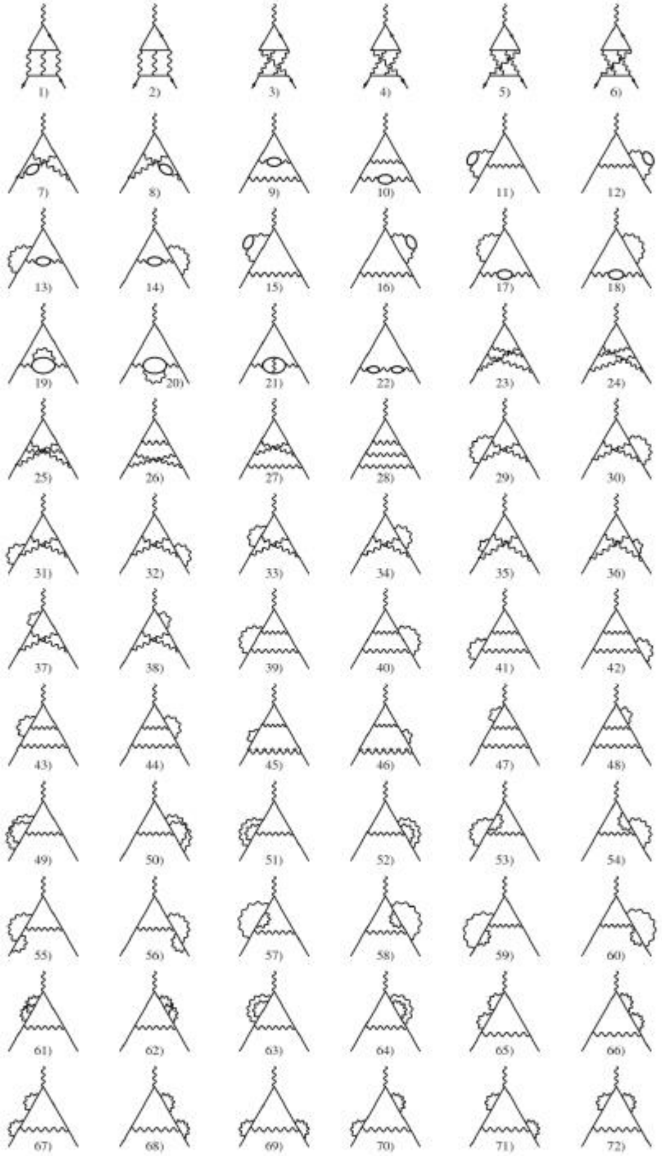
QED contribution two-loop

$$
\begin{aligned}
A_1^{(4)} &= -0.328\,478\,965\,579\,193\,78\ldots \\
A_2^{(4)}(m_\mu/m_e) &= 1.094\,258\,3111\,(84) \\
A_2^{(4)}(m_\mu/m_\tau) &= 0.000\,078\,064\,(25) \\
C_2 &= 0.765857410(27).
\end{aligned}
$$



QED contribution three-loop

$$
\begin{aligned}
A_1^{(6)} &= 1.181\,241\,456\,587\ldots \\
A_2^{(6)}(m_\mu/m_e) &= 22.868\,380\,02\,(20) \\
A_2^{(6)}(m_\mu/m_\tau) &= 0.000\,360\,51\,(21) \\
A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) &= 0.000\,527\,66\,(17) \\
C_3 &= 24.050\,509\,64\,(43).
\end{aligned}
$$



Logarithmic enhancement

the logarithmic enhancement $\ln(m_\mu/m_e) \approx 5.3$

note: It does not exist for the lightest lepton, electron.

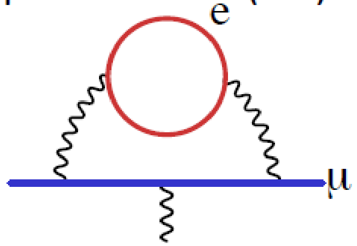
Two sources of the logarithm

1. Charge renormalization of the vacuum-polarization(VP) function

   2nd-order VP arises

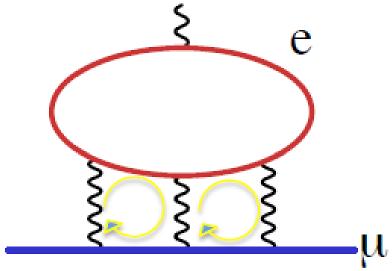
   $$\frac{2}{3}\ln(m_\mu/m_e) - \frac{5}{9} \sim 3$$

   "Renormalization Group" estimate



2. Light-by-light scattering diagram

   $$\frac{2}{3}\pi^2 \ln(m_\mu/m_e) \sim 35$$

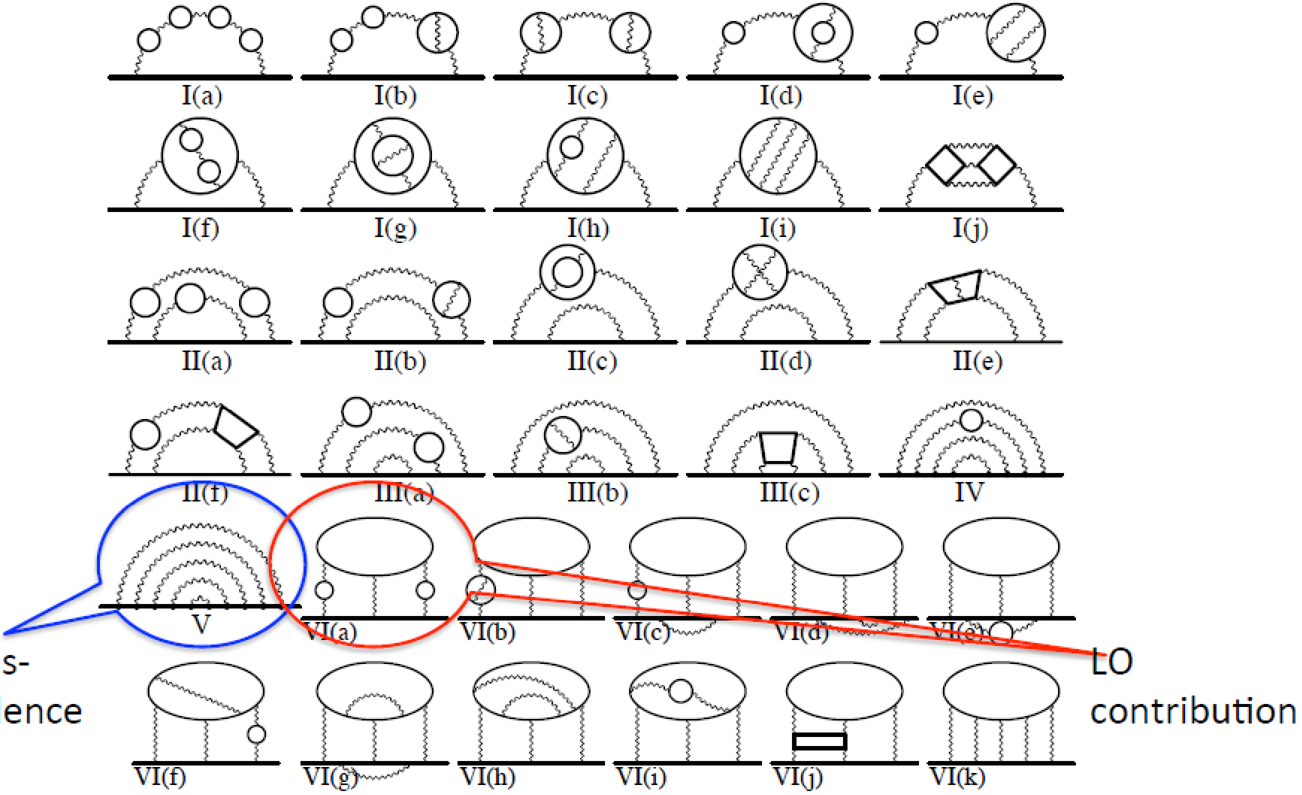


Coulomb photon loops provide the factor π²

QED contribution five-loop

12,672 Feynman vertex diagrams contribute to the 10th order .

They are classified into 32 gauge-invariant subsets over 6 sets.



No mass-dependence

LO contribution

QED contribution four- and five-loop

$$A_2^{(8)}(m_\mu/m_e) = 132.6852\ (60)$$

$$A_2^{(8)}(m_\mu/m_\tau) = 0.042\ 34\ (12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\ 72\ (4)$$

$$A_2^{(10)}(m_\mu/m_e) = 742.18\ (87)$$

$$A_2^{(10)}(m_\mu/m_\tau) = -0.068\ (5)$$

$$A_3^{(10)}(m_\mu/m_e, m_\mu/m_\tau) = 2.011\ (10)$$

QED contributions to the muon g-2 is now firmly established.

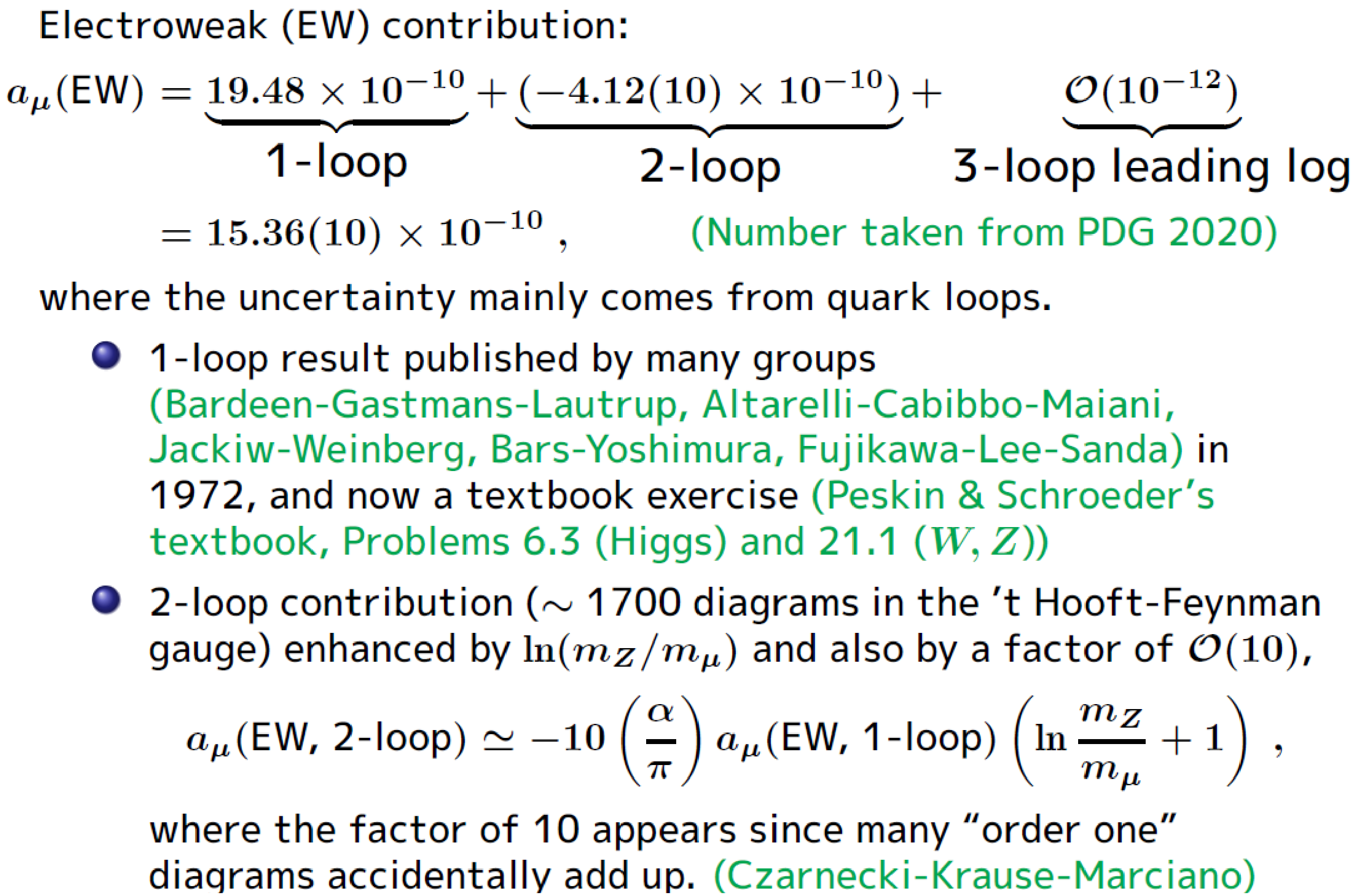
Rough estimate of the 12th-order contribution:

6th-order light-by-light x three 2nd-order vp x 10 ways

~ 20 x 3^3 x 10 $(\alpha/\pi)^6$ ~5,000 $(\alpha/\pi)^6$ ~ 0.08 x 10^{-11}

Recall the aimed goal of the on-going experiments ~ 12 x 10^{-11}

Electroweak contribution



Electroweak (EW) contribution:

$$a_\mu(\text{EW}) = \underbrace{19.48 \times 10^{-10}}_{\text{1-loop}} + \underbrace{(-4.12(10) \times 10^{-10})}_{\text{2-loop}} + \underbrace{\mathcal{O}(10^{-12})}_{\text{3-loop leading log}}$$

$$= 15.36(10) \times 10^{-10} \,,$$

(Number taken from PDG 2020)

where the uncertainty mainly comes from quark loops.

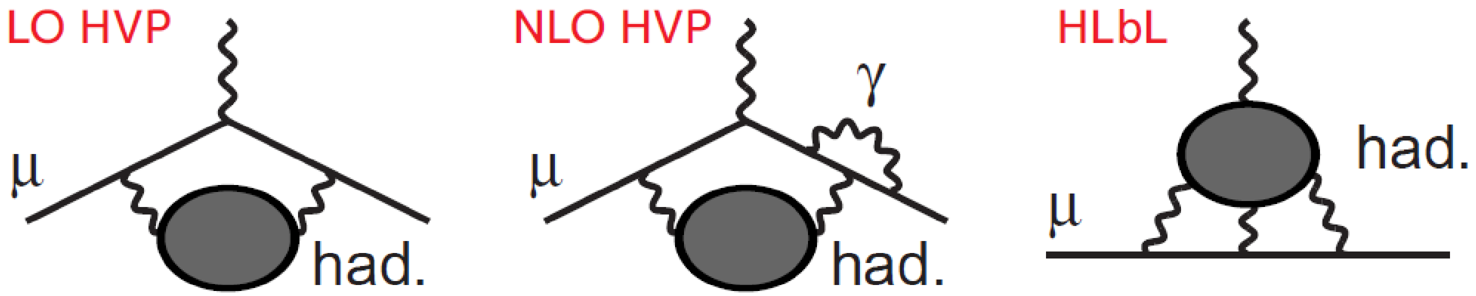
- 1-loop result published by many groups (Bardeen-Gastmans-Lautrup, Altarelli-Cabibbo-Maiani, Jackiw-Weinberg, Bars-Yoshimura, Fujikawa-Lee-Sanda) in 1972, and now a textbook exercise (Peskin & Schroeder's textbook, Problems 6.3 (Higgs) and 21.1 (W, Z))
- 2-loop contribution ($\sim$ 1700 diagrams in the 't Hooft-Feynman gauge) enhanced by $\ln(m_Z/m_\mu)$ and also by a factor of $\mathcal{O}(10)$,

$$a_\mu(\text{EW, 2-loop}) \simeq -10 \left(\frac{\alpha}{\pi}\right) a_\mu(\text{EW, 1-loop}) \left(\ln \frac{m_Z}{m_\mu} + 1\right) \,,$$

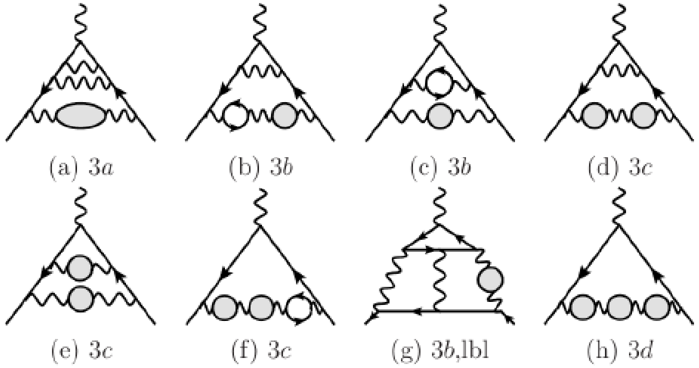
where the factor of 10 appears since many "order one" diagrams accidentally add up. (Czarnecki-Krause-Marciano)

Hadronic contribution

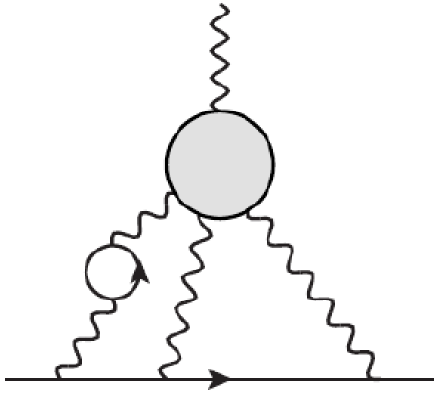
There are several hadronic contributions:



LO HVP: Leading Order Hadronic Vacuum Polarization Contribution
NLO HVP: Next-to-Leading Order HVP Contribution
HLbL: Hadronic Light-by-Light Scattering Contribution



NNLO HVP Contributions



HLbL NLO Contrib.

Hadronic contribution

Leading order (LO)

Next-to-leading order (NLO)

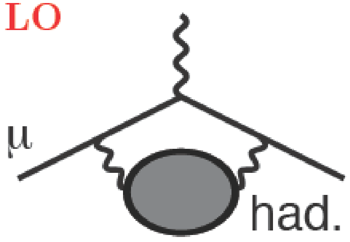
Light-by-light (LBL)

WP(2020)

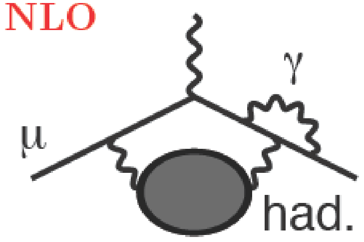
$(6\,931\ \pm\ 40) \times 10^{-11}$ $(-98.3\ \pm\ 0.7) \times 10^{-11}$ $(92\ \pm 19) \times 10^{-11}$

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ \ LO} + a_\mu^{\rm had,VP\ \ NLO} + a_\mu^{\rm had,Light-by-Light}$$

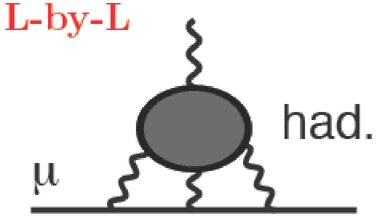
LO



NLO



L-by-L



350 ppb (0.6%)

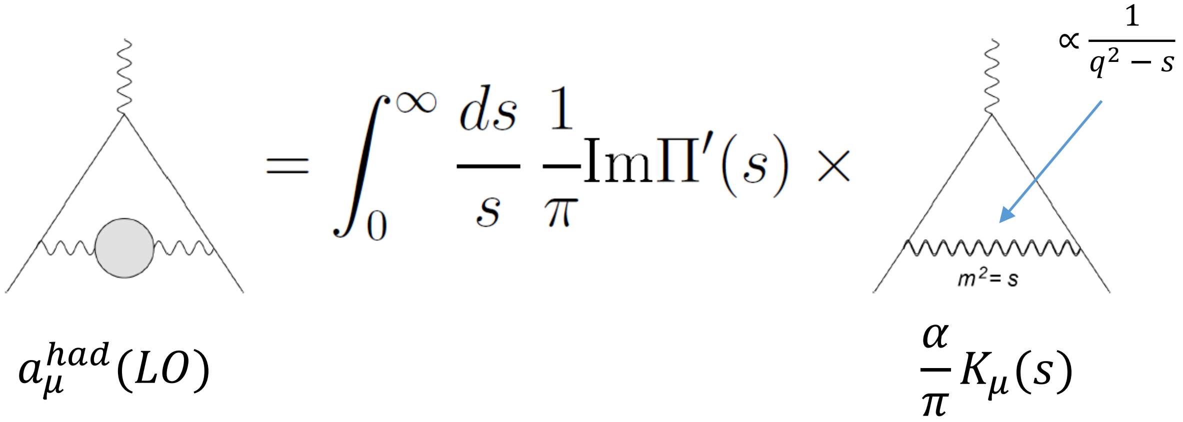
10 ppb

160 ppb

Compare to experimental accuracy of 190 ppb

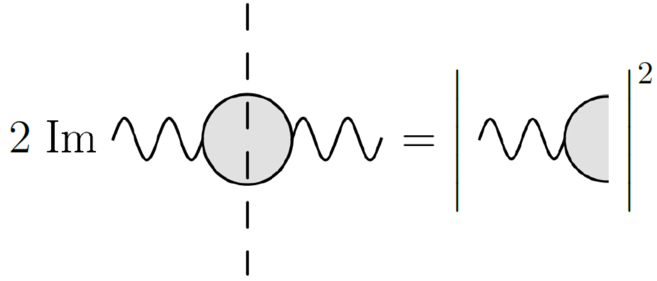
HVP: what do we need to measure

Dispersion relation:



$$= \int_0^\infty \frac{ds}{s}\,\frac{1}{\pi}\mathrm{Im}\Pi'(s) \times$$

Optical theorem:



$$\mathrm{Im}\,\Pi'(s) = \frac{s}{4\pi\alpha}\sigma^0(e^+e^- \to \gamma \to hadrons + \cdots)$$

Lets put everything together:

This is what we need to measure

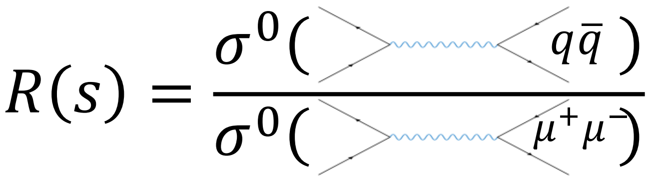
$$a_\mu^{had}(LO) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}R(s)K_\mu(s)$$

$$R(s) = \frac{\sigma^0(e^+e^- \to \gamma \to hadrons)}{4\pi\alpha^2/3s}$$

$\sigma^0(e^+e^- \to \mu^+\mu^-)$

$s = (\text{c.m. energy})^2$

R(s)



$$R(s) = \frac{\sigma^0(\quad q\bar{q}\ )}{\sigma^0(\quad \mu^+\mu^-)}$$

In the zeroth order of QCD and zero quark masses:

$$R^{(0)}(s) = 3\sum_f q_f^2$$

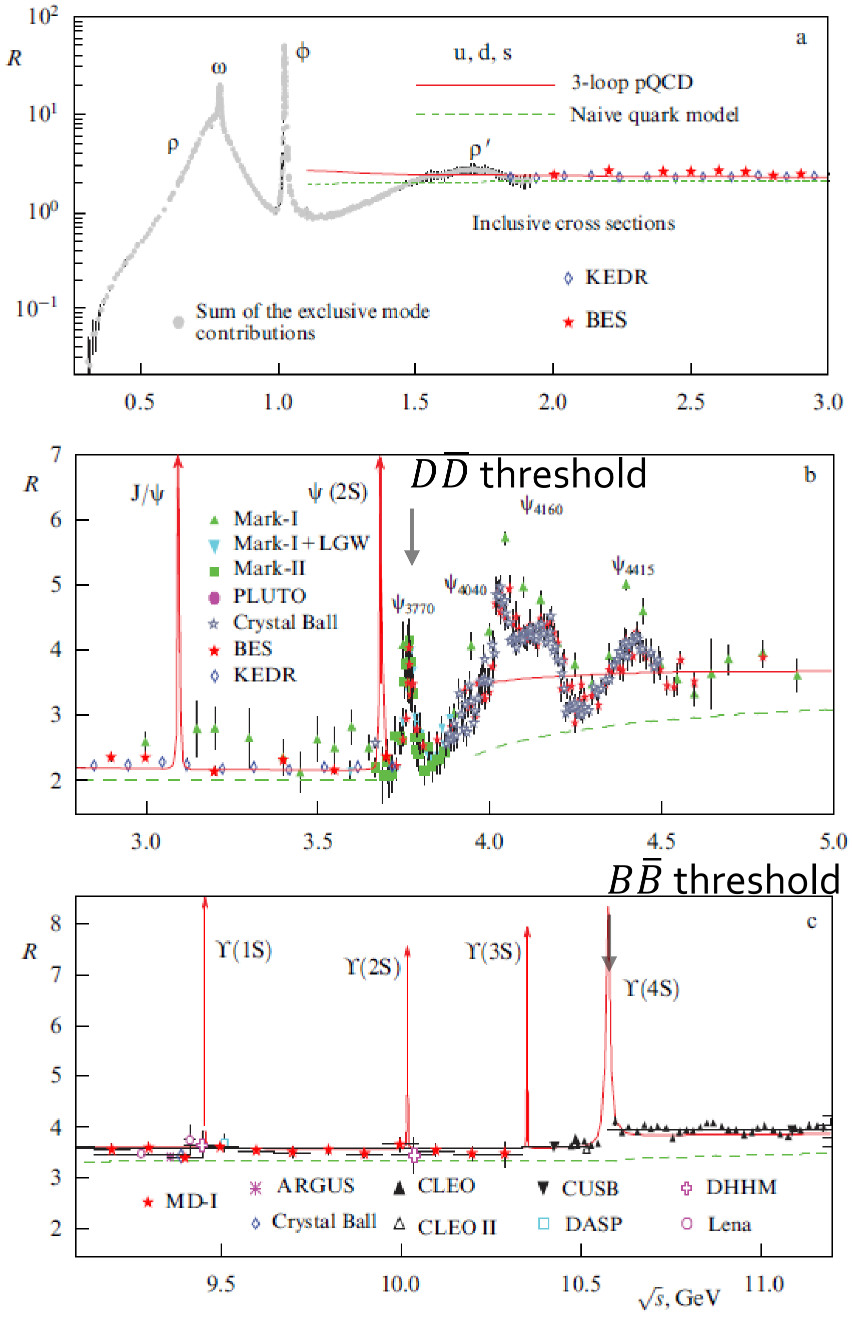
$$R(u,d,s) = \frac{6}{3}$$

$$R(u,d,s,c) = \frac{10}{3}$$

$$R(u,d,s,c,b) = \frac{11}{3}$$

Full pQCD calculation includes NNLO contribution, quark masses, running α_s,…

Good agreement of data vs pQCD at $\sqrt{s} > 2$ GeV and away from resonances



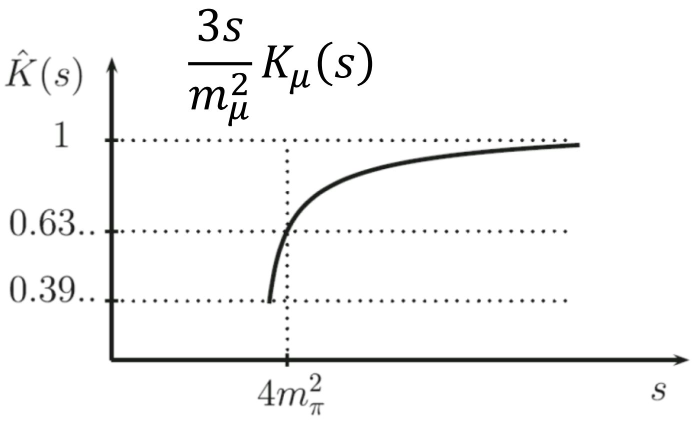
$D\bar{D}$ threshold

$B\bar{B}$ threshold

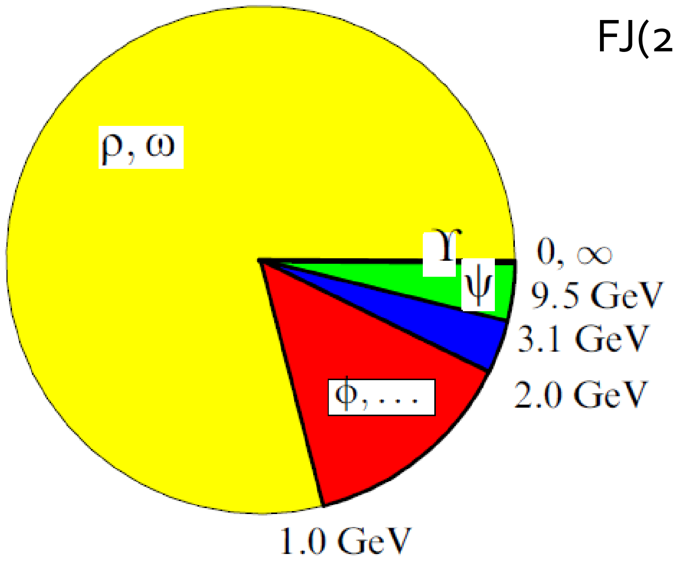
Contribution of various energies

In a_μ^{had} integral, the main contribution comes from low energies

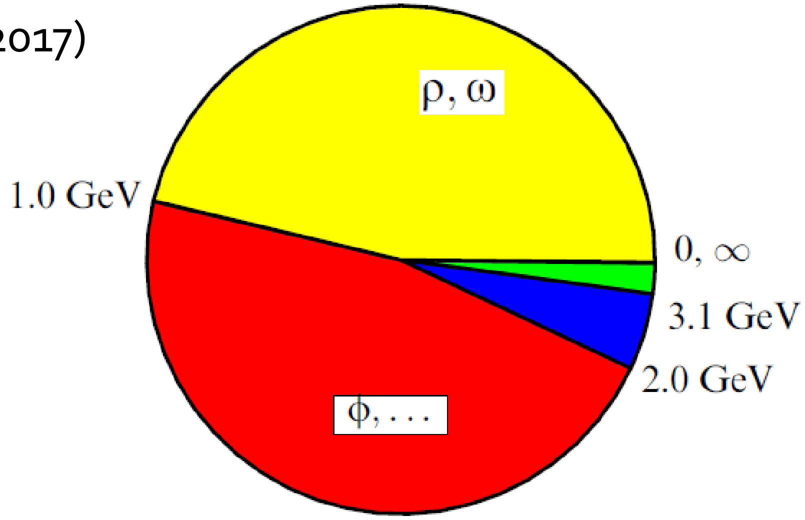
$$a_\mu^{had}(LO) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} R(s) K_\mu(s) \sim \int \frac{R(s)}{s^2} ds$$



FJ(2017)



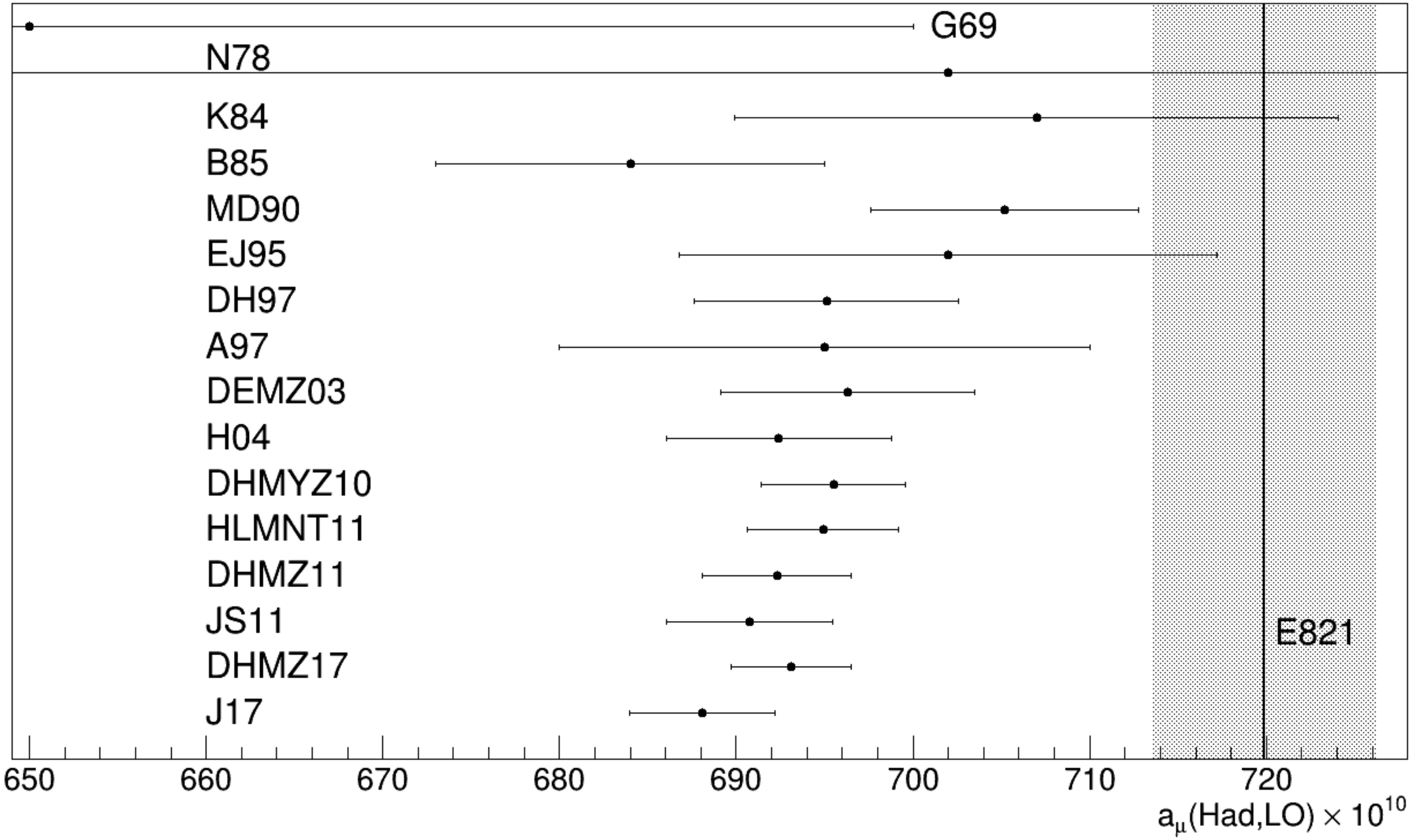
Contribution to the integral



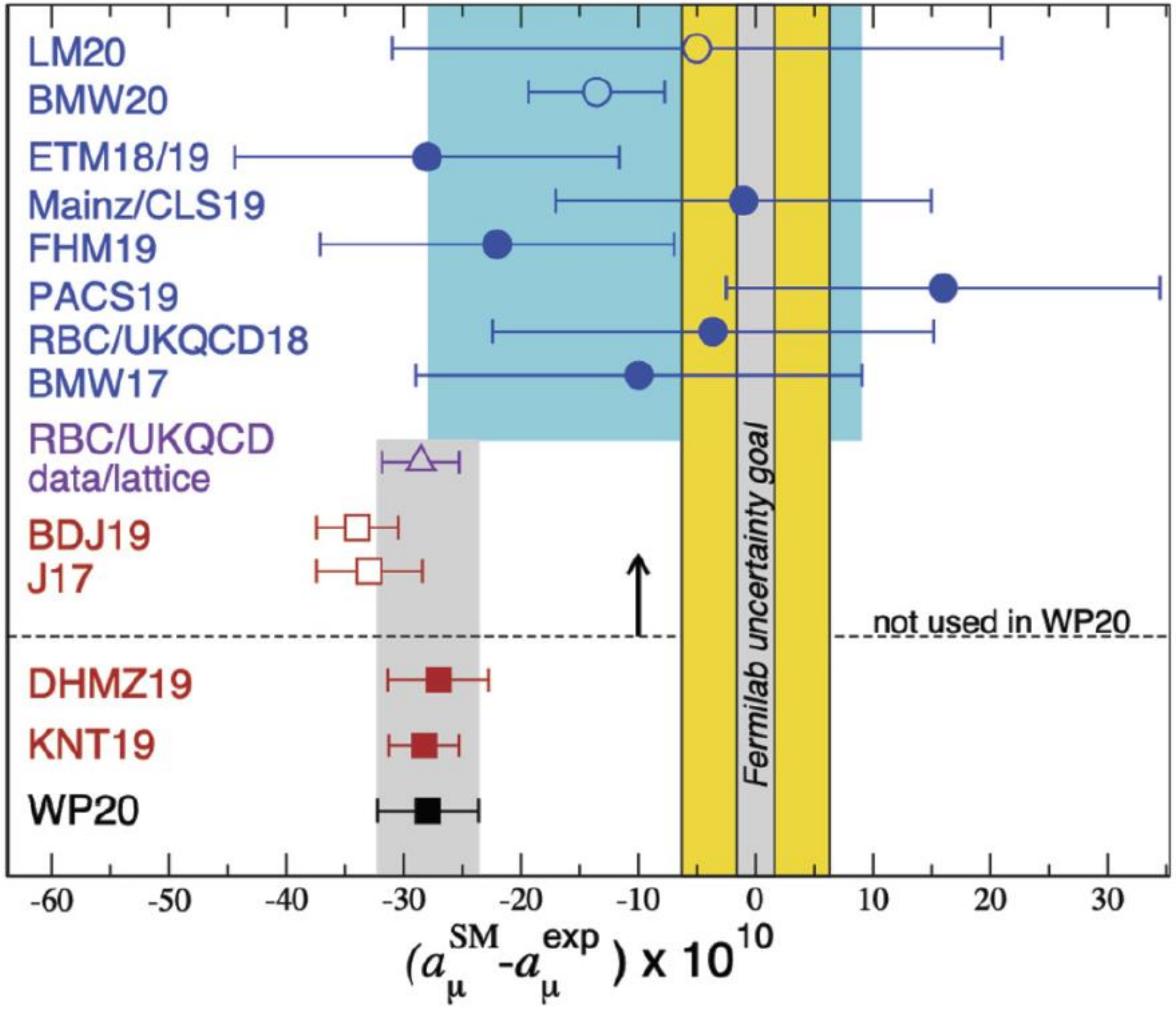
Contribution to the error of integral

When we measure R(s) in order to calculate hadronic contribution to a_μ, we are focused at low energies $\sqrt{s} \lesssim 2$ GeV

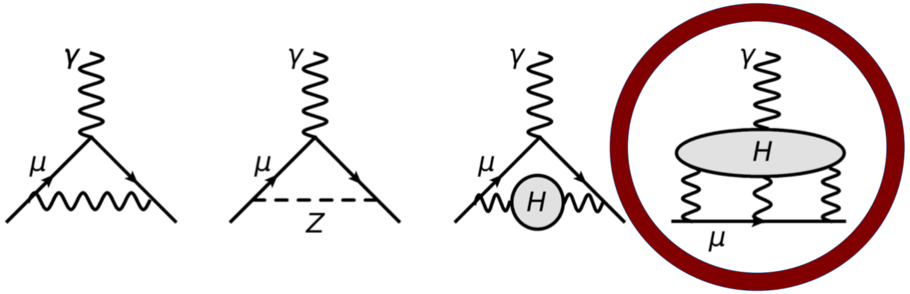
Hadronic vacuum polarization contribution



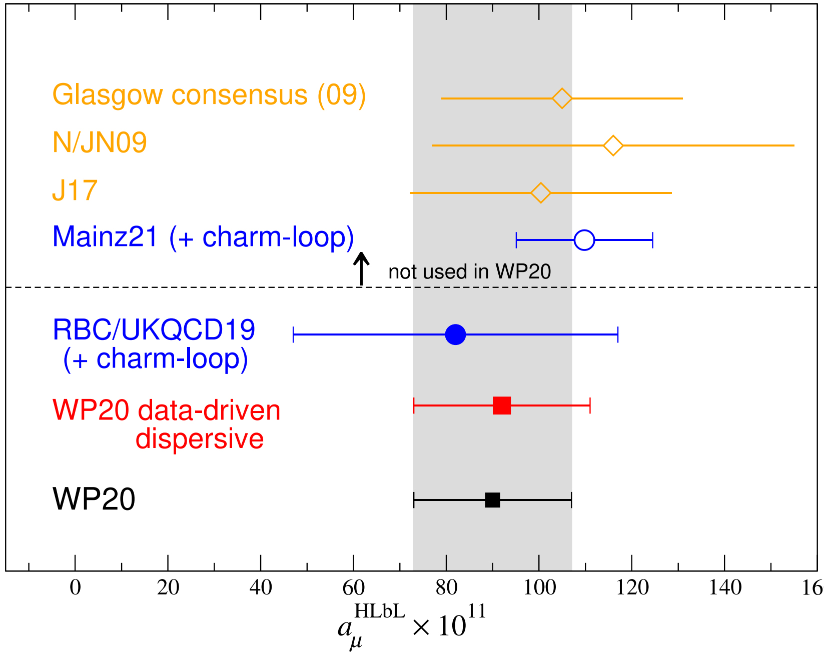
Hadronic vacuum polarization contribution



Hadronic light-by-light contributions

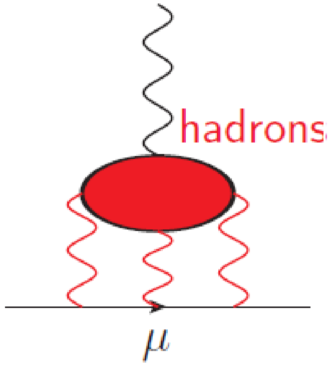


- Hadronic light-by-light has been particularly difficult in the past
  - Not calculable in QCD
  - Not directly measurable
  - Relied on model-dependent calculations
- Two developments
  - advancement in lattice calculations
  - data-driven approaches to check the models
- All approaches are in good agreement

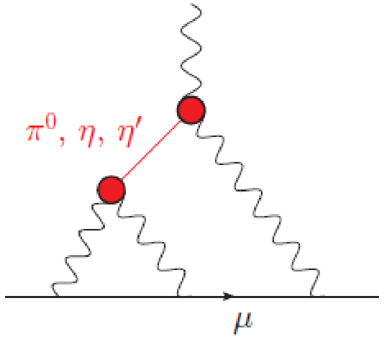


Вклады в Hlbl

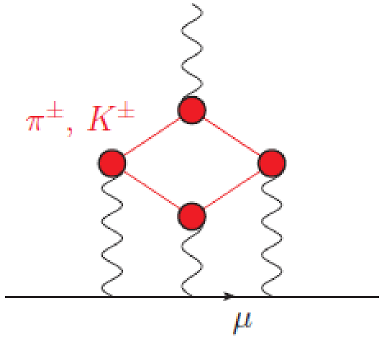
$$
\begin{aligned}
a_\mu^{had;LBL} = & \ (9.545[6.468+1.487+1.590]\pm 1.240 + && (\pi^0,\eta,\eta') \\
& 0.755[0.189+0.519+0.047]\pm 0.271 + && (a_1,f_1,f_1') \\
& -0.598[-0.017-0.296-0.285]\pm 0.120 + && (a_0,f_0,f_0') \\
& 0.11[0.079+0.007+0.022+0.002]\pm 0.01 + && (f_2',f_2,a_2',a_2) \\
& -2.0\pm 0.5 + && (\pi - loop) \\
& 2.23\pm 0.4 + && (quark - loop) \\
& 0.3\pm 0.2)\times 10^{-10} && (NLO) \\
= & \ (10.34\pm 2.88)\times 10^{-10}.
\end{aligned}
$$



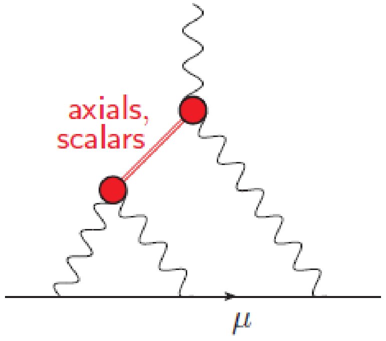
- different contributions calculated or estimated (in 10^{-11}):



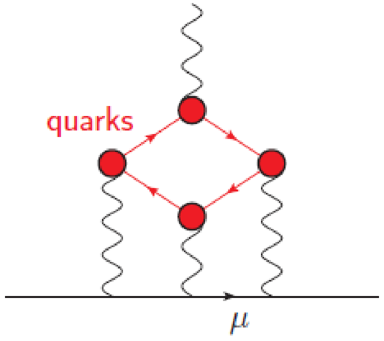
94±4



–24±1



5±7



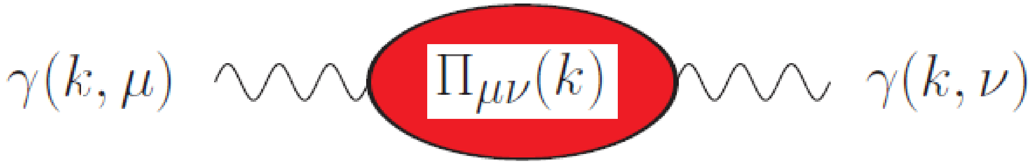
18±10

$\longrightarrow$ increasing systematic control over HLbL using dispersion-theoretical approach

Aoyama et al. 2020

HVP structure

- photon two-point function:



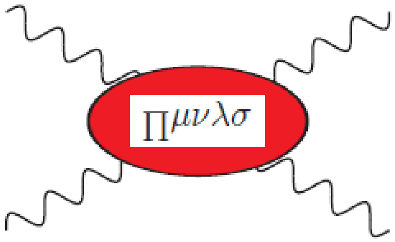
  - ▷ one single independent momentum k
  - ▷ symmetric rank-2 tensor: two structures $g_{\mu\nu}$, $k_\mu k_\nu$
  - ▷ scalar invariant can depend on one single invariant k^2
- gauge invariance: $k^\mu \Pi_{\mu\nu}(k) = 0 = k^\nu \Pi_{\mu\nu}(k)$

$$\boxed{\Pi_{\mu\nu}(k) = (k^2 g_{\mu\nu} - k_\mu k_\nu)\Pi(k^2)}$$

⟶ Lorentz + gauge invariance reduce HVP to one single function of a single variable!

Hlbl structure

Colangelo, Hoferichter, Procura, Stoffer 2014, 2015

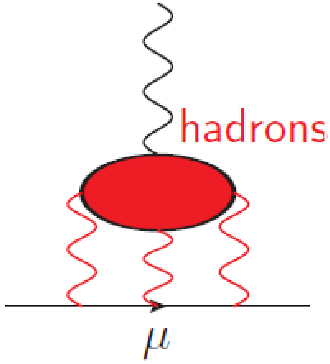


- HLbL tensor $\Pi^{\mu\nu\lambda\sigma}$: Lorentz invariance

  $\longrightarrow$ 138 (136) scalar functions Eichmann et al. 2014
- gauge invariance: Bardeen, Tung 1968; Tarrach 1975

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma}\Pi_i$$

$\longrightarrow$ 7 distinct structures, 47 related by crossing



- master formula:

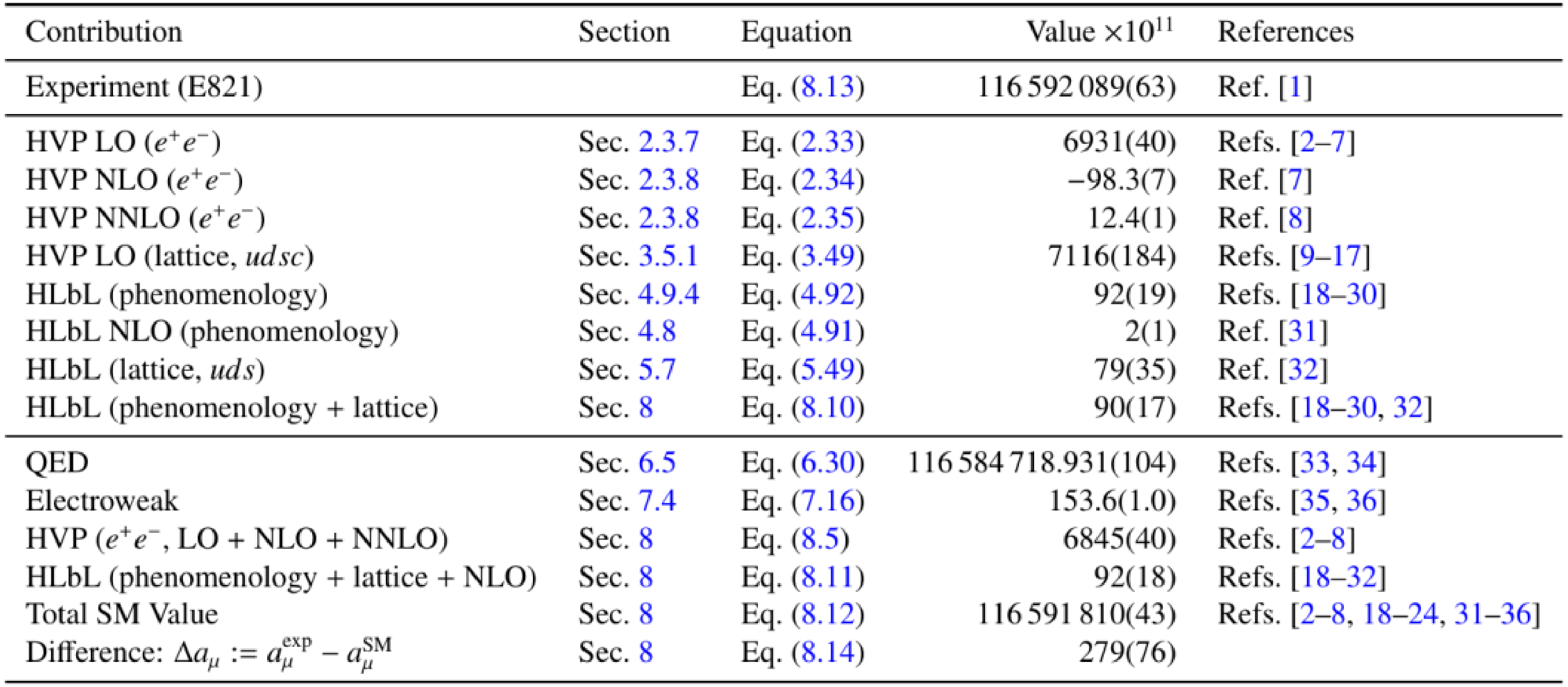
$$a_\mu^{\text{HLbL}} = -e^6 \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{\sum_{i=1}^{12}\hat{T}_i(q_1,q_2;p)\hat{\Pi}_i(q_1,q_2,-q_1-q_2)}{q_1^2q_2^2(q_1+q_2)^2[(p+q_1)^2-m_\mu^2][(p-q_2)^2-m_\mu^2]}$$

- $\hat{T}_i$: known kernels

  $\hat{\Pi}_i$: dispersively ↔ measurable form factors / scatt. amplitudes

a_μ в Стандартной модели

From Table 1 of the White Paper



Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

HVP: Hadronic Vacuum Polarization contribution

HLbL: Hadronic Light-by-Light contribution

Data for HVP calculation

How well do we need to measure R(s)

From the White Paper (Physics Reports 887 (2020) 1):

$$a_\mu^{\text{had}}(LO) = 693.1(4.0) \times 10^{-10}$$

The expected final precision of the Fermilab measurement

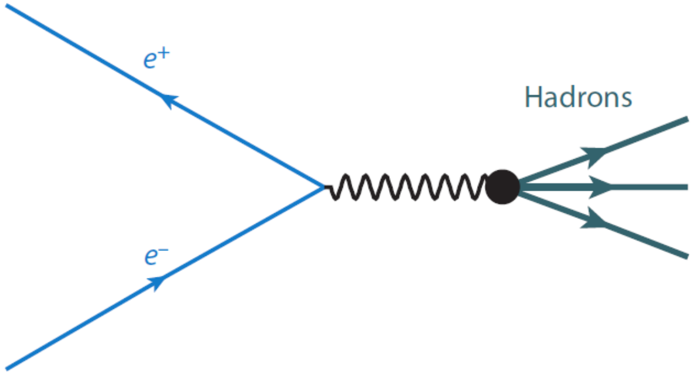
$$\Delta a_\mu = 1.6 \times 10^{-10}$$

We need to know $R(s)$ to 0.23% to match Fermilab precision

Now the hadronic contribution is known to 0.57%

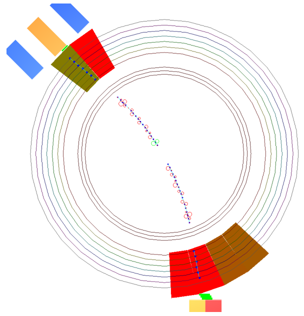
Measurement techniques:

Direct vs ISR

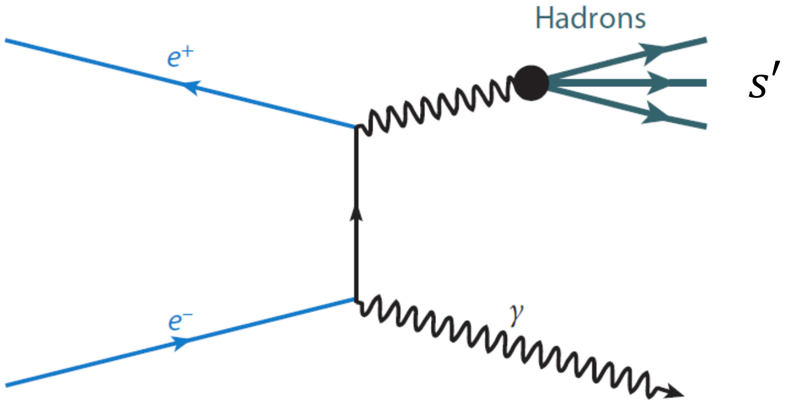


Direct measurement (Energy scan)

At fixed s: $\sigma_{e^+e^-\to H}(s) \sim N_H/L$
Data is taken at different s

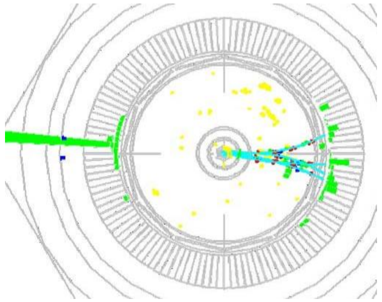


VEPP-2M: CMD-2, SND
VEPP-2000: CMD-3, SND2k



ISR (Initial State Radiation)

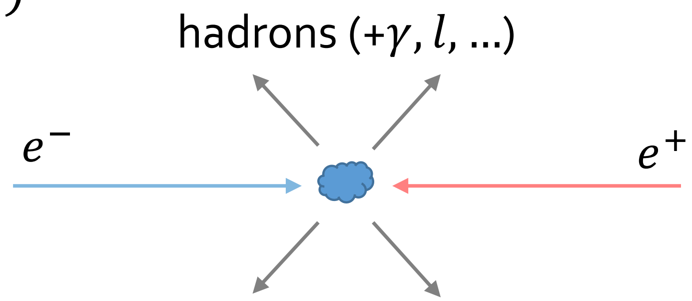
$\sigma_{e^+e^-\to H}(s') \sim \frac{dN_{H+\gamma}/ds'}{L\cdot dW/ds'}$
Data is taken at fixed $s > s'$



KLOE, BABAR, BES-III, CLEO

Energy scan approach

Direct measurement of $\sigma(e^+e^- \to hadrons)$ (energy scan approach):



- performed at electron-positron collider
- collect data at different beam energy
- at each energy point: select final states with hadrons, subtract background and normalize to luminosity

Number of selected events

Number of background events

$$\sigma = \frac{N_{obs} - N_{bg}}{\varepsilon \cdot \int \mathcal{L} dt}$$

Detection efficiency:
- kinematical limits of detector (fiducial volume) – detector never has 4π coverage
- detector response

Luminosity integral
- measured by selection of monitoring events with known cross section

Exclusive vs inclusive measurement

Detection efficiency is (usually) calculated using MC simulation

- In order to calculated ε, we need to know the energy and angular distributions of final particles (including all correlations)

$$\sigma = \frac{N_{obs} - N_{bg}}{\varepsilon \cdot \int \mathcal{L} dt}$$

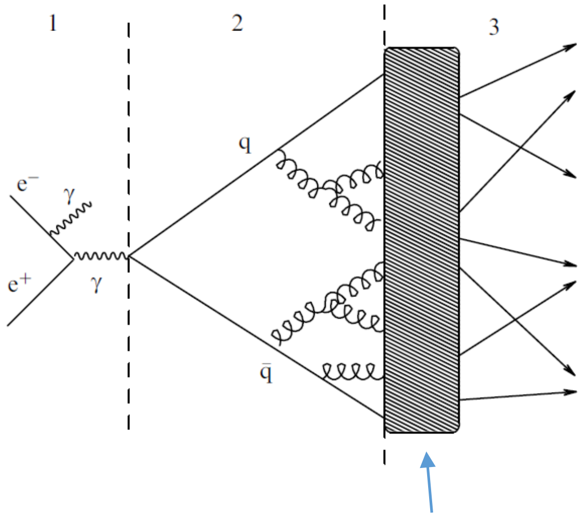
For high energies, where multiplicity is large enough, there are effective models of hadronization, which describe data reasonably well

At low energy the detection efficiency varies significantly between different final states and different paths of hadronization (intermediate states)

At low energies we have to measure cross section for each possible final state separately and then calculate sum to get R (***exclusive approach***)

At high energy we can measure total cross section directly (***inclusive approach***)

Final state



Intermediate states

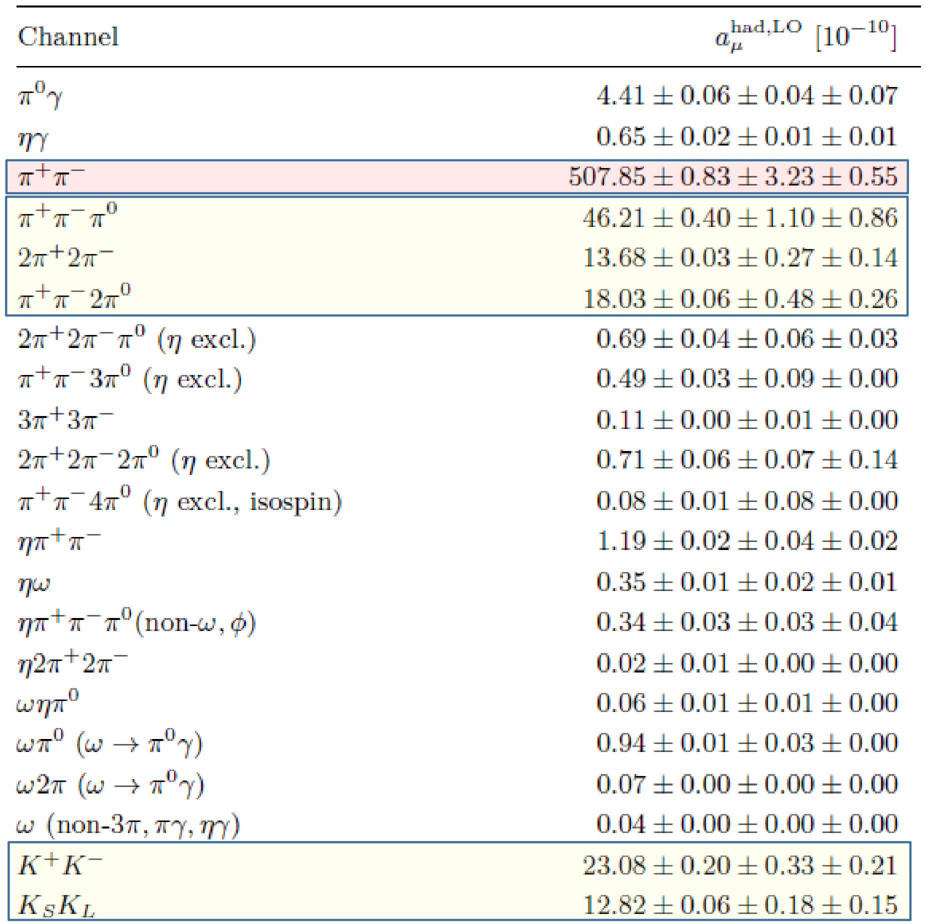
The practical boundary between two approaches in $\sqrt{s} = 2$ GeV.

The $a_\mu^{had}(LO)$ calculation is mostly based on exclusive measurements.

Contribution of exclusive hadronic cross sections to a_μ

In exclusive approach, we calculate a_μ integral for each final state and sum them:

$$a_\mu^{had}(LO) = \sum_{X=\pi^0\gamma,\pi^+\pi^-,\dots} a_\mu^X(LO) = \sum_X \frac{1}{4\pi^3}\int \sigma^0(e^+e^- \to X)K_\mu(s)ds$$

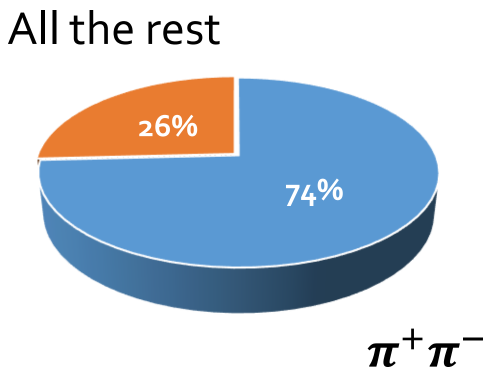


Channel	$a_\mu^{\mathrm{had,LO}}$ $[10^{-10}]$
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$

From DHMZ'19

The larger the contribution, the better relative precision is required

$e^+e^- \to \pi^+\pi^-$ is by far the most challenging and has got the most attention (74% of total hadronic contribution!)



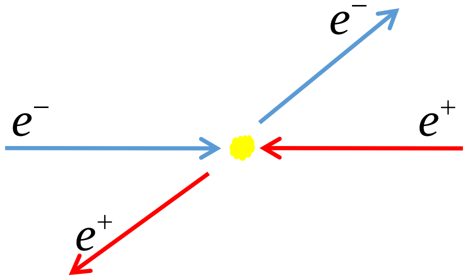
Luminosity measurement

We need to know luminosity integral in order to normalize the measured hadronic cross section.

For that we use *monitoring process* with known cross section

$$\int \mathcal{L}dt = \frac{N_{obs} - N_{bg}}{\varepsilon \cdot \sigma_{known}}$$

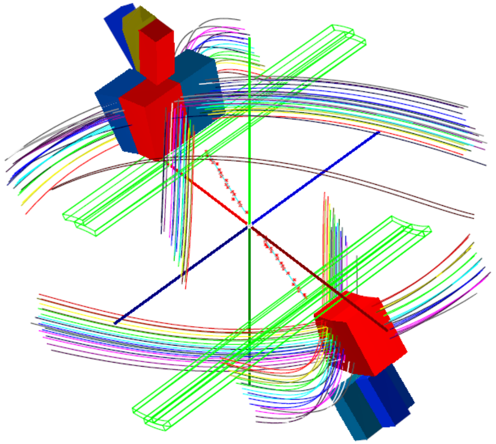
The most popular monitoring process is
large angle Bhabha scattering $e^+e^- \to e^+e^-$:
easily identifiable, large cross section



Other good processes for luminosity measurement:

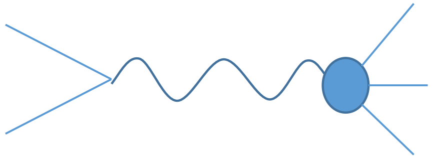
$e^+e^- \to \mu^+\mu^-$	*Has many advantages, but relatively small cross section and large background*
$e^+e^- \to \gamma\gamma$	*Natural for final states with neutrals*
$e^+e^- \to e^+e^-\gamma$ $e^+e^- \to e^+e^-\gamma\gamma$	*Often used for online measurement*

All these are QED processes – the cross section can be calculated



$e^+e^- \to e^+e^-$ in CMD-3

Radiative corrections



We want to measure $e^+e^- \to H$, but these events are accompanied by similar events where photons are emitted by any of the particles.

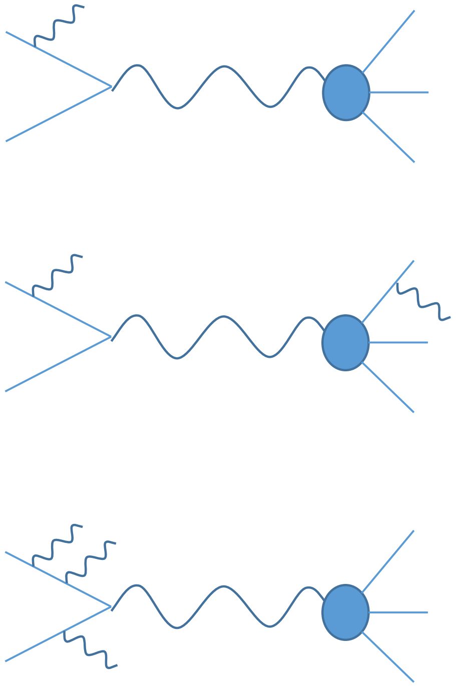
Radiation of high-energy γ is suppresses by α, but radiation of soft photons is enhanced.

Radiation changes both the cross-section and the kinematics of the final state:

$$\sigma = \frac{N_{obs} - N_{bg}}{\varepsilon(\delta) \cdot (1+\delta) \cdot \int \mathcal{L} dt}$$

And we have to calculate radiative corrections to the cross section of monitoring process as well

Radiative processes



ISR FSR

Initial *Final*

state radiation

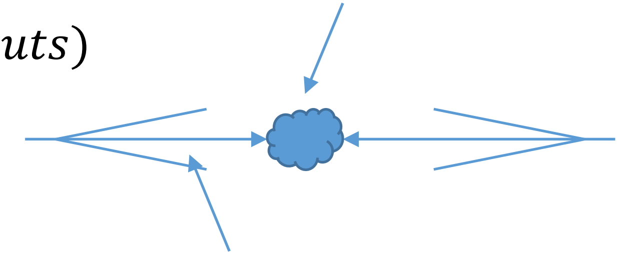
How to calculate radiative corrections

Main idea: allow each initial particle to emit any number of photons (jets).
The amount of energy carried by photons is described by structure function.

$$\sigma_{vis}(s) = \int_0^1 dx_1 dx_2 D(x_1, s) D(x_2, s) \sigma_0(x_1 x_2 s) \cdot \Theta(cuts)$$

we measure this

we want to know this



Hard process at $s' = x_1 x_2 s$

photon “jet” $D(x, s)$

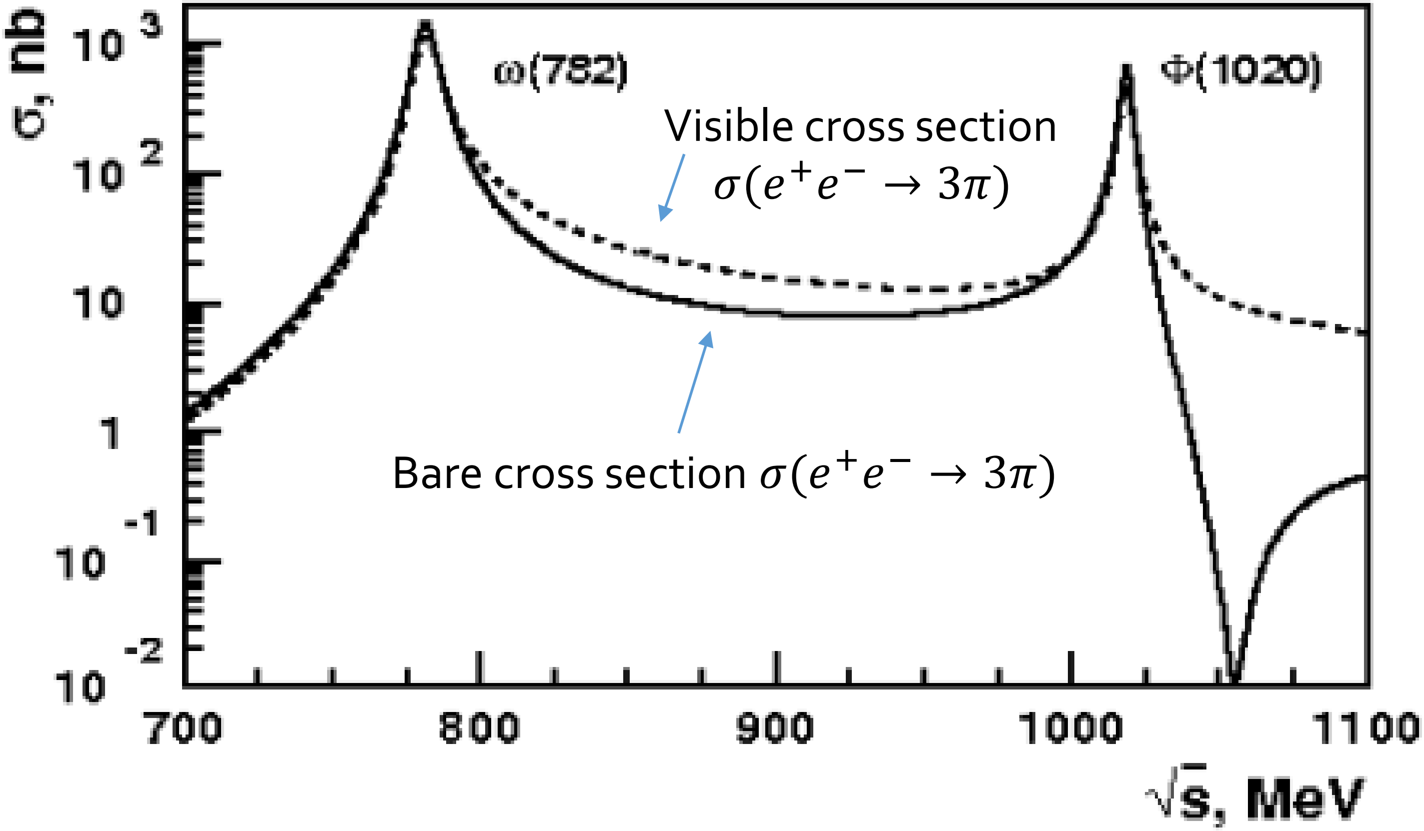
The radiative correction depends on the measured cross-section – need to use iterative procedure.

Structure functions are known to high precision (<0.1%). Main limitation is from kinematics: we don’t take into account angular distribution of photons in the jet.

This approach is ok for ~1% measurements and is typically used for multi-hadron events.

Typical value for radiative corrections is ~10% (can be much larger near narrow resonances)

Example:
$e^+e^- \to \pi^+\pi^-\pi^0$



Visible cross section $\sigma(e^+e^- \to 3\pi)$

Bare cross section $\sigma(e^+e^- \to 3\pi)$

Radiative corrections for precise measurements

Calculation of radiative corrections for high-precision final states (e^+e^-, $\mu^+\mu^-$, $\pi^+\pi^-$, $\gamma\gamma$,…) is much more complicated. Usually, it is implemented as MC generator and used together with the full detector simulation for proper evaluation of detector efficiency

Extensive review: Eur.Phys.J. C66 (2010) 585-686

MCGPJ (VEPP-2000)

1 real γ (from any particle) + jets along all particles

BABAYAGA (e^+e^-)

1 real γ + $n\gamma$ generated iteratively by emitting one γ at a time

PHOKHARA (KLOE, BABAR)

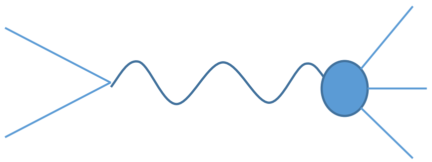
1 ISR γ + 1 real γ + soft

Many final states, intended for ISR measurements

These generators include ISR, FSR, virtual corrections, vacuum polarization and (partially) interference between various contributions.

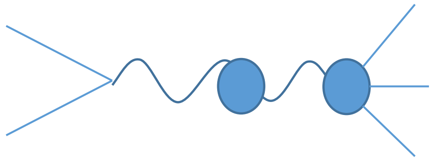
FSR from hadrons is model-dependent, e.g., assume point-like pions.

Vacuum polarization



$\sigma^0(e^+e^- \to \gamma \to X)$

In a_μ calculation



$\sigma(e^+e^- \to \gamma^* \to X)$

In experiment

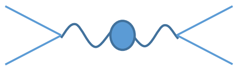
In the calculation of a_μ, we assume the lowest order photon propagator $1/q^2$. But the real propagator includes higher order effects (loop corrections): $1/(q^2 - \Pi(q^2))$. Therefore the measured cross section have to be corrected:

$$\sigma^0(e^+e^- \to X) = \sigma(e^+e^- \to X) \times \frac{|\alpha(s)|^2}{\alpha^2}$$

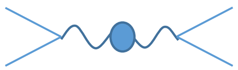
The running fine structure constant is also calculated via dispersion relation based on R(s):

$$\Delta\alpha_{had}(s) = -\frac{\alpha s}{3\pi}\int_0^\infty \frac{R(s')}{s'(s-s'-i0)}ds'$$

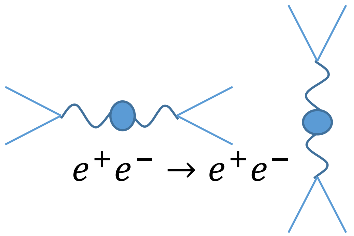
Nice way to avoid this correction is to use $e^+e^- \to \mu^+\mu^-$ for luminosity measurement



$e^+e^- \to X$



$e^+e^- \to \mu^+\mu^-$



$e^+e^- \to e^+e^-$

From measured cross section to input to a_μ calculation

“Visible” cross section
$\sigma(e^+e^-(\gamma) \rightarrow X(\gamma))$

Here we correct for all detector effects

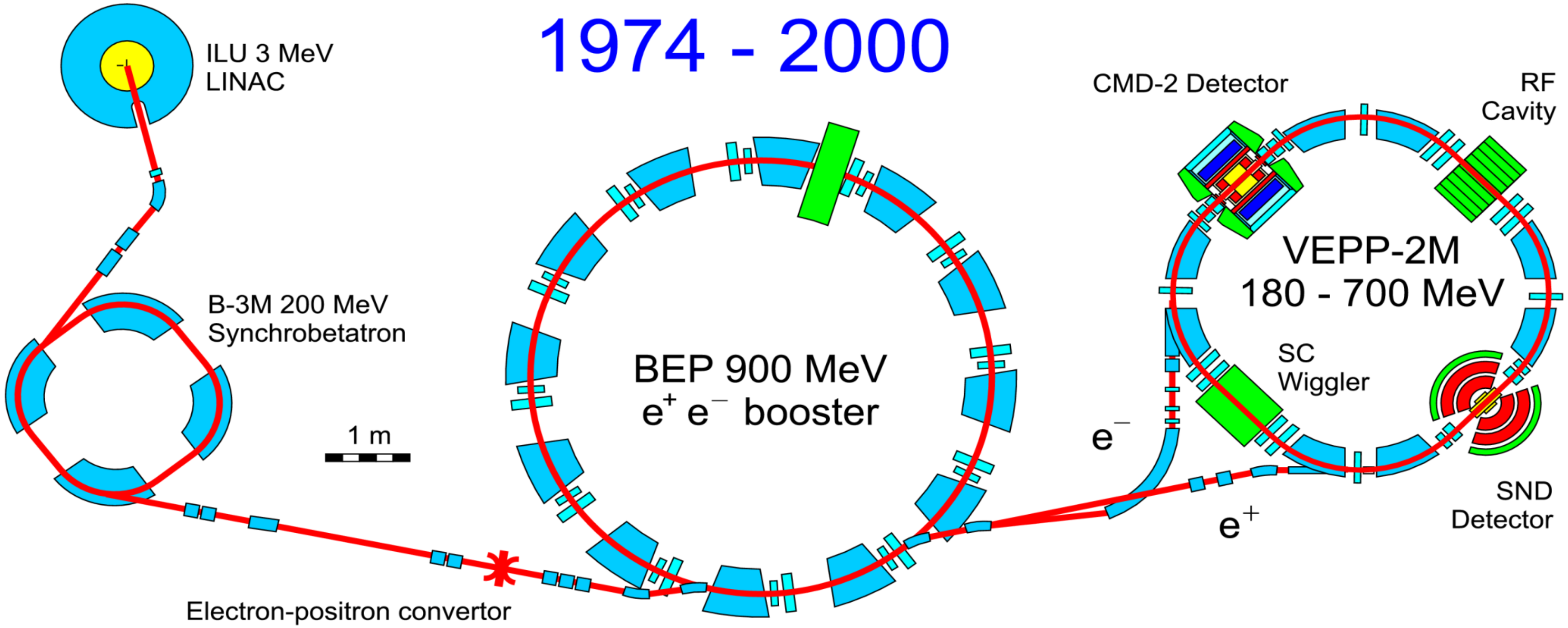
Adjust for radiative corrections (ISR, FSR)
$\sigma(e^+e^- \rightarrow X)$

This one is used to get parameters of the resonances (mass, width,…)

Adjust for vacuum polarization and return back FSR
$\sigma^0(e^+e^- \rightarrow X(\gamma))$

This one is used in the a_μ integral

VEPP-2M (1993-2000)



Energy range: 0.36 – 1.4 GeV

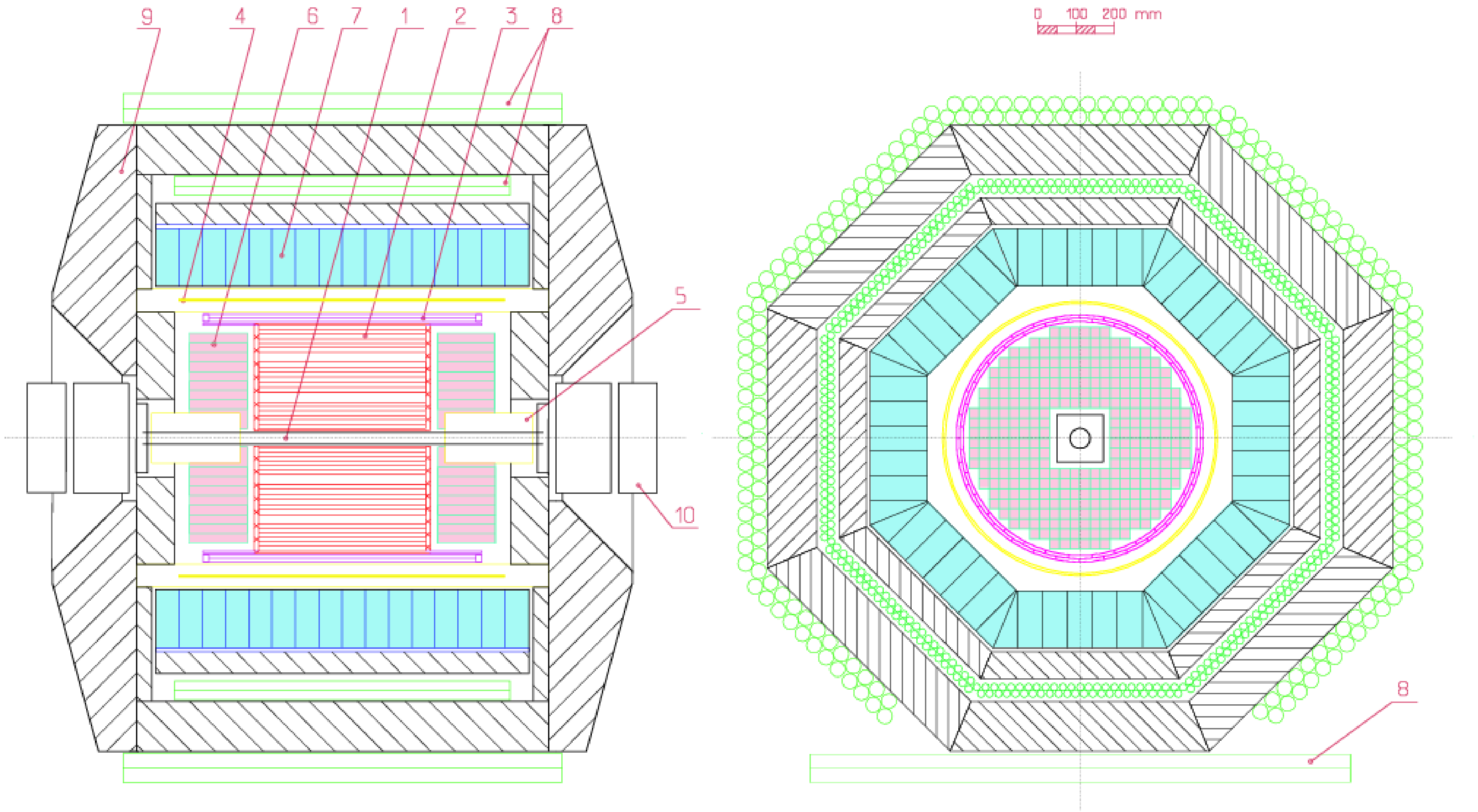
Luminosity up to 5*10^{30} 1/cm^{2}s

Lets set the scale:

$\sigma(e^+e^- \to \pi^+\pi^-)$ at ρ peak (0.77 GeV) ~ 1000 nb

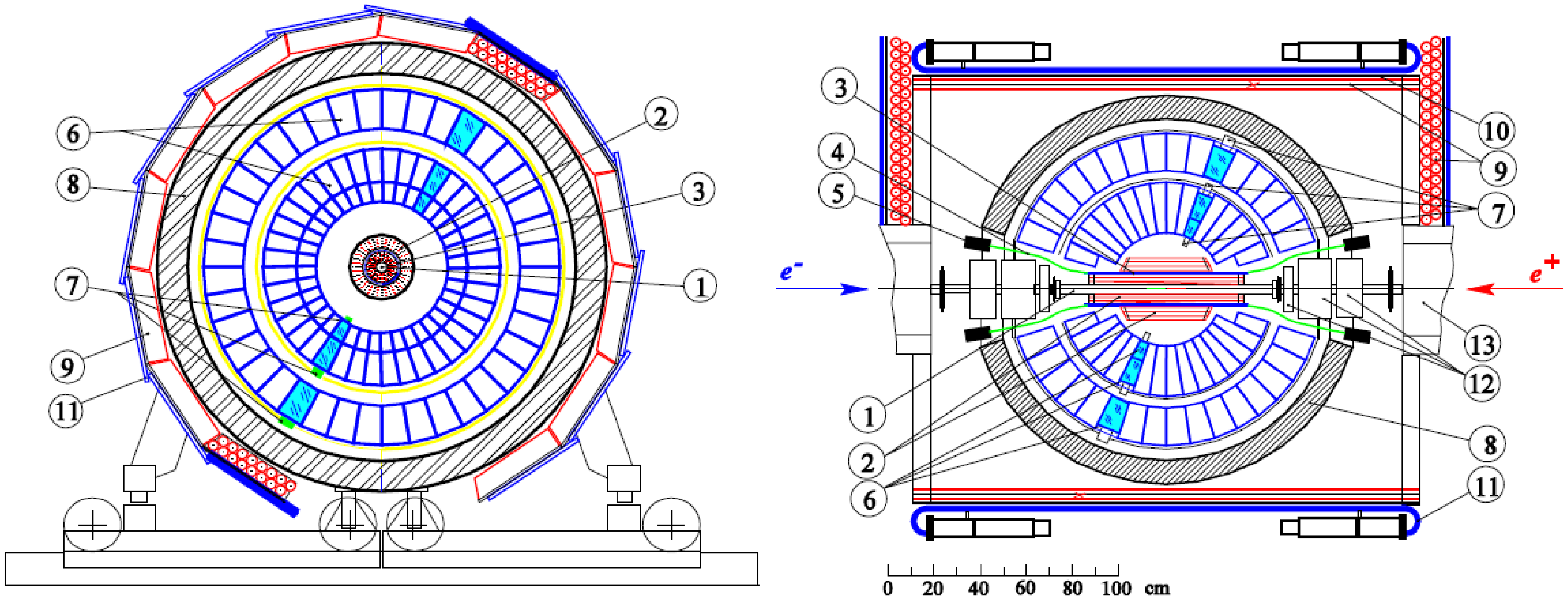
$L = 10^{30}\ cm^{-2}s^{-1}$ corresponds to 1 Hz for σ = 1000 nb

CMD-2



1 - vacuum chamber
2 - drift chamber
3 - **Z**-chamber
4 - main solenoid
5 - compensating magnet
6 - **BGO** endcap calorimeter
7 - **CsI** barrel calorimeter
8 - muon range system
9 - iron yoke
10 - storage ring lenses

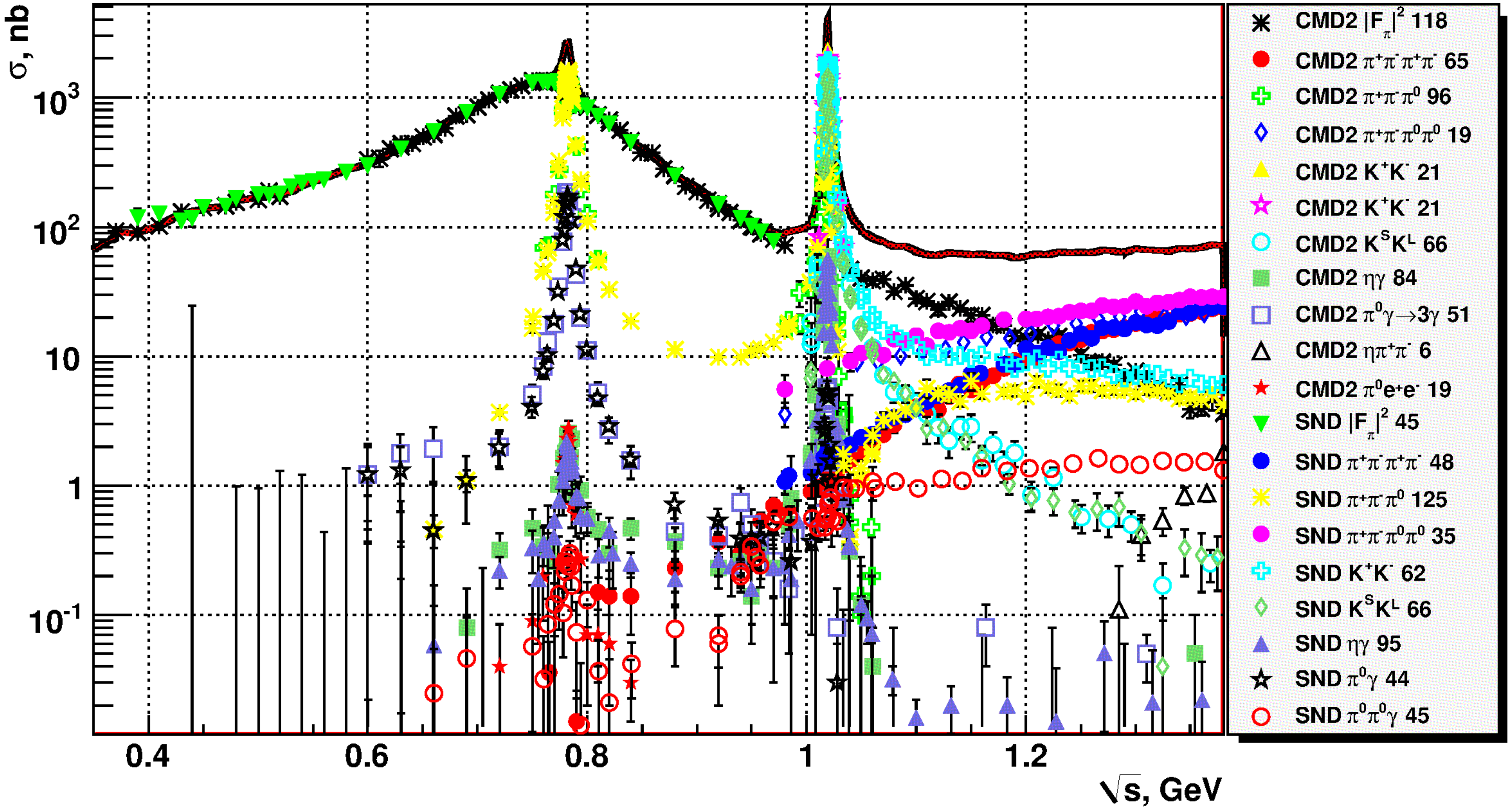
SND



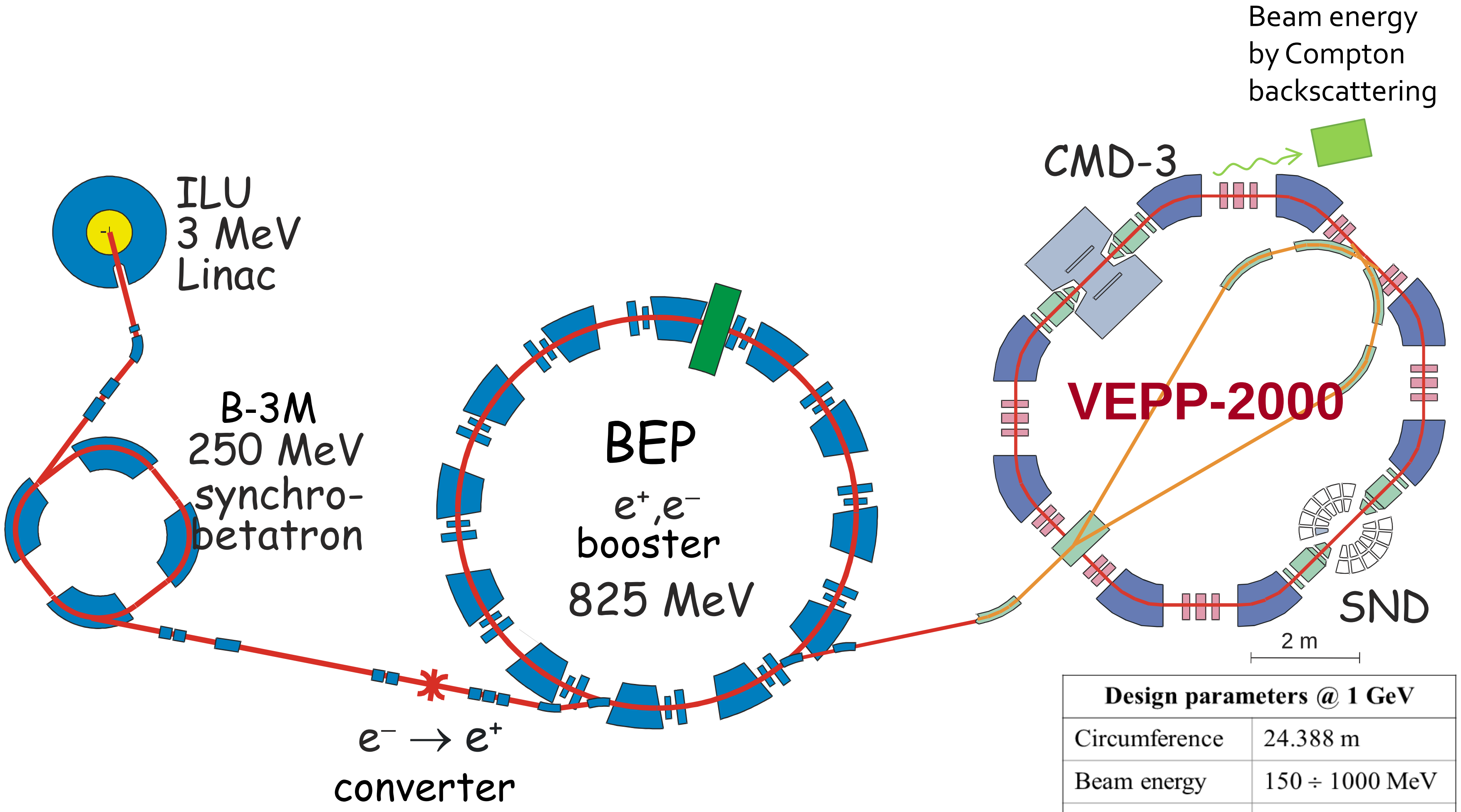
- No magnetic field
- Spherical three-layer NaI calorimeter
- Small drift chamber around interaction point

Optimized for neutral processes (e.g., $\pi^0\gamma$)

Overview of VEPP-2M measurements



VEPP-2000 (2011-2013)



C.m. energy range is 0.32-2.0 GeV

Unique optics – “round beams”

Experiments CMD-3 and SND started by the end of 2010

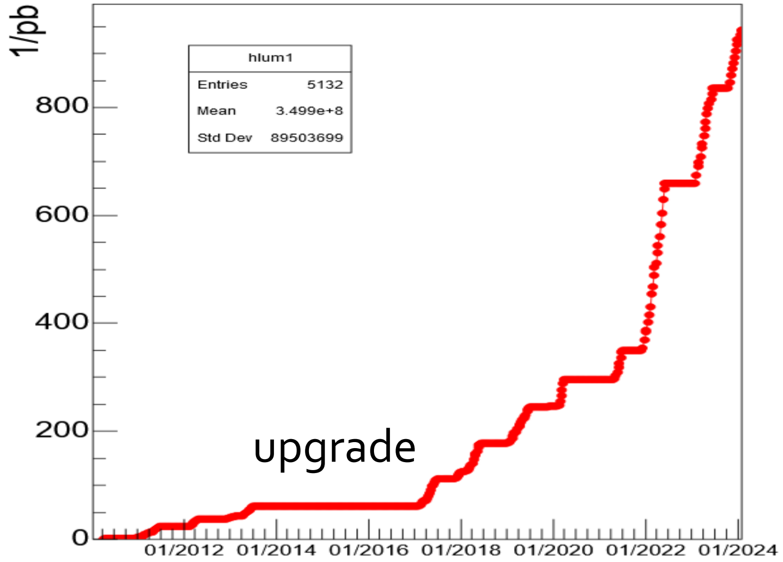
Design parameters @ 1 GeV	
Circumference	24.388 m
Beam energy	150 ÷ 1000 MeV
N of bunches	1×1
N of particles	1×10^{11}
Betatron tunes	4.14 / 2.14
Beta*	8.5 cm
BB parameter	0.1
Luminosity	1×10^{32} cm^{-2}s^{-1}

VEPP-2000 (2011-)

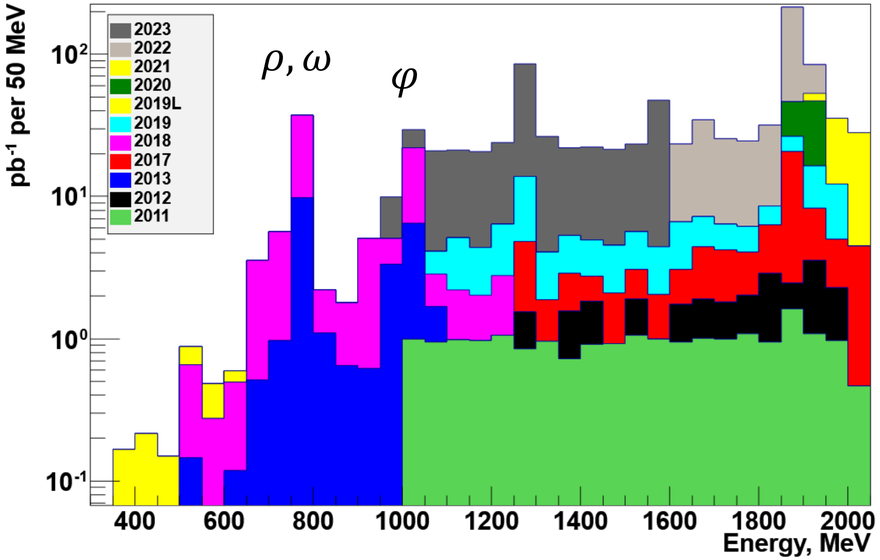
250 m beamline

VEPP-2000

New injection complex



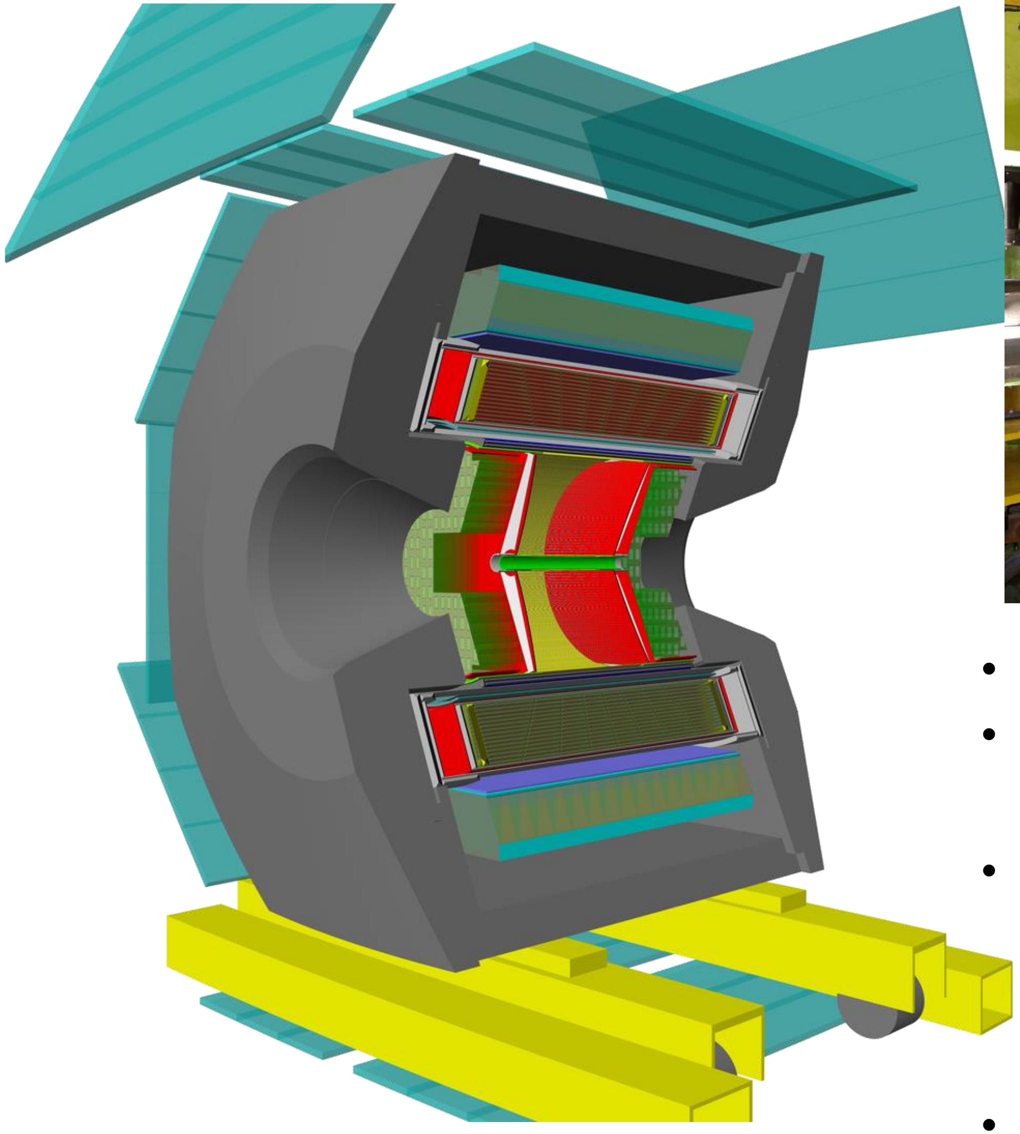
История набора интеграла светимости

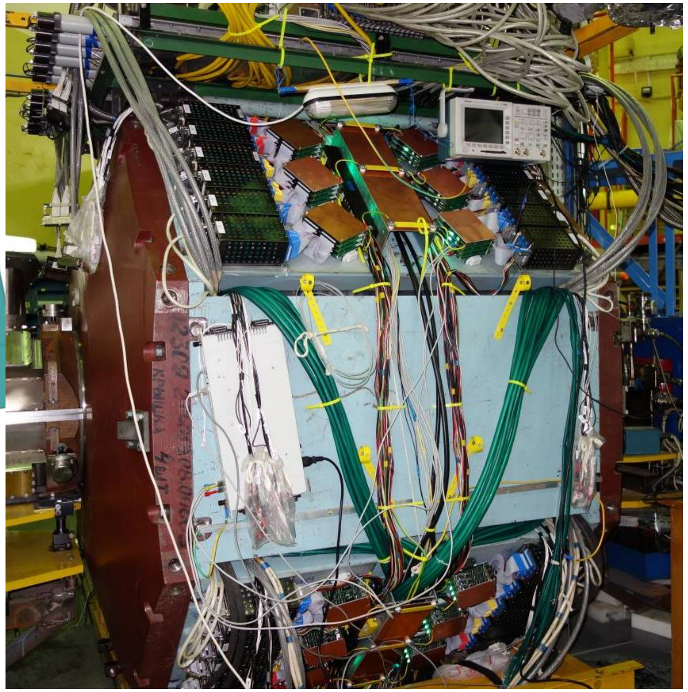


Распределение набранного интеграла светимости по энергии

CMD-3 Detector

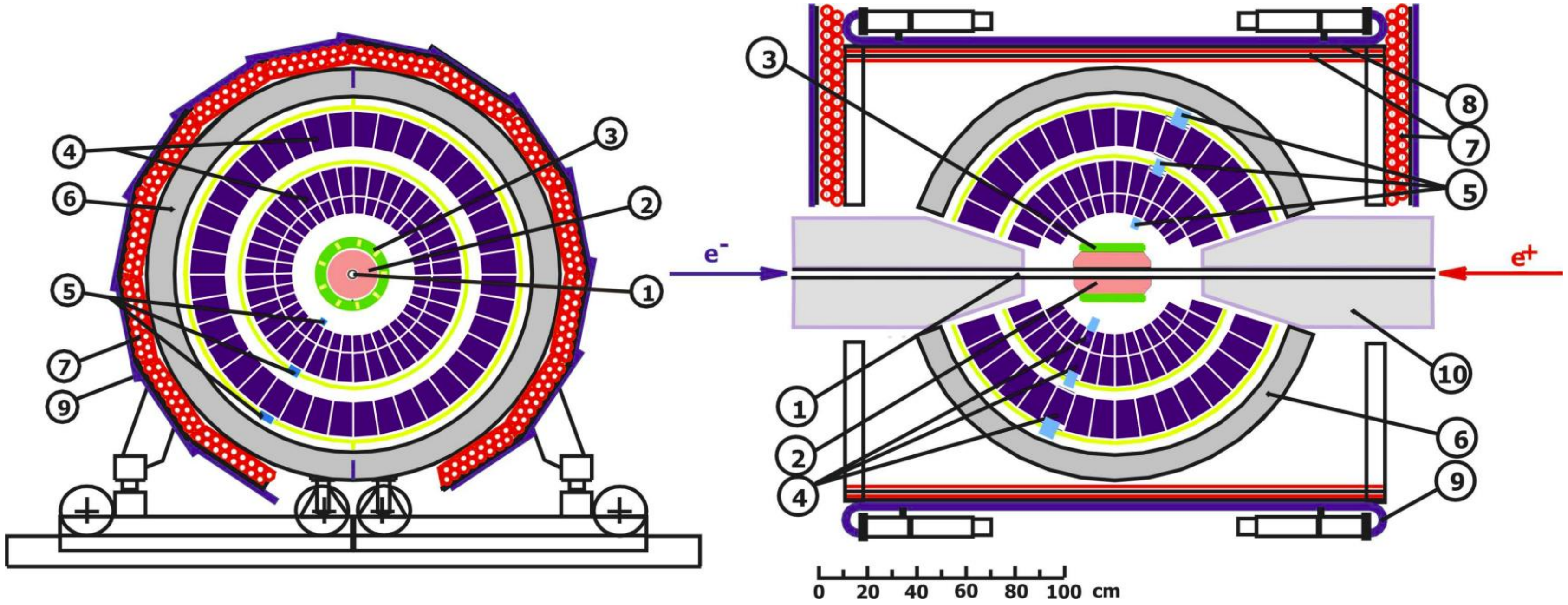
*Cryogenic Magnetic Detector





- Magnetic field 1.0-1.3 T
- Drift chamber
  - $\sigma_{R\varphi} \sim 100\ \mu, \sigma_z \sim 2-3$ mm
- EM calorimeter (LXE, CsI, BGO), 13.5 X_0
  - $\sigma_E/E \sim 3\% - 10\%$
  - $\sigma_\Theta \sim 5$ mrad
- TOF
- Muon counters

SND

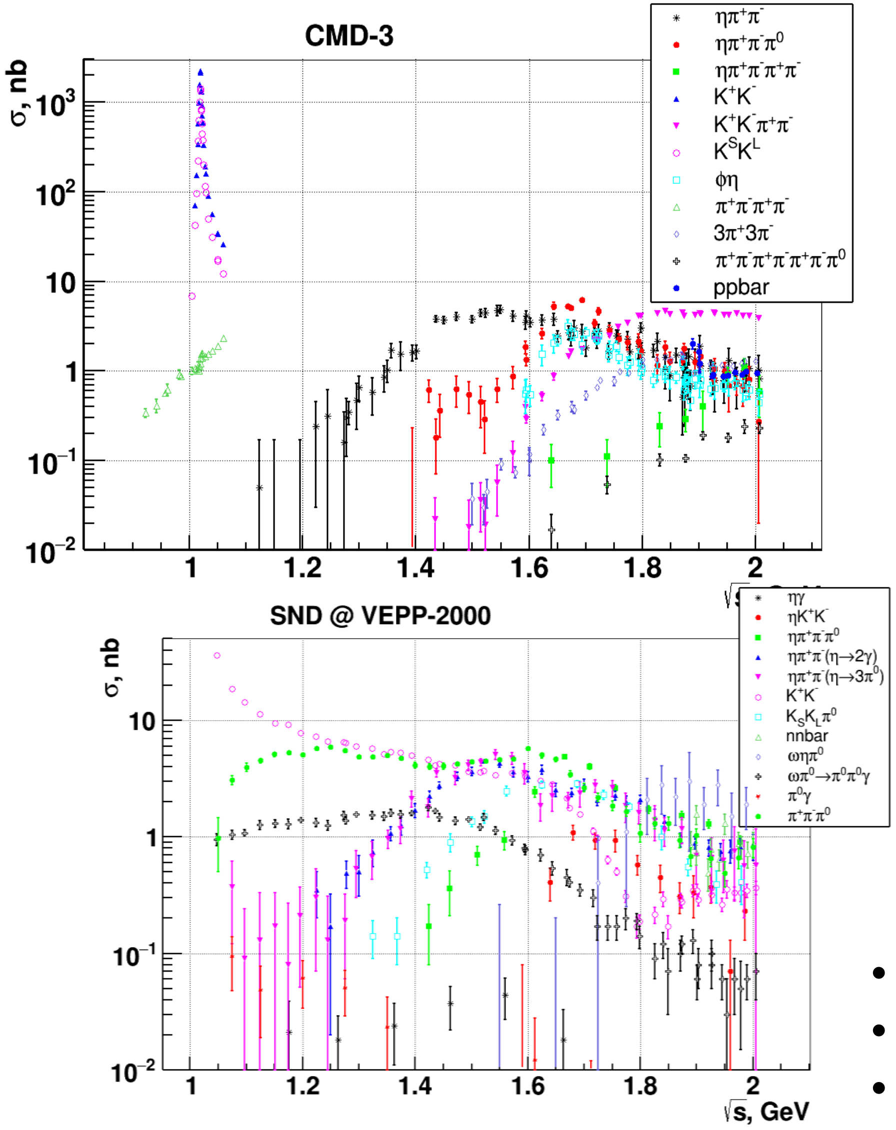


1 – beam pipe
2 – tracking system
3 – aerogel
4 – NaI(Tl) crystals
5 – phototriodes
6 – muon absorber
7–9 – muon detector
10 – focusing solenoid

Advantages compared to previous SND:

- new system - Cherenkov counter (n=1.05, 1.13)
  e/π separation E<450 MeV
  π/K separation E<1 GeV
- new drift chamber
  better tracking
  better determination of solid angle

Measurements at VEPP-2000



Final states under analysis at CMD-3

Signature	Final states (preliminary, published)
2 charged	$\pi^+\pi^-$, K^+K^-, K_SK_L, $p\overline{p}$
2 charged + γ's	$\pi^+\pi^-\gamma$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-2\pi^0$, $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $\pi^+\pi^-\eta$, $\pi^+\pi^-\pi^0\eta$, $\pi^+\pi^-2\pi^0\eta$, $K^+K^-\pi^0$, $K^+K^-2\pi^0$, $K^+K^-\eta$, $K_SK_L\pi^0$, $K_SK_L\eta$
4 charged	$2(\pi^+\pi^-)$, $K^+K^-\pi^+\pi^-$, $K_SK^\pm\pi^\mp$
4 charged + γ's	$2(\pi^+\pi^-)\pi^0$, $2\pi^+2\pi^-2\pi^0$, $\pi^+\pi^-\eta$, $\pi^+\pi^-\omega$, $2\pi^+2\pi^-\eta$, $K^+K^-\omega$, $K_SK^\pm\pi^\mp\pi^0$
6 charged	$3(\pi^+\pi^-)$, $K_SK_S\pi^+\pi^-$
6 charged + γ's	$3(\pi^+\pi^-)\pi^0$
Neutral	$\pi^0\gamma$, $2\pi^0\gamma$, $3\pi^0\gamma$, $\eta\gamma$, $\pi^0\eta\gamma$, $2\pi^0\eta\gamma$
Other	$n\overline{n}$, $\pi^0e^+e^-$, ηe^+e^-
Rare decays	η', $D^*(2007)^0$

- More final states compare to VEPP-2M
- 1-2 order of magnitude more data
- The experiments are collecting data

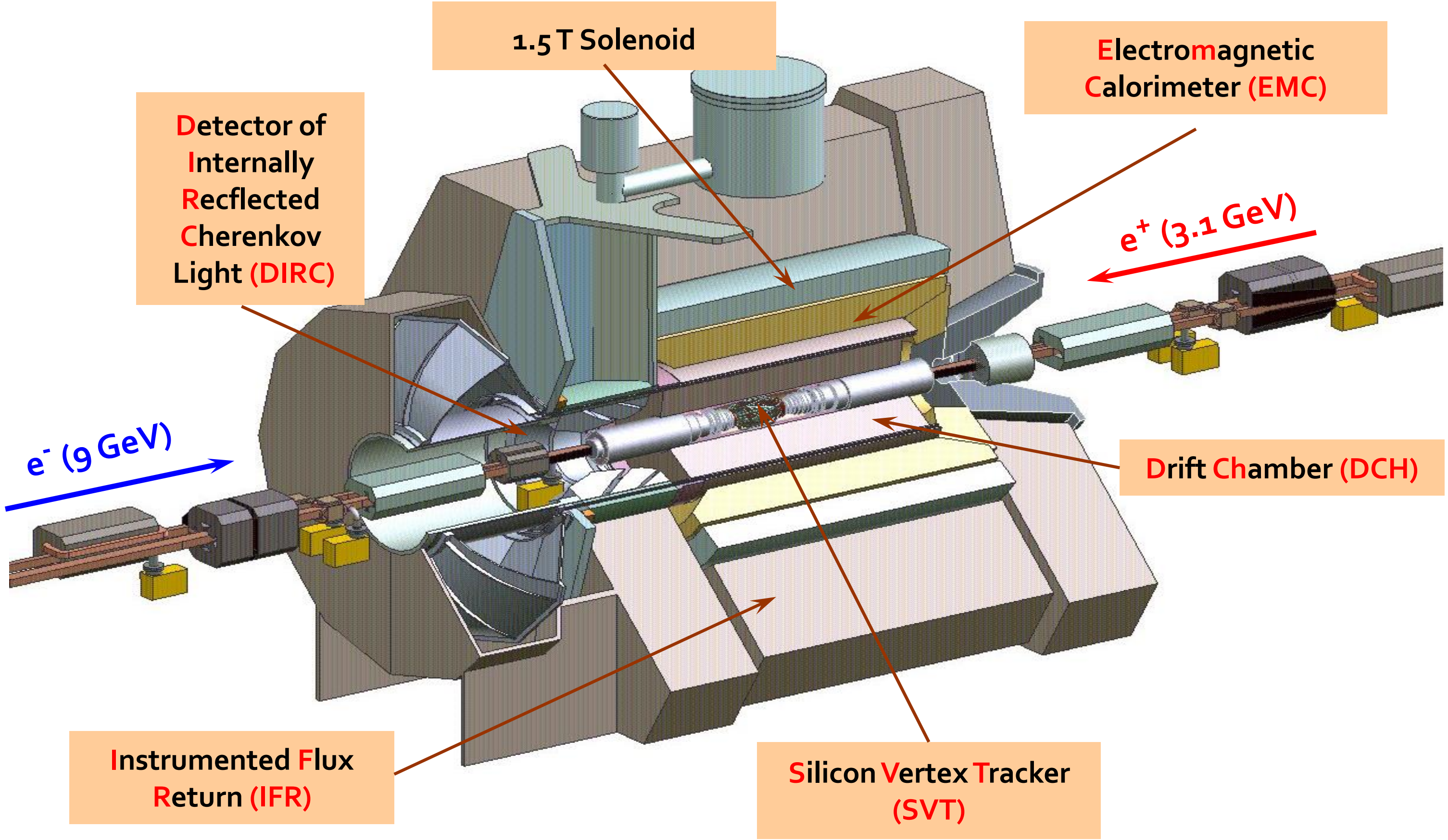
BABAR experiment (1999-2008)

PEP-II asymmetric e^+e^- collider at SLAC

9 GeV e^- and 3.1 GeV e^+

About 500 fb^{-1} collected in 1999-2008

Comprehensive program of ISR measurements, using a data sample of 469 fb-1 collected at and near $\Upsilon(4S)$ (10.58 GeV)



1.5 T Solenoid

Electromagnetic Calorimeter (EMC)

Detector of Internally Recflected Cherenkov Light (DIRC)

e^+ (3.1 GeV)

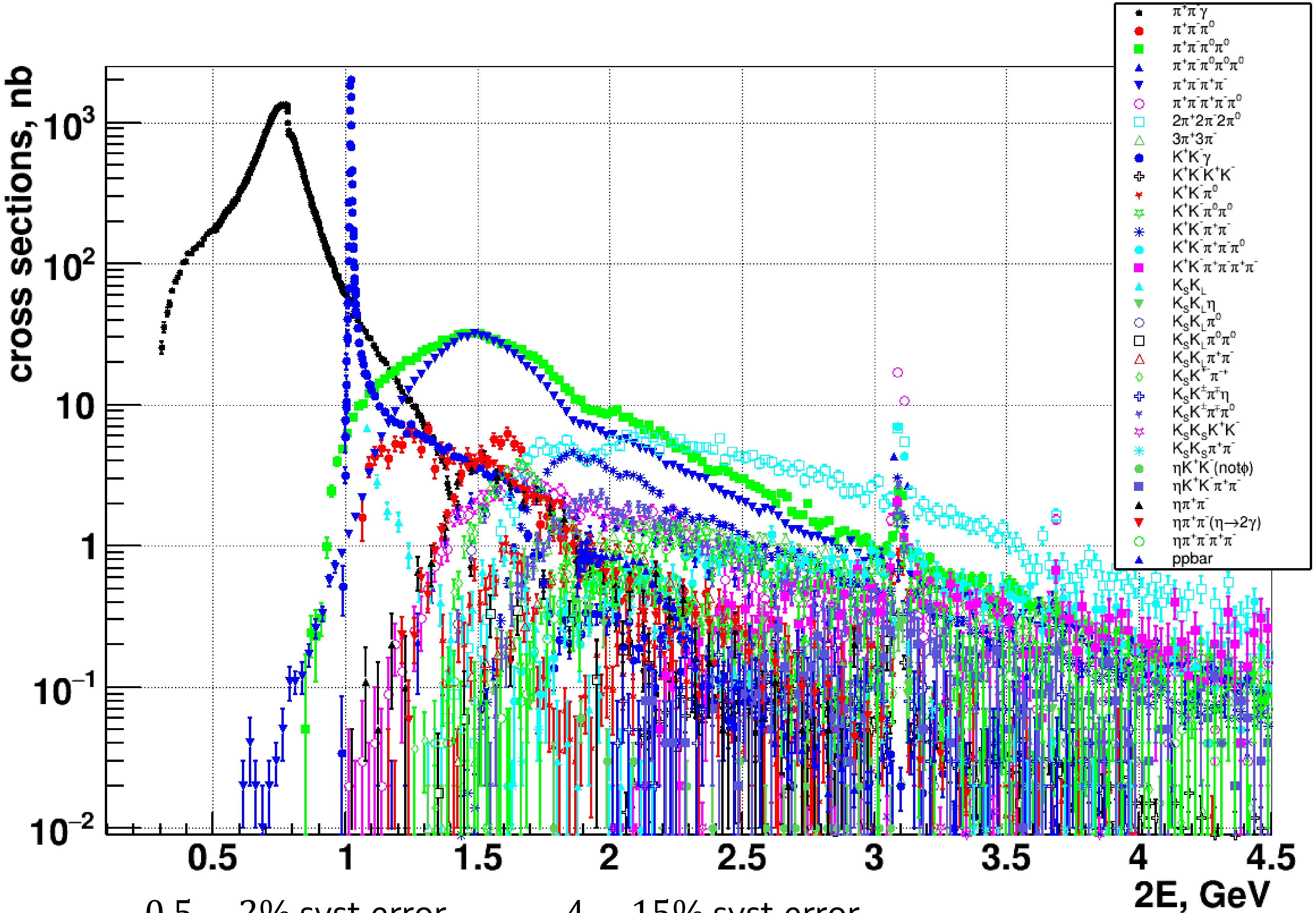
e^- (9 GeV)

Drift Chamber (DCH)

Instrumented Flux Return (IFR)

Silicon Vertex Tracker (SVT)

BABAR

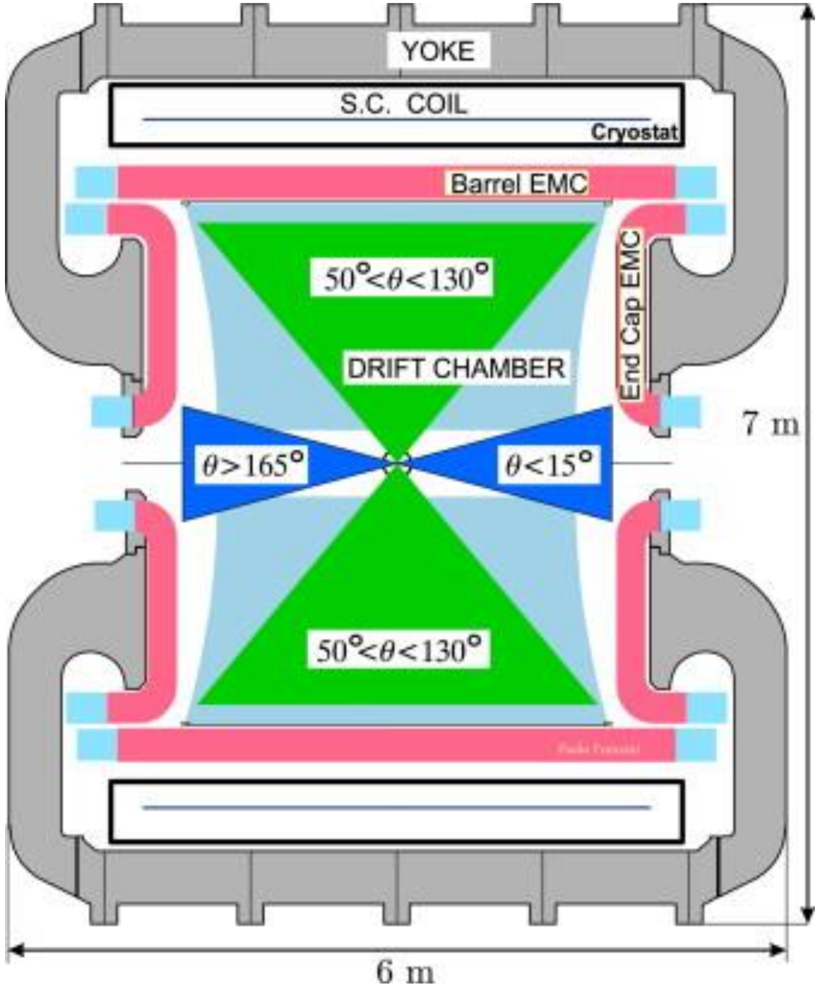


0.5 – 2% syst.error

4 – 15% syst.error

BABAR measurements are mostly tagged

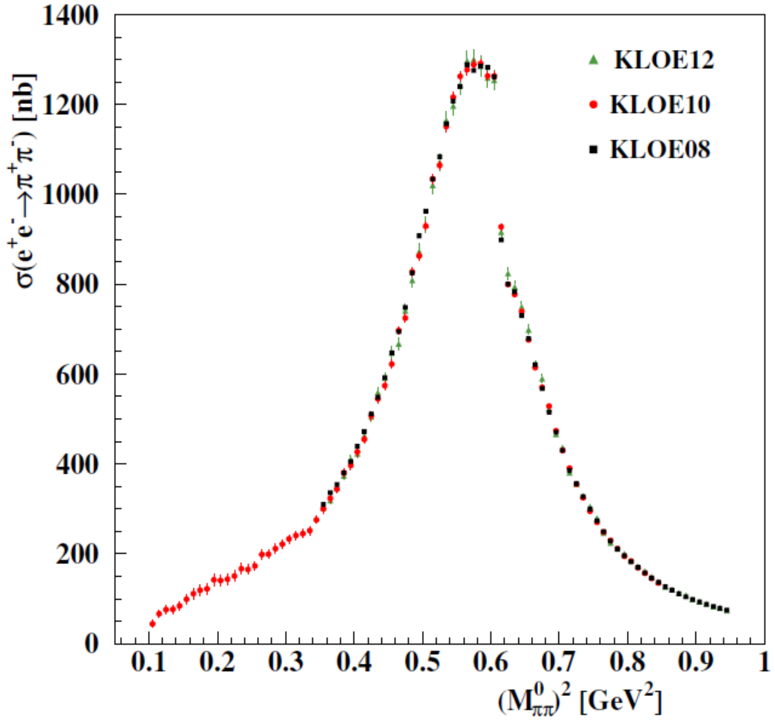
KLOE (2000-2006)



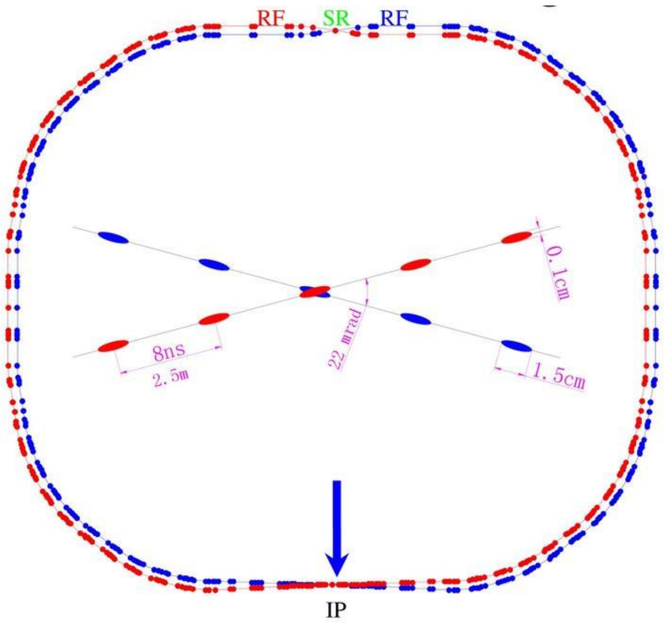
Installed at the DAFNE phi-factory

Mostly collected data at $\phi(1020)$ meson

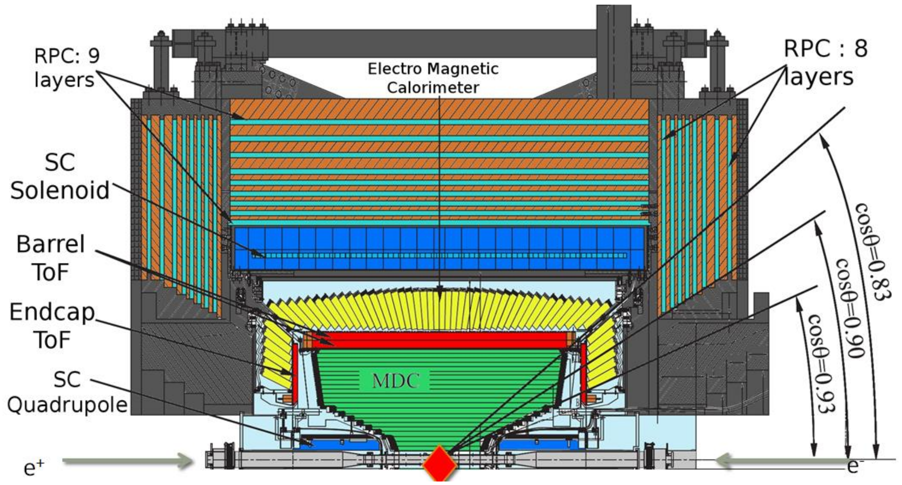
ISR measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$, both tagged and untagged



BES-III



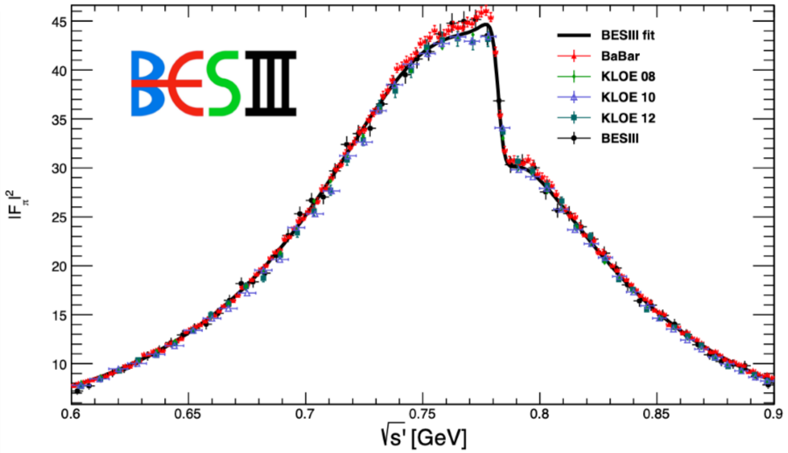
Beam energy:
1.0-2.3GeV
Luminosity:
1×10^{33} cm^{-2}s^{-1}
Optimum energy:
1.89 GeV
Energy spread:
5.16×10^{-4}
No. of bunches:
93
Bunch length:
1.5 cm
Total current:
0.91 A
SR mode:
0.25A @ 2.5 GeV



BEPC-II collider covers c.m.energy range from 2 to 5 GeV
"$c\tau$-factory"

BES-III detector is taking data
(and there were BES and BES-II before)

Tagged ISR measurement
$\sigma(e^+e^- \to \pi^+\pi^-)$

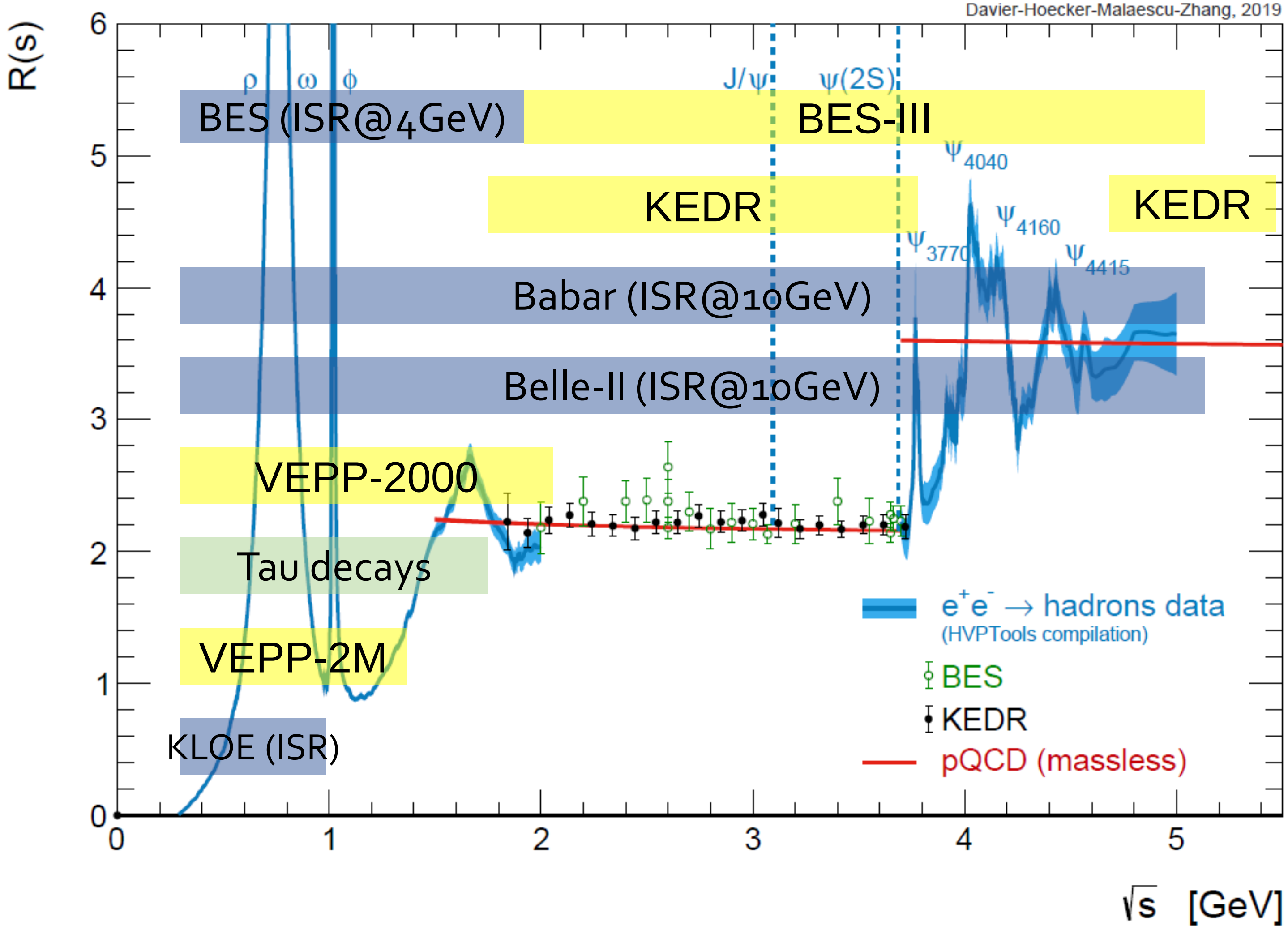


Statistics is limited compare to BaBar

Variety of ISR approaches

	Tagged ISR	Untagged ISR
Normalization to e^+e^-	KLOE-2010 ($\pi^+\pi^-$) BABAR (most channels)	KLOE-2005 ($\pi^+\pi^-$) KLOE-2008 ($\pi^+\pi^-$) BABAR ($p\bar{p}$)
Normalization to $\mu^+\mu^-(\gamma)$	BABAR ($\pi^+\pi^-$)* BES-III ($\pi^+\pi^-$) CLEO-c ($\pi^+\pi^-$)	KLOE-2012 ($\pi^+\pi^-$)

Where the measurements are done



Energy scan

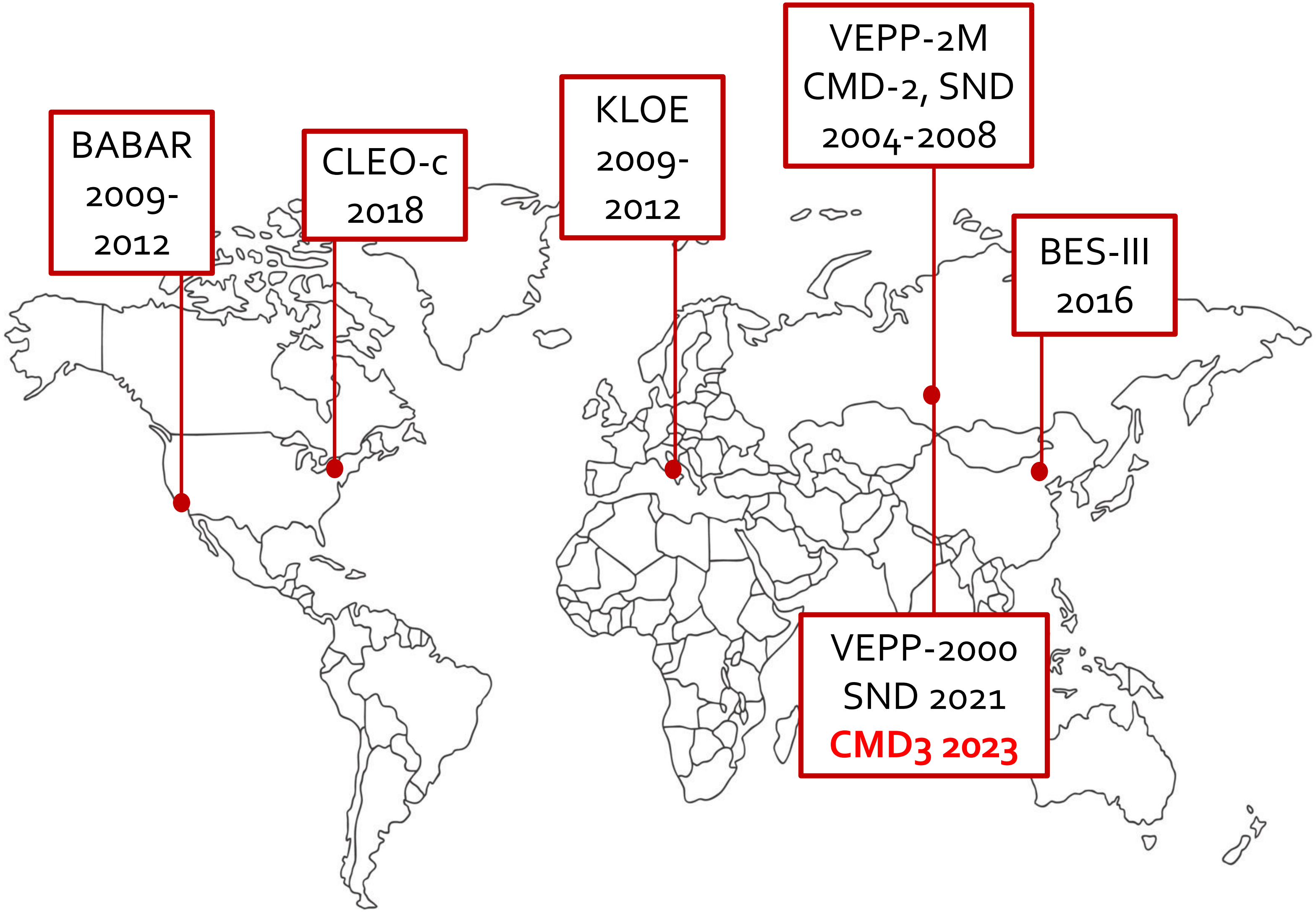
ISR

Tau decays

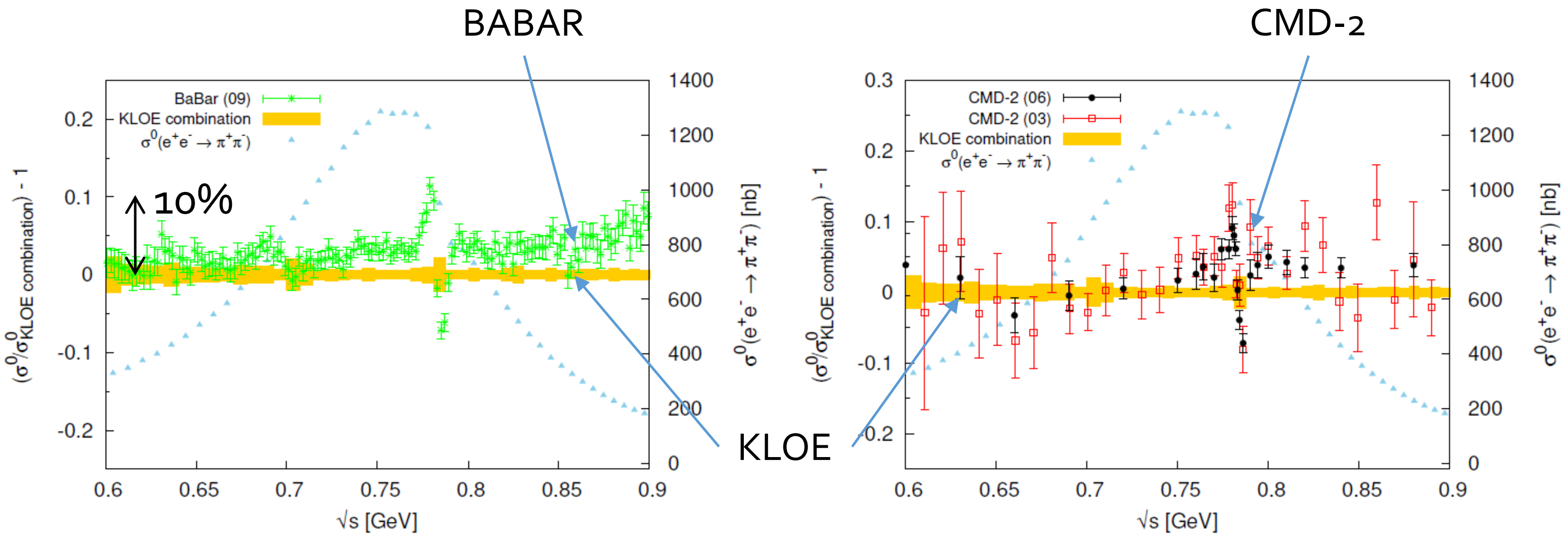
$$e^+e^- \to \pi^+\pi^-$$

Measurements of $e^+e^- \to \pi^+\pi^-$

There are several measurements of $\sigma(e^+e^- \to \pi^+\pi^-)$ with sub-percent systematic accuracy



Tensions in $e^+e^- \to \pi^+\pi^-$ data

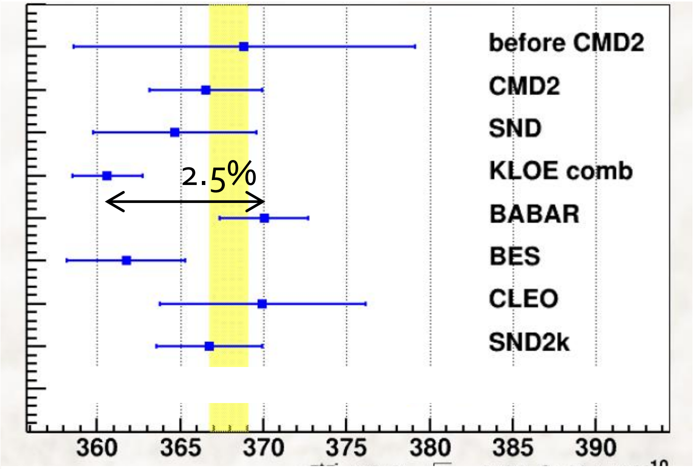


There are few-% discrepancies between various sub-% measurements of $\sigma(e^+e^- \to \pi^+\pi^-)$ *Unexplained*

WP2020: scale factor for $\Delta a_\mu(Had; LO)$

CMD-3 goal: new high statistics low systematics measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ via energy scan

$$a_\mu^{had}(LO; 2\pi, 0.6 < \sqrt{s} < 0.88\ \text{GeV})$$



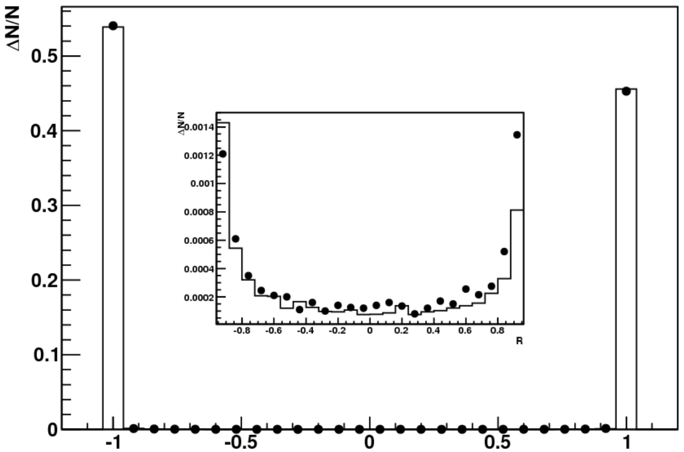
$$\frac{1}{4\pi^3}\int_{0.6}^{0.88} \sigma^0(e^+e^- \to \pi^+\pi^-)K_\mu(s)ds$$

$e^+e^- \to \pi^+\pi^-$ at SND (2021)

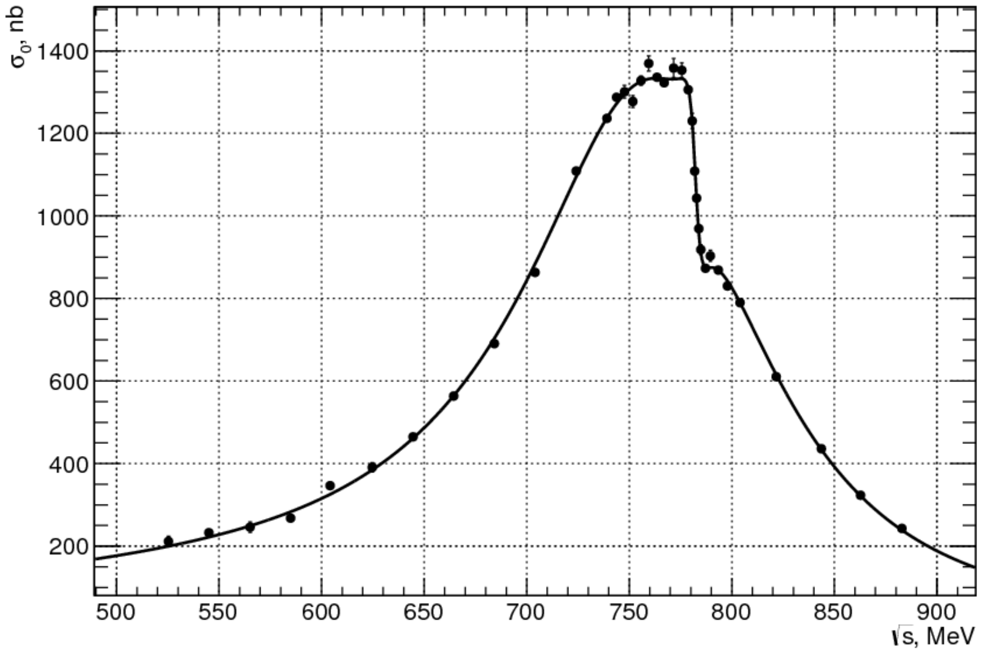
First measurement of
$e^+e^- \to \pi^+\pi^-$
at VEPP-2000

The analysis is based on $4.7\ \mathrm{pb}^{-1}$ data recorded in 2013 (1/10 full SND data set)

π/e separation using ML (BDT)



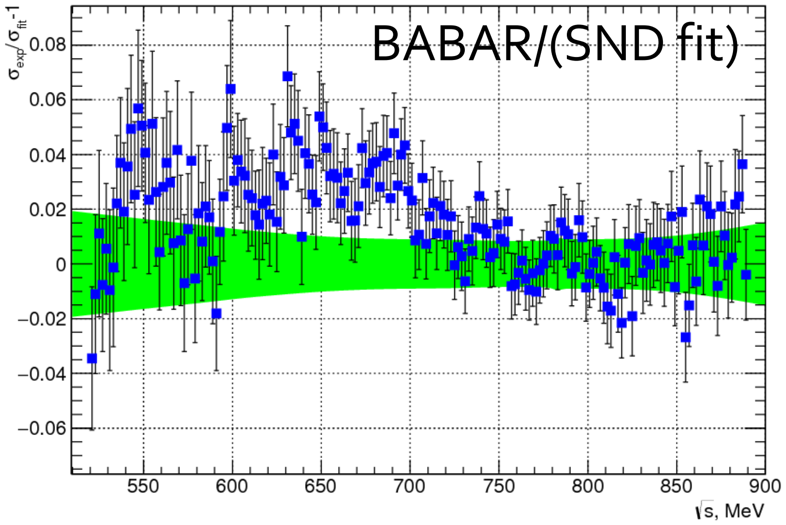
JHEP 2021,113 (2021)

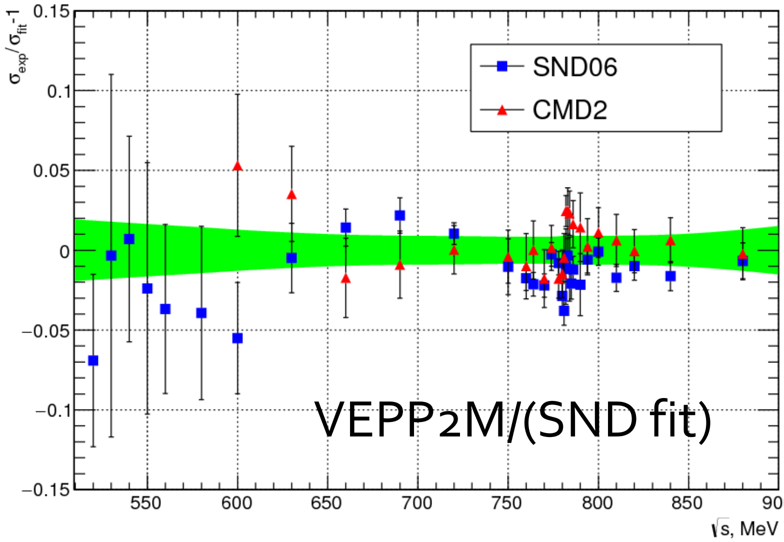


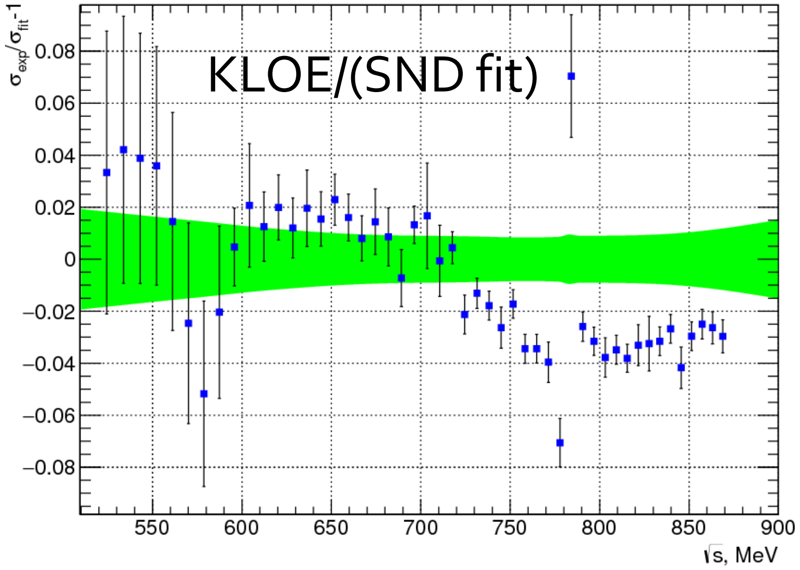
Systematic uncertainty on the cross section (%)

Source	**< 0.6 GeV**	**0.6 - 0.9 GeV**
Trigger	0.5	0.5
Selection criteria	0.6	0.6
e/π separation	0.5	0.1
Nucl. interaction	0.2	0.2
Theory	0.2	0.2
Total	**0.9**	**0.8**

$e^+e^- \to \pi^+\pi^-$ at SND (2021): comparison to other measurements







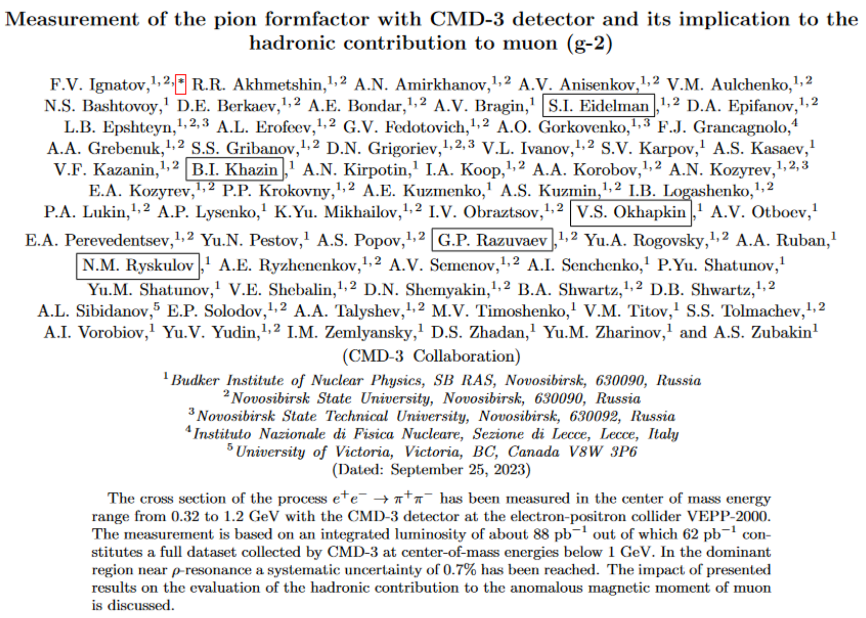
$0.53 < \sqrt{s} < 0.88$ GeV

	$a_\mu(\pi^+\pi^-) \times 10^{10}$
SND & VEPP-2000	$409.8 \pm 1.4 \pm 3.9$
SND & VEPP-2M	$406.5 \pm 1.7 \pm 5.3$
BABAR	$413.6 \pm 2.0 \pm 2.3$
KLOE	$403.4 \pm 0.7 \pm 2.5$

CMD-3 measurement of $e^+e^- \to \pi^+\pi^-$ (2023)

Publications

arXiv:2309.12910



Measurement of the pion formfactor with CMD-3 detector and its implication to the hadronic contribution to muon (g-2)

F.V. Ignatov,[1,2,*] R.R. Akhmetshin,[1,2] A.N. Amirkhanov,[1,2] A.V. Anisenkov,[1,2] V.M. Aulchenko,[1,2]
N.S. Bashtovoy,[1] D.E. Berkaev,[1,2] A.E. Bondar,[1,2] A.V. Bragin,[1] S.I. Eidelman,[1,2] D.A. Epifanov,[1,2]
L.B. Epshteyn,[1,2,3] A.L. Erofeev,[1,2] G.V. Fedotovich,[1,2] A.O. Gorkovenko,[1,3] F.J. Grancagnolo,[4]
A.A. Grebenuk,[1,2] S.S. Gribanov,[1,2] D.N. Grigoriev,[1,2,3] V.L. Ivanov,[1,2] S.V. Karpov,[1] A.S. Kasaev,[1]
V.F. Kazanin,[1,2] B.I. Khazin,[1] A.N. Kirpotin,[1] I.A. Koop,[1,2] A.A. Korobov,[1,2] A.N. Kozyrev,[1,2,3]
E.A. Kozyrev,[1,2] P.P. Krokovny,[1,2] A.E. Kuzmenko,[1] A.S. Kuzmin,[1,2] I.B. Logashenko,[1,2]
P.A. Lukin,[1,2] A.P. Lysenko,[1] K.Yu. Mikhailov,[1,2] I.V. Obraztsov,[1,2] V.S. Okhapkin,[1] A.V. Otboev,[1]
E.A. Perevedentsev,[1,2] Yu.N. Pestov,[1] A.S. Popov,[1,2] G.P. Razuvaev,[1,2] Yu.A. Rogovsky,[1,2] A.A. Ruban,[1]
N.M. Ryskulov,[1] A.E. Ryzhenenkov,[1,2] A.V. Semenov,[1,2] A.I. Senchenko,[1] P.Yu. Shatunov,[1]
Yu.M. Shatunov,[1] V.E. Shebalin,[1,2] D.N. Shemyakin,[1,2] B.A. Shwartz,[1,2] D.B. Shwartz,[1,2]
A.L. Sibidanov,[5] E.P. Solodov,[1,2] A.A. Talyshev,[1,2] M.V. Timoshenko,[1] V.M. Titov,[1] S.S. Tolmachev,[1,2]
A.I. Vorobiov,[1] Yu.V. Yudin,[1,2] I.M. Zemlyansky,[1] D.S. Zhadan,[1] Yu.M. Zharinov,[1] and A.S. Zubakin[1]
(CMD-3 Collaboration)

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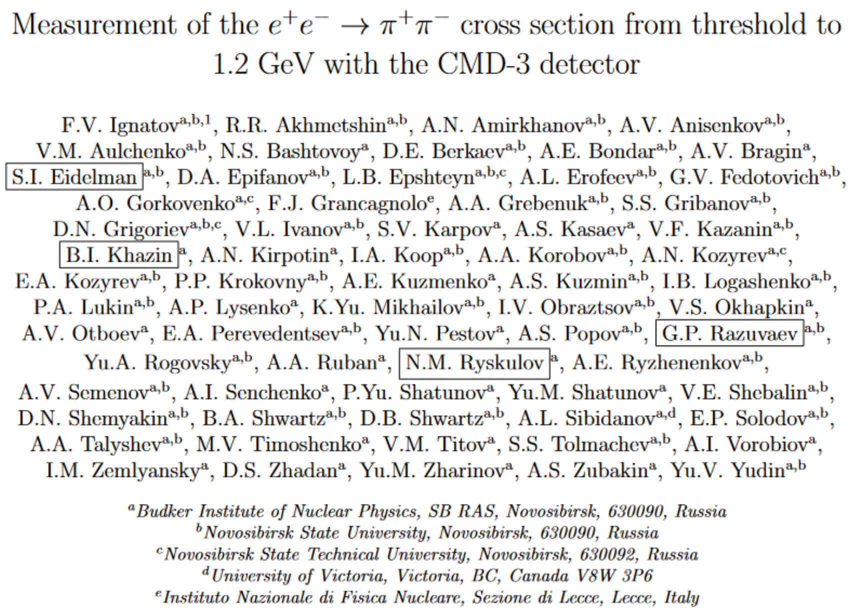
(Dated: September 25, 2023)

The cross section of the process $e^+e^- \to \pi^+\pi^-$ has been measured in the center of mass energy range from 0.32 to 1.2 GeV with the CMD-3 detector at the electron-positron collider VEPP-2000. The measurement is based on an integrated luminosity of about 88 pb^{-1} out of which 62 pb^{-1} constitutes a full dataset collected by CMD-3 at center-of-mass energies below 1 GeV. In the dominant region near ρ-resonance a systematic uncertainty of 0.7% has been reached. The impact of presented results on the evaluation of the hadronic contribution to the anomalous magnetic moment of muon is discussed.

Submitted to PRL

arXiv:2302.08834



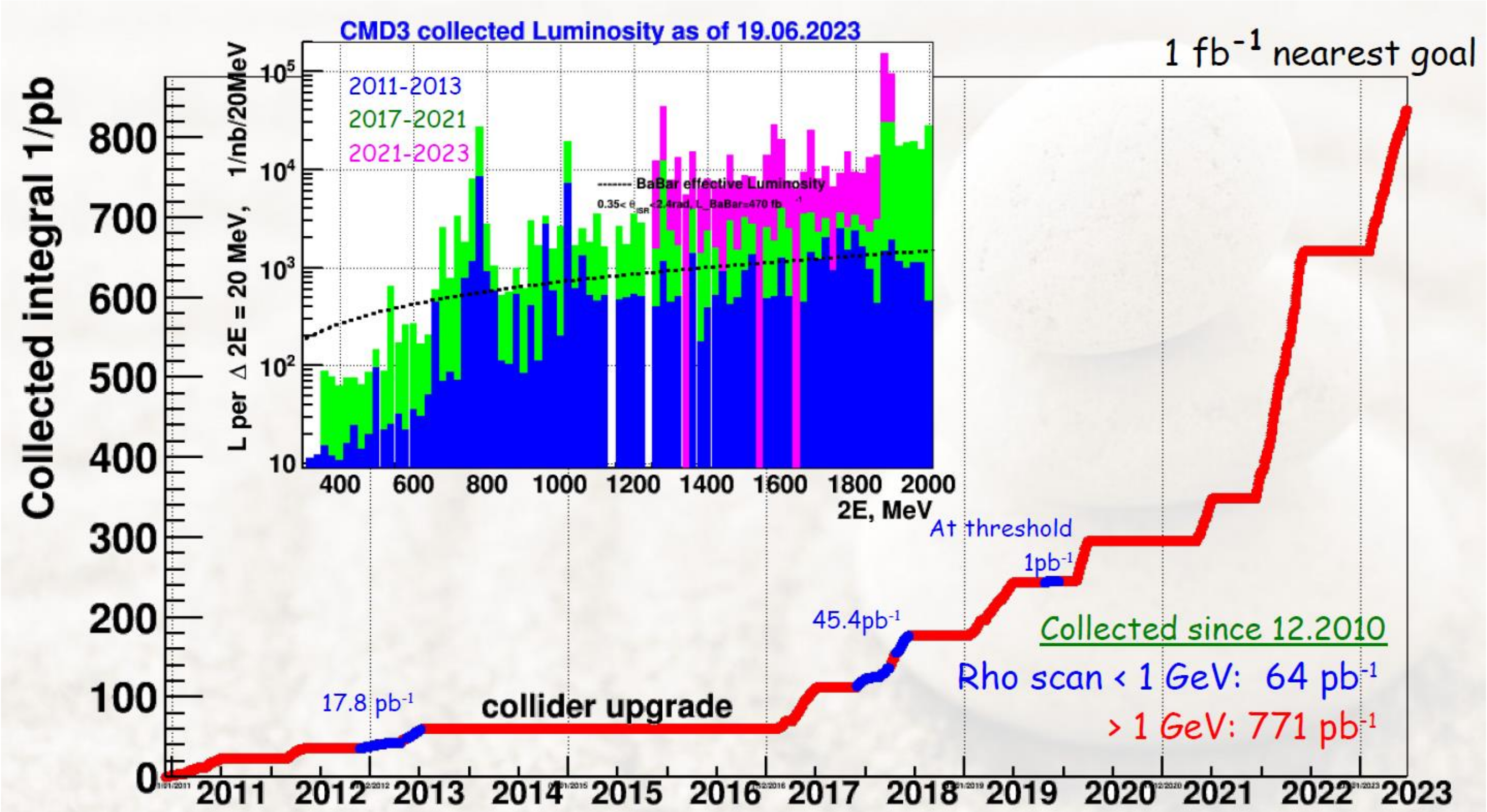
Measurement of the $e^+e^- \to \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector

F.V. Ignatov[a,b,1], R.R. Akhmetshin[a,b], A.N. Amirkhanov[a,b], A.V. Anisenkov[a,b],
V.M. Aulchenko[a,b], N.S. Bashtovoy[a], D.E. Berkaev[a,b], A.E. Bondar[a,b], A.V. Bragin[a],
S.I. Eidelman[a,b], D.A. Epifanov[a,b], L.B. Epshteyn[a,b,c], A.L. Erofeev[a,b], G.V. Fedotovich[a,b],
A.O. Gorkovenko[a,c], F.J. Grancagnolo[e], A.A. Grebenuk[a,b], S.S. Gribanov[a,b],
D.N. Grigoriev[a,b,c], V.L. Ivanov[a,b], S.V. Karpov[a], A.S. Kasaev[a], V.F. Kazanin[a,b],
B.I. Khazin[a], A.N. Kirpotin[a], I.A. Koop[a,b], A.A. Korobov[a,b], A.N. Kozyrev[a,c],
E.A. Kozyrev[a,b], P.P. Krokovny[a,b], A.E. Kuzmenko[a], A.S. Kuzmin[a,b], I.B. Logashenko[a,b],
P.A. Lukin[a,b], A.P. Lysenko[a], K.Yu. Mikhailov[a,b], I.V. Obraztsov[a,b], V.S. Okhapkin[a],
A.V. Otboev[a], E.A. Perevedentsev[a,b], Yu.N. Pestov[a], A.S. Popov[a,b], G.P. Razuvaev[a,b],
Yu.A. Rogovsky[a,b], A.A. Ruban[a], N.M. Ryskulov[a], A.E. Ryzhenenkov[a,b],
A.V. Semenov[a,b], A.I. Senchenko[a], P.Yu. Shatunov[a], Yu.M. Shatunov[a], V.E. Shebalin[a,b],
D.N. Shemyakin[a,b], B.A. Shwartz[a,b], D.B. Shwartz[a,b], A.L. Sibidanov[a,d], E.P. Solodov[a,b],
A.A. Talyshev[a,b], M.V. Timoshenko[a], V.M. Titov[a], S.S. Tolmachev[a,b], A.I. Vorobiov[a],
I.M. Zemlyansky[a], D.S. Zhadan[a], Yu.M. Zharinov[a], A.S. Zubakin[a], Yu.V. Yudin[a,b]

[a] *Budker Institute of Nuclear Physics, SB RAS, Novosibirsk, 630090, Russia*
[b] *Novosibirsk State University, Novosibirsk, 630090, Russia*
[c] *Novosibirsk State Technical University, Novosibirsk, 630092, Russia*
[d] *University of Victoria, Victoria, BC, Canada V8W 3P6*
[e] *Instituto Nazionale di Fisica Nucleare, Sezione di Lecce, Lecce, Italy*

Submitted to PRD

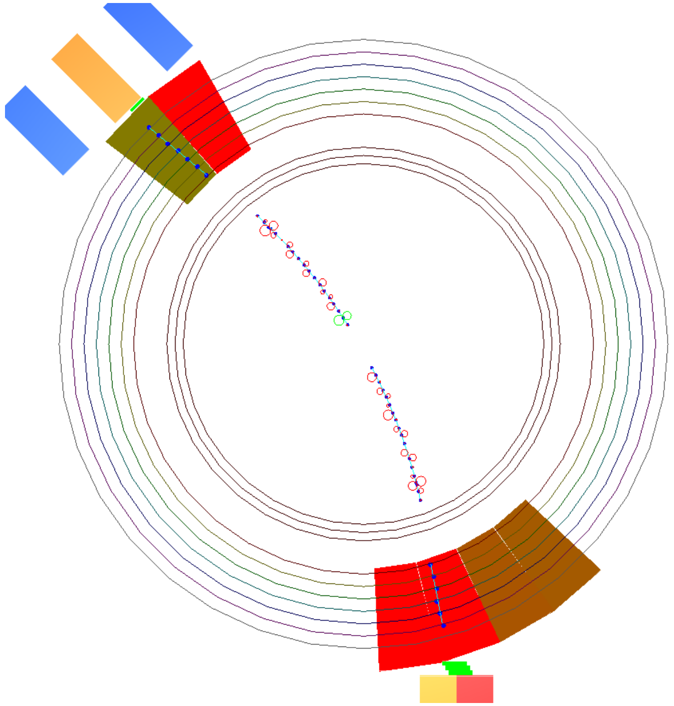
CMD-3 collected data



The result is based on 3 data taking seasons: 2013, 2018, 2020

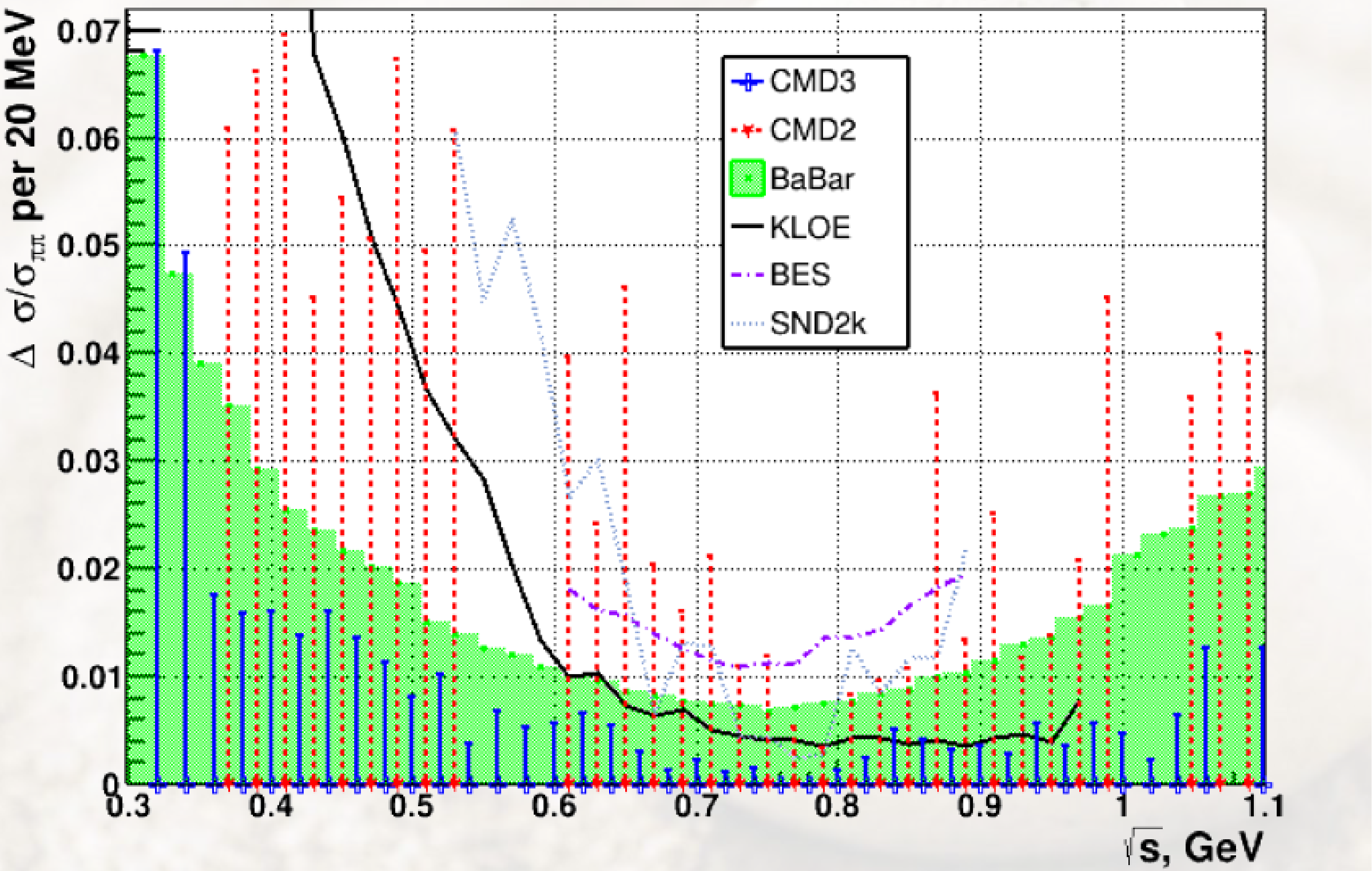
Features of CMD-3 measurement

- World-largest statistics
  - 34 000 000 $e^+e^- \to \pi^+\pi^-$
  - 3 700 000 $e^+e^- \to \mu^+\mu^-$
  - 44 000 000 $e^+e^- \to e^+e^-$
- Many built-in cross checks
  - 3 methods for final states indentification
  - 2 methods for angle measurement
  - Measurement of $\sigma(e^+e^- \to \mu^+\mu^-)$
  - Measurement of charge asymmetry
- Very detailed study of potential systematics



Example of $e^+e^- \to \pi^+\pi^-$ event

Statistical precision of CMD-3 data



based on L = 61.9 pb^{-1} at $\sqrt{s}$ < 1 GeV (+25.7 pb^{-1}, 1.0-1.2 GeV)

CMD-3 $e^+e^- \to \pi^+\pi^-$ analysis

Select events with 2 back-to-back tracks in the detector at large angle:

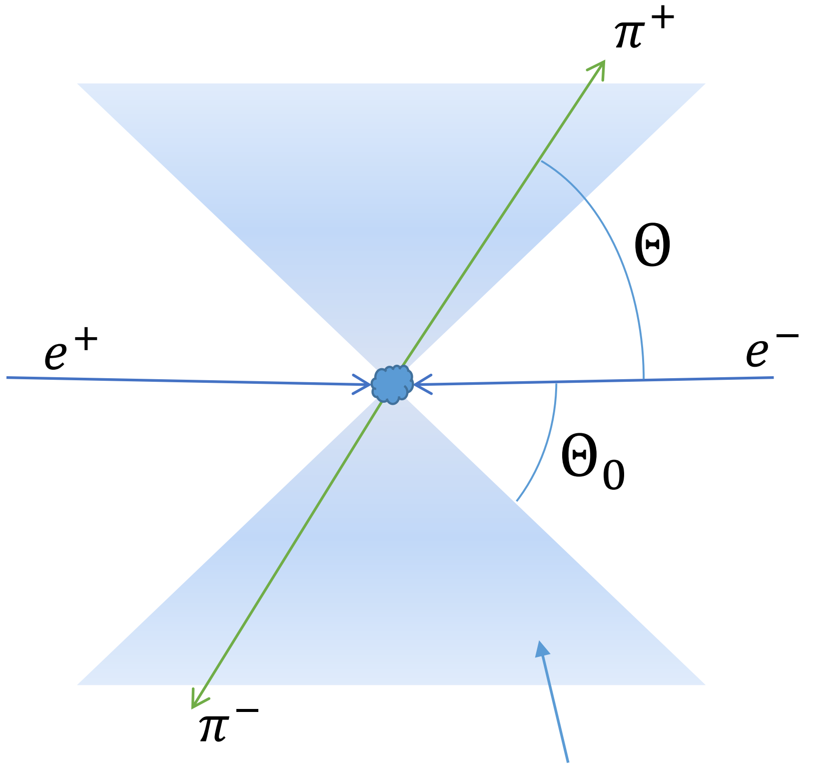
$$e^+e^- \to e^+e^-, \mu^+\mu^-, \pi^+\pi^-$$

and cosmic background

Key pieces of analysis to reach high precision:

- $e/\mu/\pi$ separation
- radiative corrections
- fiducial volume
- detection efficiency corrections

$e^+e^- \to e^+e^-, \mu^+\mu^-, \pi^+\pi^-$; cosmic bg



fiducial volume
$\Theta_0 \le \Theta_{avr} \le (\pi - \Theta_0)$
$\Theta_0 = 1.0 \dots 1.4$

$$\sigma(\pi^+\pi^-) = \frac{\pi\alpha^2}{3s}\beta_\pi^3 \cdot |F_\pi|^2$$

$$|F_\pi|^2 = \left(\frac{N_{\pi\pi}}{N_{ee}} - \Delta_{bg}\right) \cdot \frac{\sigma_{ee}^0 \cdot (1+\delta_{ee}) \cdot \varepsilon_{ee}}{\sigma_{\pi\pi}^0 \cdot (1+\delta_{\pi\pi}) \cdot \varepsilon_{\pi\pi}}$$

measured   Born cross-section   Detection efficiencies
Radiative corrections

Three methods of separation of $e^+e^-, \mu^+\mu^-, \pi^+\pi^-$

Separation (counting) of $e^+e^-, \mu^+\mu^-, \pi^+\pi^-$ events is based on

a) **momenta** of two particles

b) or **energy deposition** in LXe calorimeter

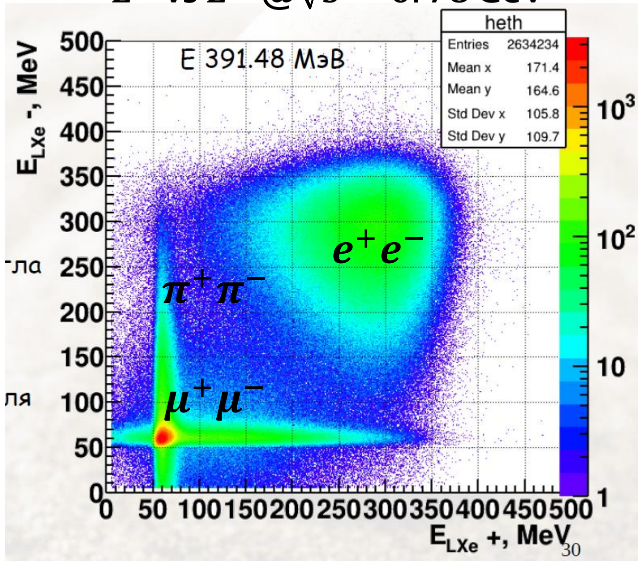
$$-\ln L = -\sum_{bins} n_i \ln\left[\sum_{a=ee,\mu\mu,\pi\pi,bg} N_a f_a(X^+, X^-)\right] + \sum_a N_a$$

$$X = P \text{ or } E$$

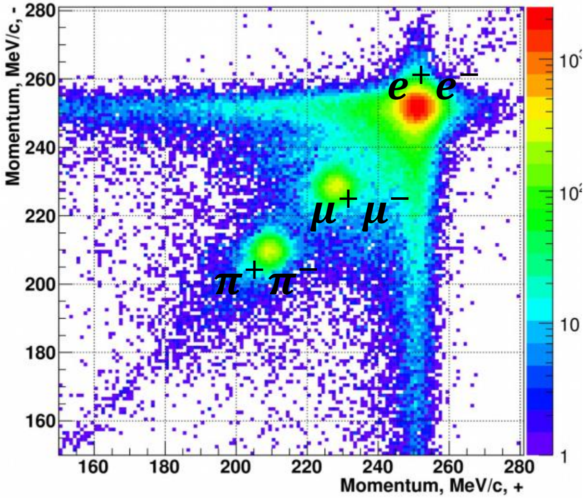
± sign reflects energy deposition and momentum of particle with corresponding charge

Independent check by **angular distribution**

E^- vs E^+ @$\sqrt{s} = 0.78$ GeV



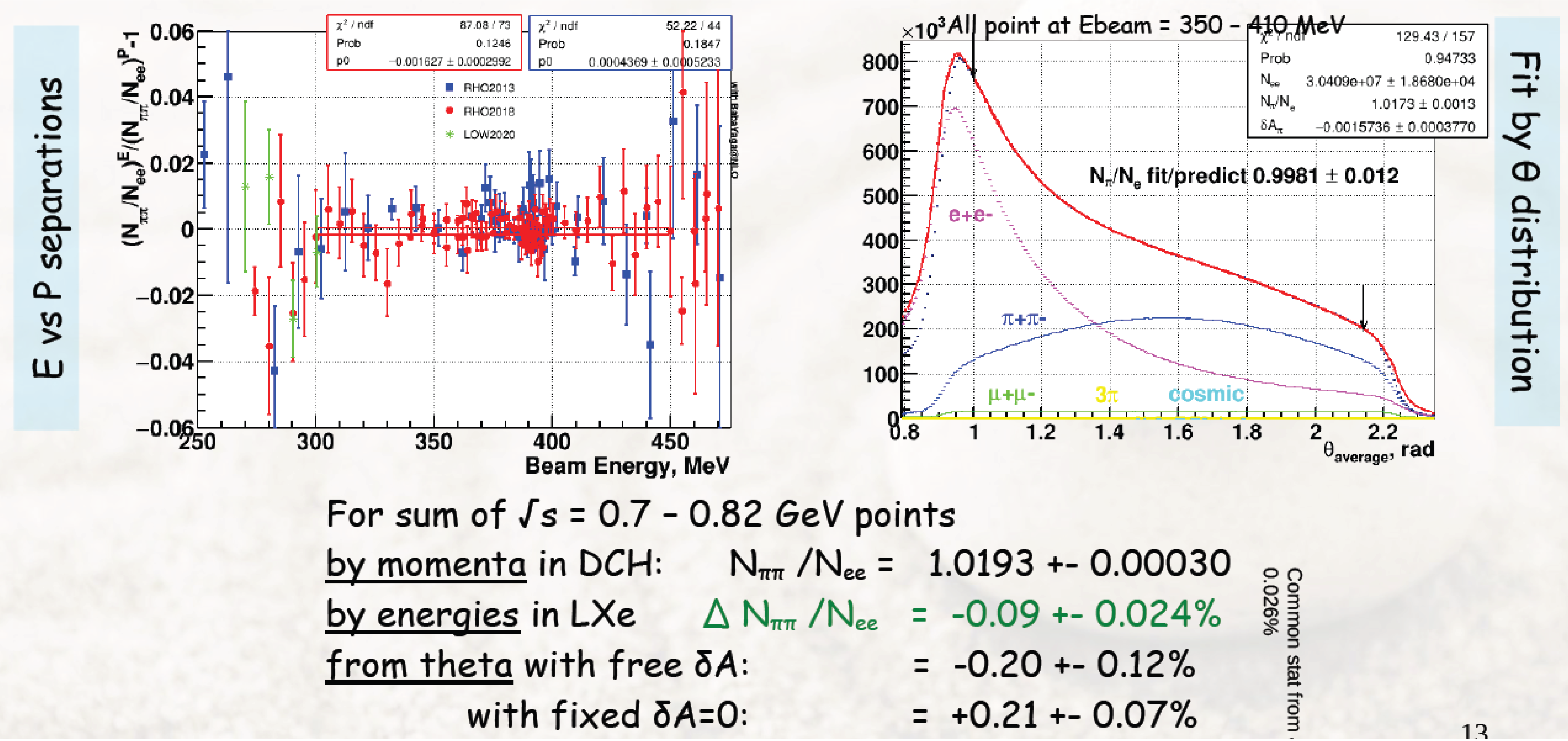
P^- vs P^+ @$\sqrt{s} = 0.5$ GeV



Unique feature of CMD-3: three independent methods to measure $N_{\pi\pi}/N_{ee}$!

Comparison

Three methods agree to 0.2%!



For sum of √s = 0.7 - 0.82 GeV points

by momenta in DCH: $N_{\pi\pi}/N_{ee}$ = 1.0193 +- 0.00030

by energies in LXe $\Delta N_{\pi\pi}/N_{ee}$ = -0.09 +- 0.024%

from theta with free δA: = -0.20 +- 0.12%

with fixed δA=0: = +0.21 +- 0.07%

Common stat from 0.026%

CMD-3 $e^+e^- \to \pi^+\pi^-$ analysis: radiative corrections

Measurement of $e^+e^- \to \pi^+\pi^-$ requires high precision calculation of radiative corrections.

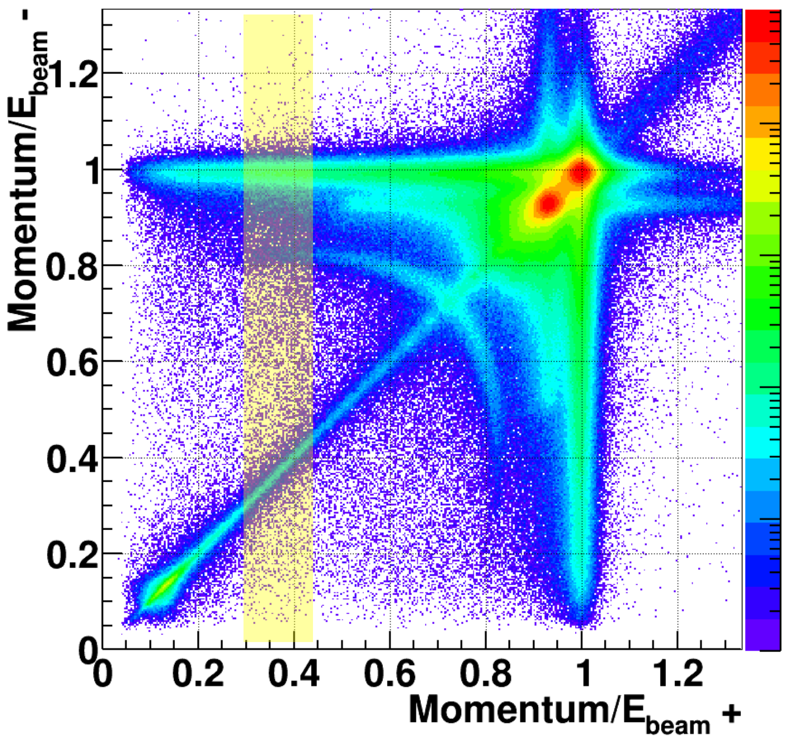
We use two high-precision MC generators for $e^+e^- \to e^+e^-$:

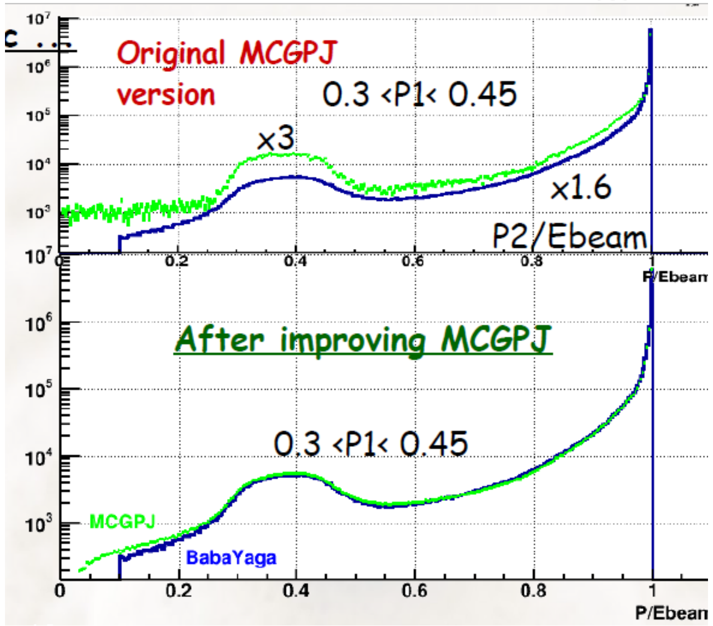
- MCGPJ generator (0.2%)
- BaBaYaga@NLO (0.1%)

With high statistics we've observed inconsistencies in tails of distributions, which were traced to particulars of MCGPJ generator

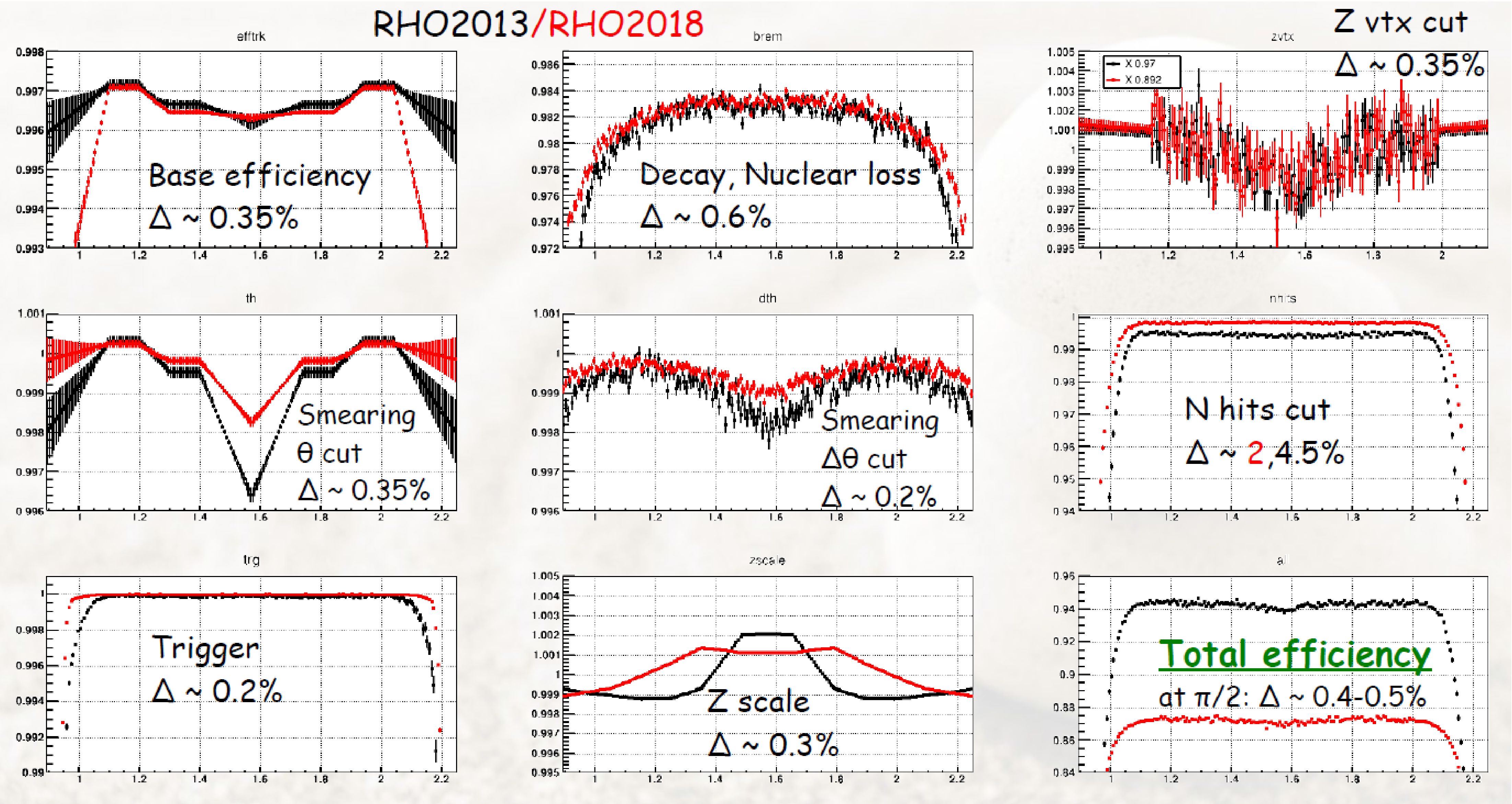
After improvements, tails of e^+e^- spectra still differ by few %, which limits the precision to O(0.1%)

NNLO MC generator for $e^+e^- \to e^+e^-$ is needed for higher precision





Efficiency corrections



RHO2013/RHO2018

Base efficiency
Δ ~ 0.35%

Decay, Nuclear loss
Δ ~ 0.6%

Z vtx cut
Δ ~ 0.35%

Smearing
θ cut
Δ ~ 0.35%

Smearing
Δθ cut
Δ ~ 0.2%

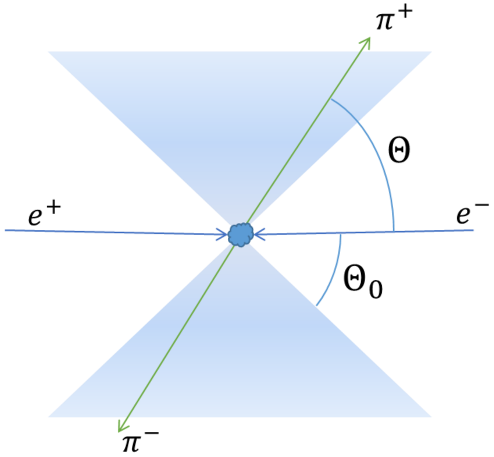
N hits cut
Δ ~ 2,4.5%

Trigger
Δ ~ 0.2%

Z scale
Δ ~ 0.3%

Total efficiency
at π/2: Δ ~ 0.4-0.5%

Measurement of polar angle

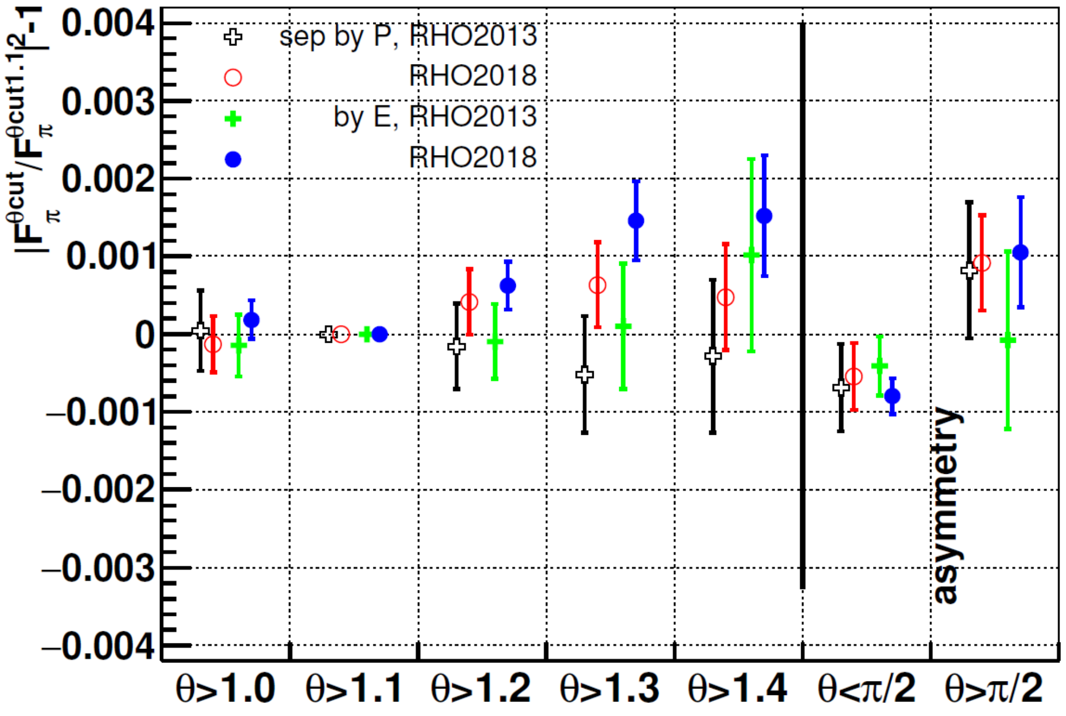


Θ angle is measured by drift chamber via charge division

Two detector systems with strips readout, LXe calorimeter and Z-chamber, are used for precise calibration and monitoring of DC

We need to precisely know the fiducial volume (Θ_0 cut).

$$|F_\pi|^2 = \left(\frac{N_{\pi\pi}}{N_{ee}} - \Delta_{bg}\right) \cdot \frac{\sigma^0_{ee} \cdot (1+\delta_{ee}) \cdot \varepsilon_{ee}}{\sigma^0_{\pi\pi} \cdot (1+\delta_{\pi\pi}) \cdot \varepsilon_{\pi\pi}}$$



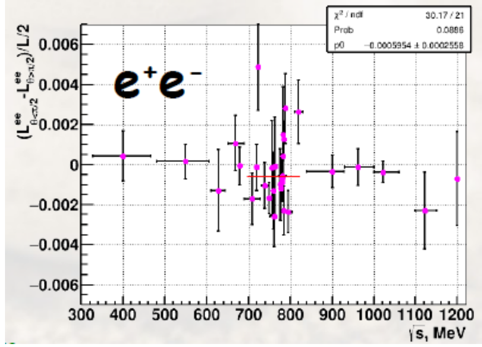
Dependence of result on Θ_0 cut

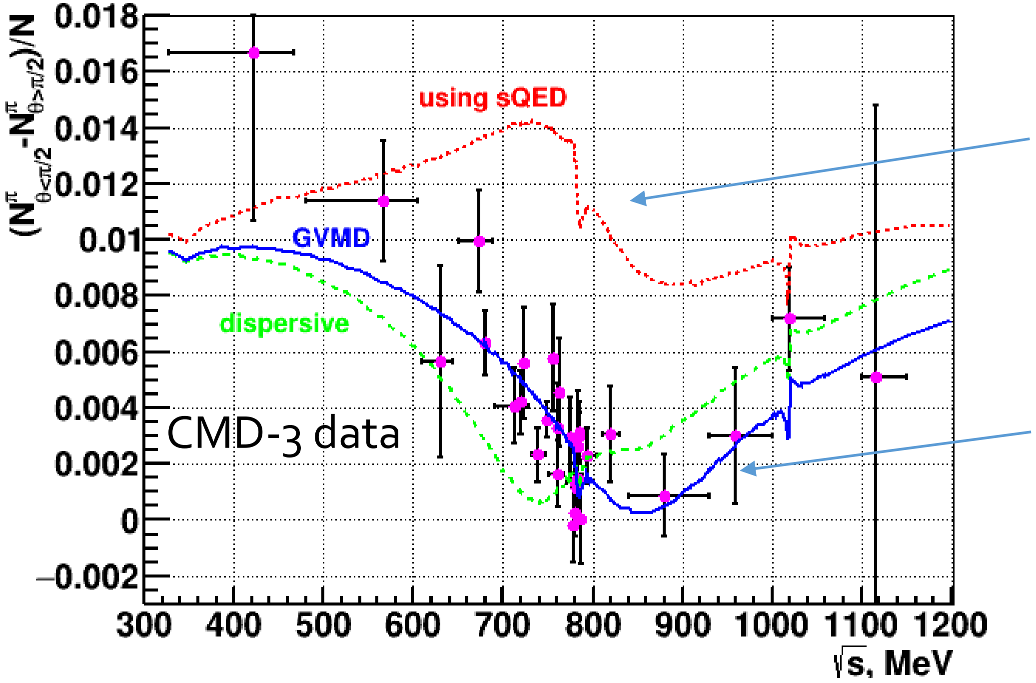
Factor 10 smaller compared to CMD-2, SND2k!

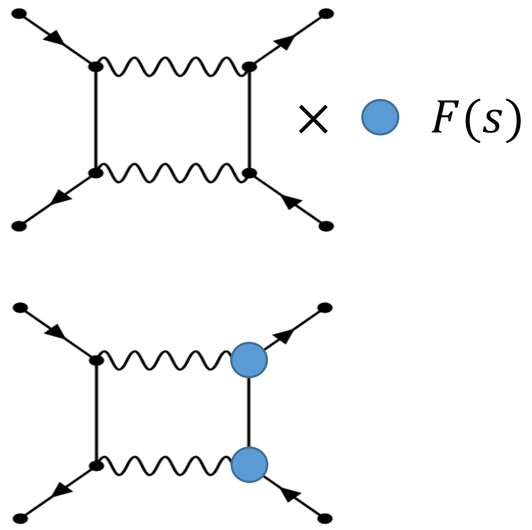
Charge asymmetry in $e^+e^- \to \pi^+\pi^-$

Charge asymmetry in $e^+ e^- \to \pi^+ \pi^-$ is due to interference between ISR/FSR and between one- and two-photon exchange

$$A = \left(N^{\pi}_{\Theta<\pi/2} - N^{\pi}_{\Theta>\pi/2}\right)/N$$







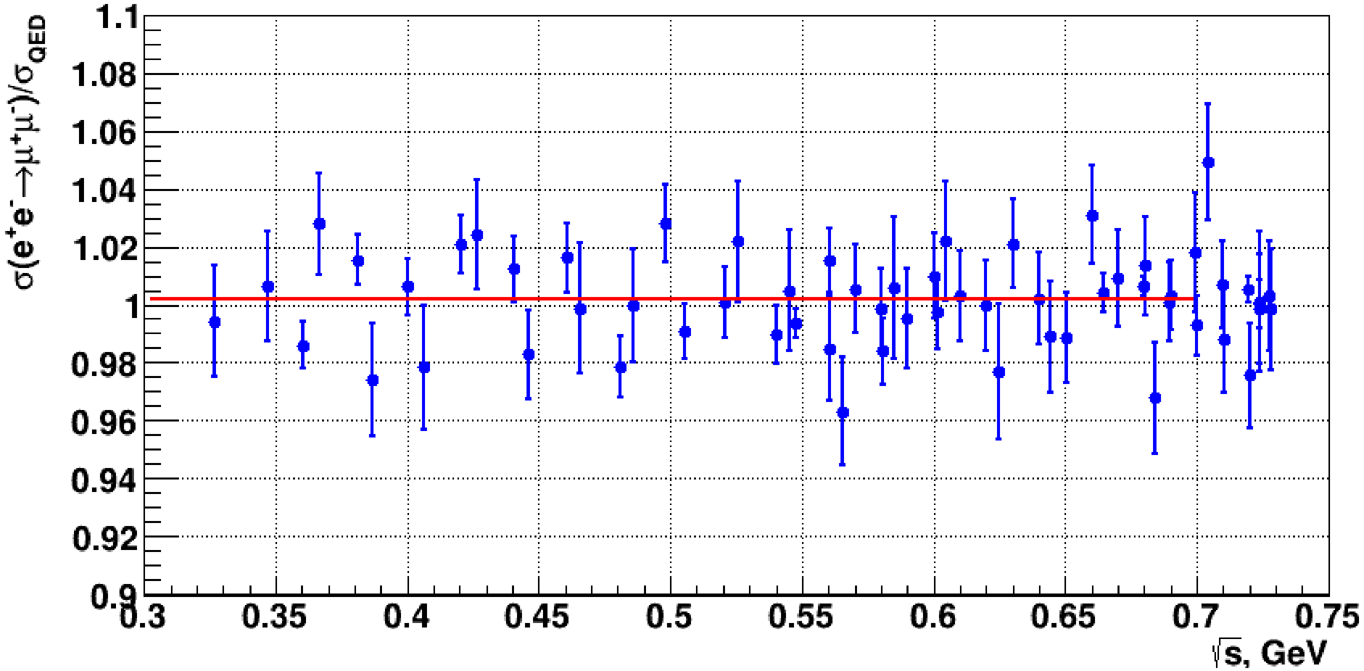
The theoretical model by Lee, Ignatov, PLB 833 (2022) 137283 (GVDM) describes well the CMD-3 data

Recent calculation in dispersive formalism Colangelo et al., JHEP 08 (2022) 295 confirms the effect.

Measurement of $e^+e^- \to \mu^+\mu^-$

$e^+e^- \to \mu^+\mu^-$ events are identified as a by-product of analysis, which allows to measure $\sigma(e^+e^- \to \mu^+\mu^-)$ and compare it to QED prediction

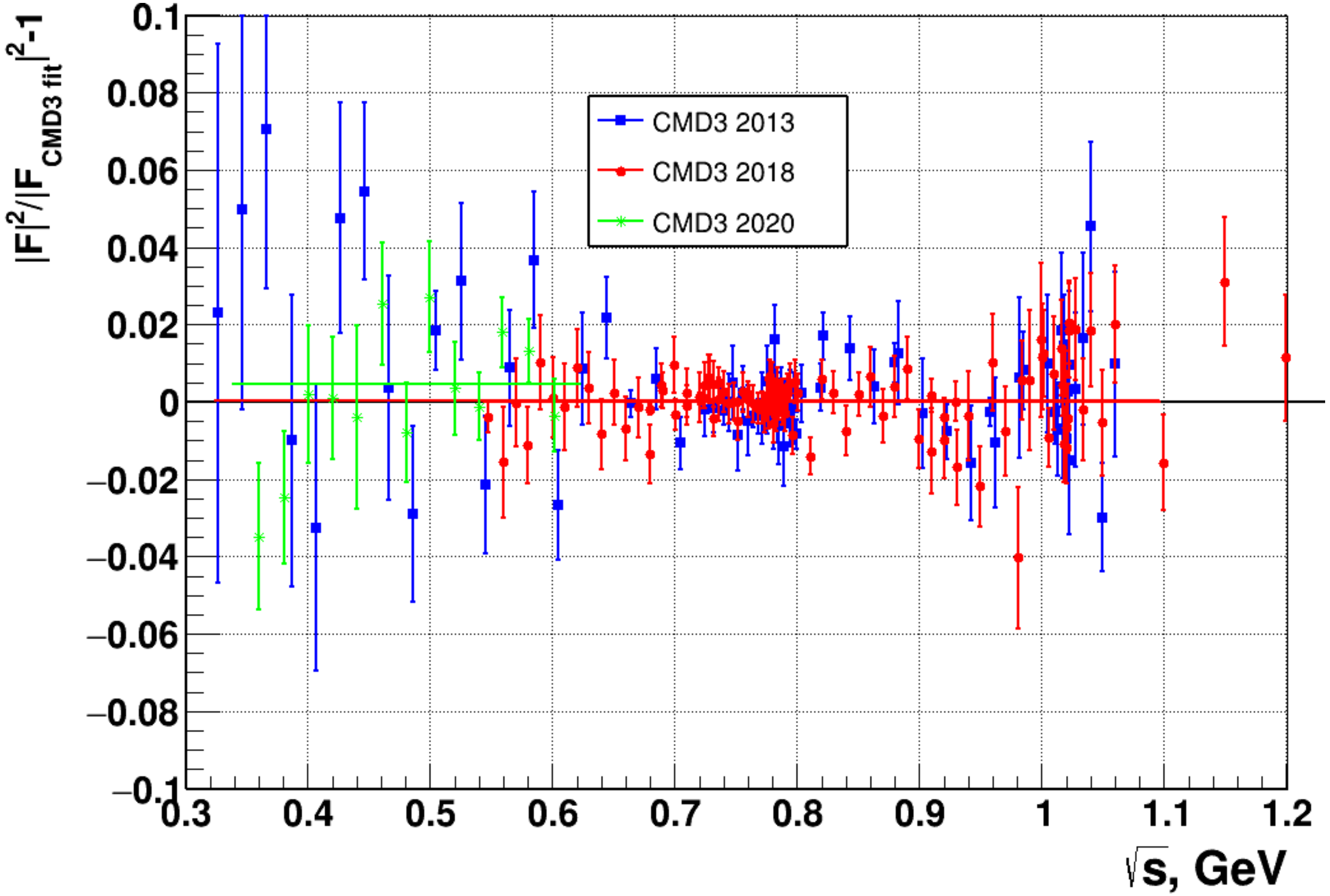
$\sigma(e^+e^- \to \mu^+\mu^-)_{CMD3}/\sigma(e^+e^- \to \mu^+\mu^-)_{QED}$



+0.17 ± 0.16 %

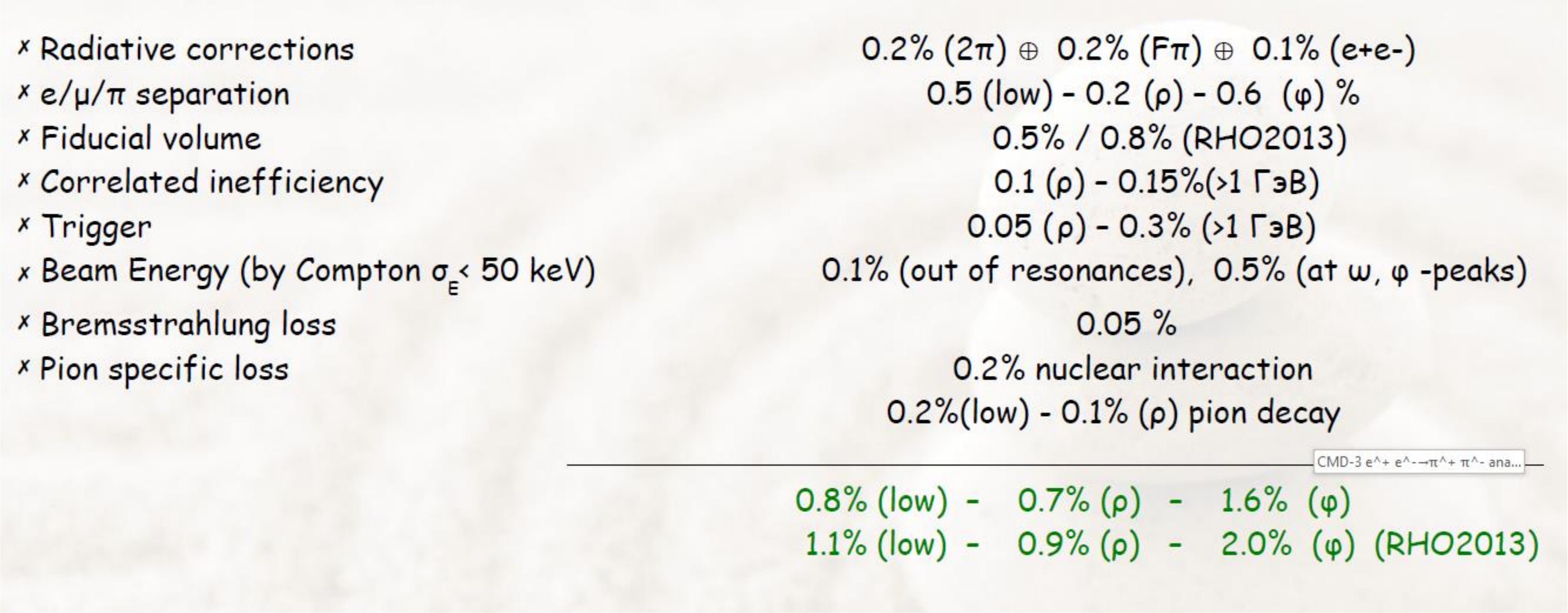
Powerful cross-check of $\sigma(e^+e^- \to \pi^+\pi^-)$ measurement! All ingredients are tested: event separation, detection efficiencies, radiative corrections.

Comparison of data taking seasons



Results based on 2013, 2018 and 2020 data only agree to ~0.1%!
The detector performance and run conditions were significantly different for these runs.

Systematic errors



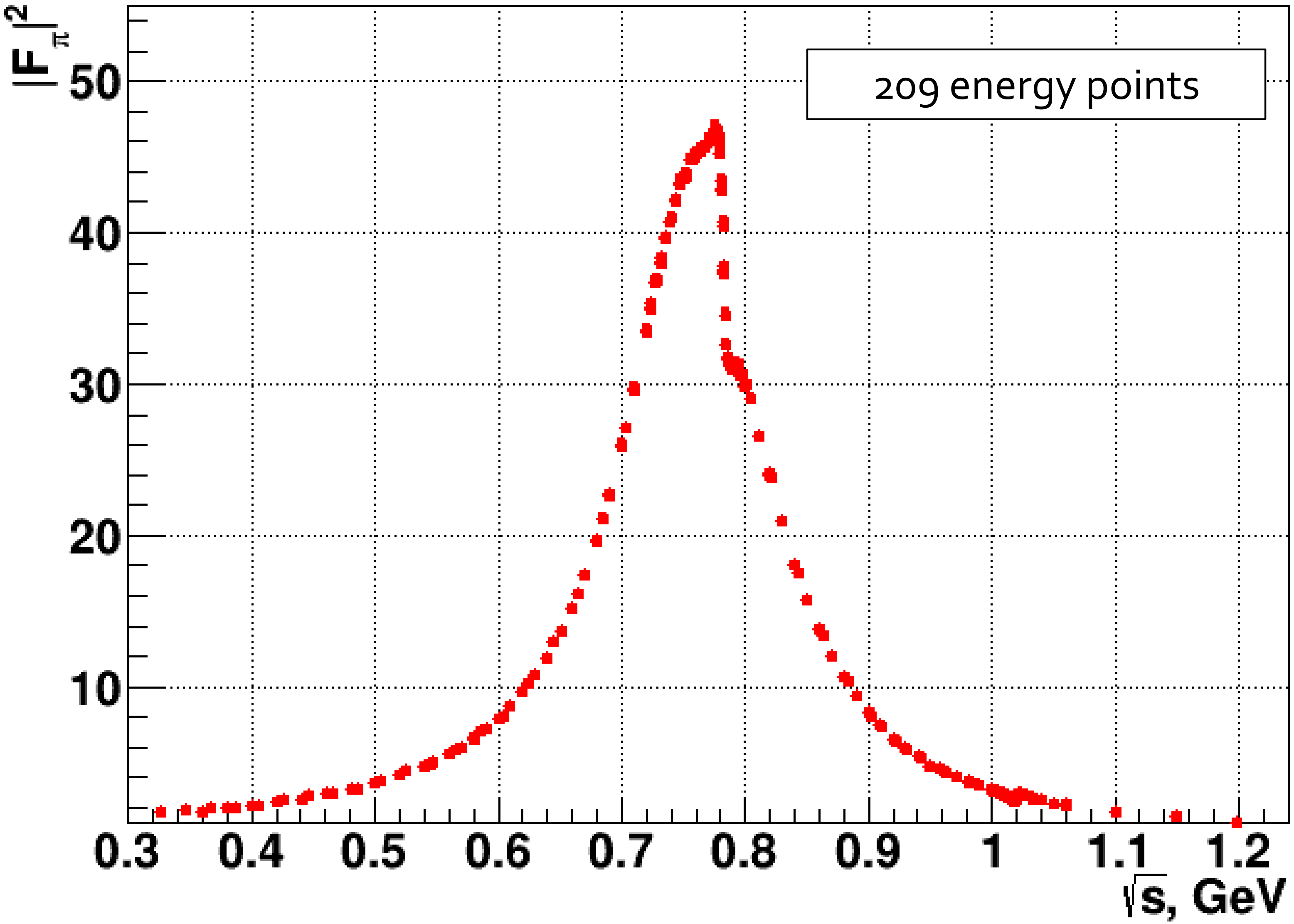
✗ Radiative corrections	0.2% (2π) ⊕ 0.2% (Fπ) ⊕ 0.1% (e+e-)
✗ e/μ/π separation	0.5 (low) - 0.2 (ρ) - 0.6 (φ) %
✗ Fiducial volume	0.5% / 0.8% (RHO2013)
✗ Correlated inefficiency	0.1 (ρ) - 0.15%(>1 ГэВ)
✗ Trigger	0.05 (ρ) - 0.3% (>1 ГэВ)
✗ Beam Energy (by Compton σ_E< 50 keV)	0.1% (out of resonances), 0.5% (at ω, φ -peaks)
✗ Bremsstrahlung loss	0.05 %
✗ Pion specific loss	0.2% nuclear interaction 0.2%(low) - 0.1% (ρ) pion decay

0.8% (low) - 0.7% (ρ) - 1.6% (φ)

1.1% (low) - 0.9% (ρ) - 2.0% (φ) (RHO2013)

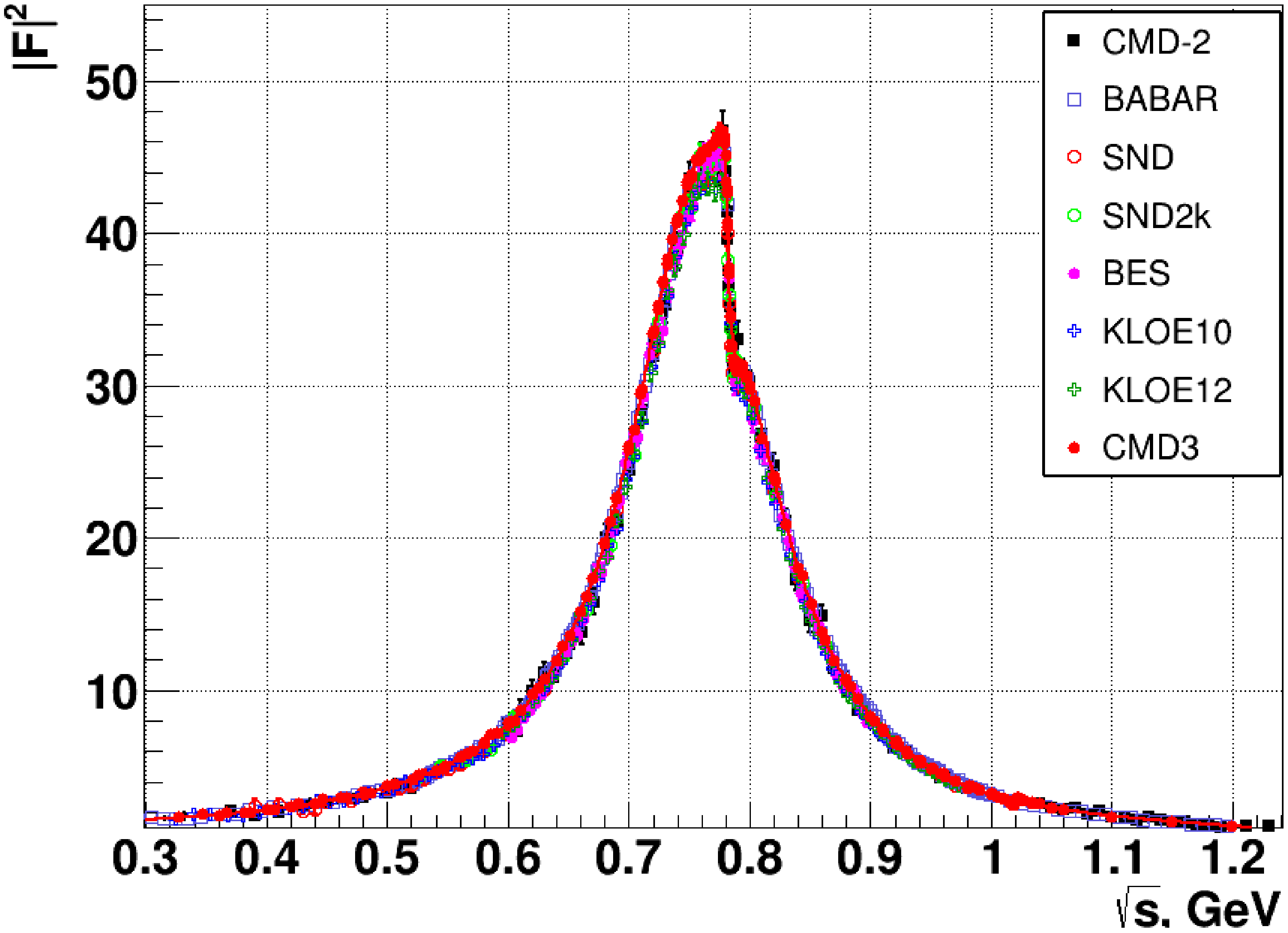
Conservative estimate

Measurement of $e^+e^- \to \pi^+\pi^-$ at CMD-3



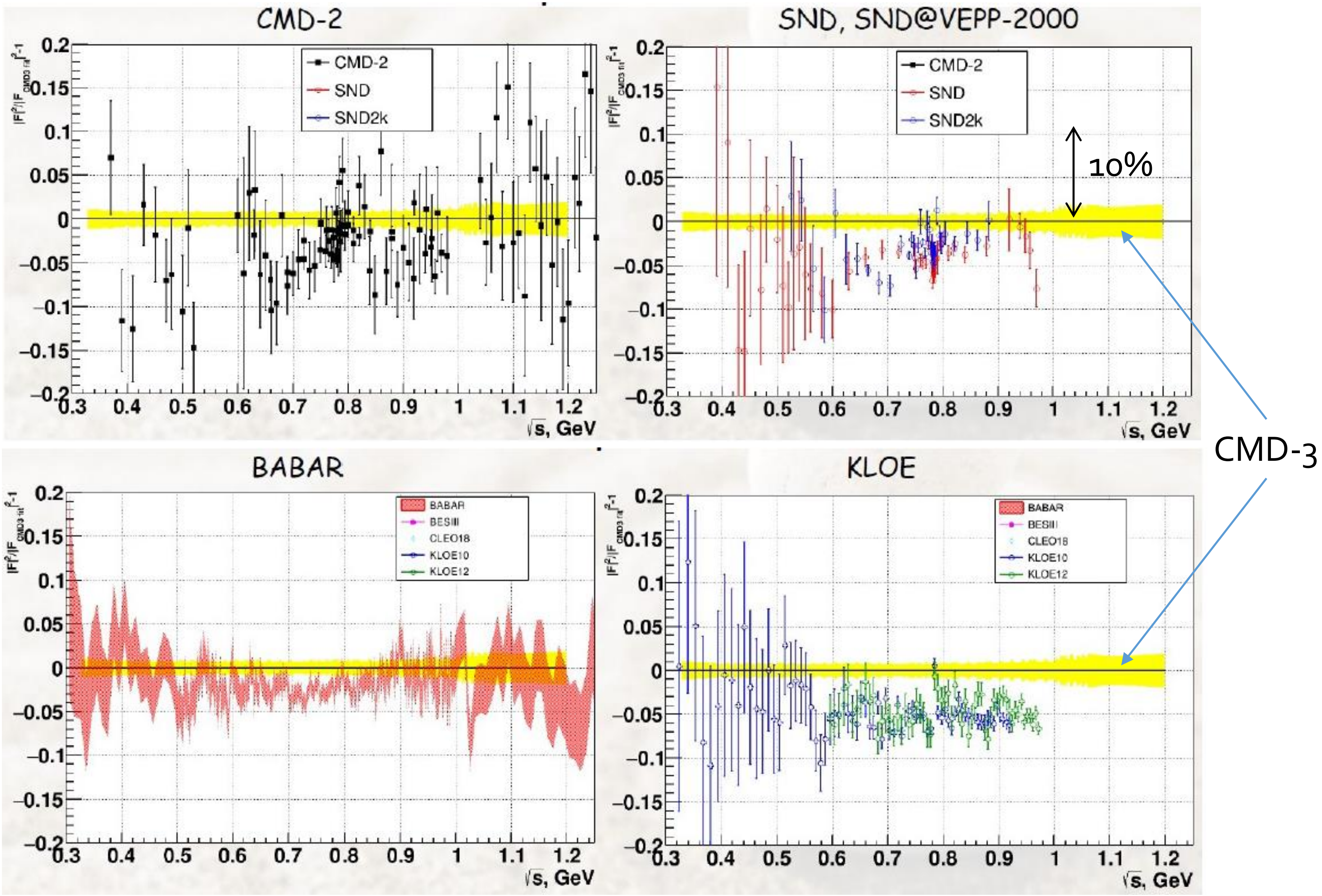
Comparison to other measurements

At first glance, they looks close to each other…

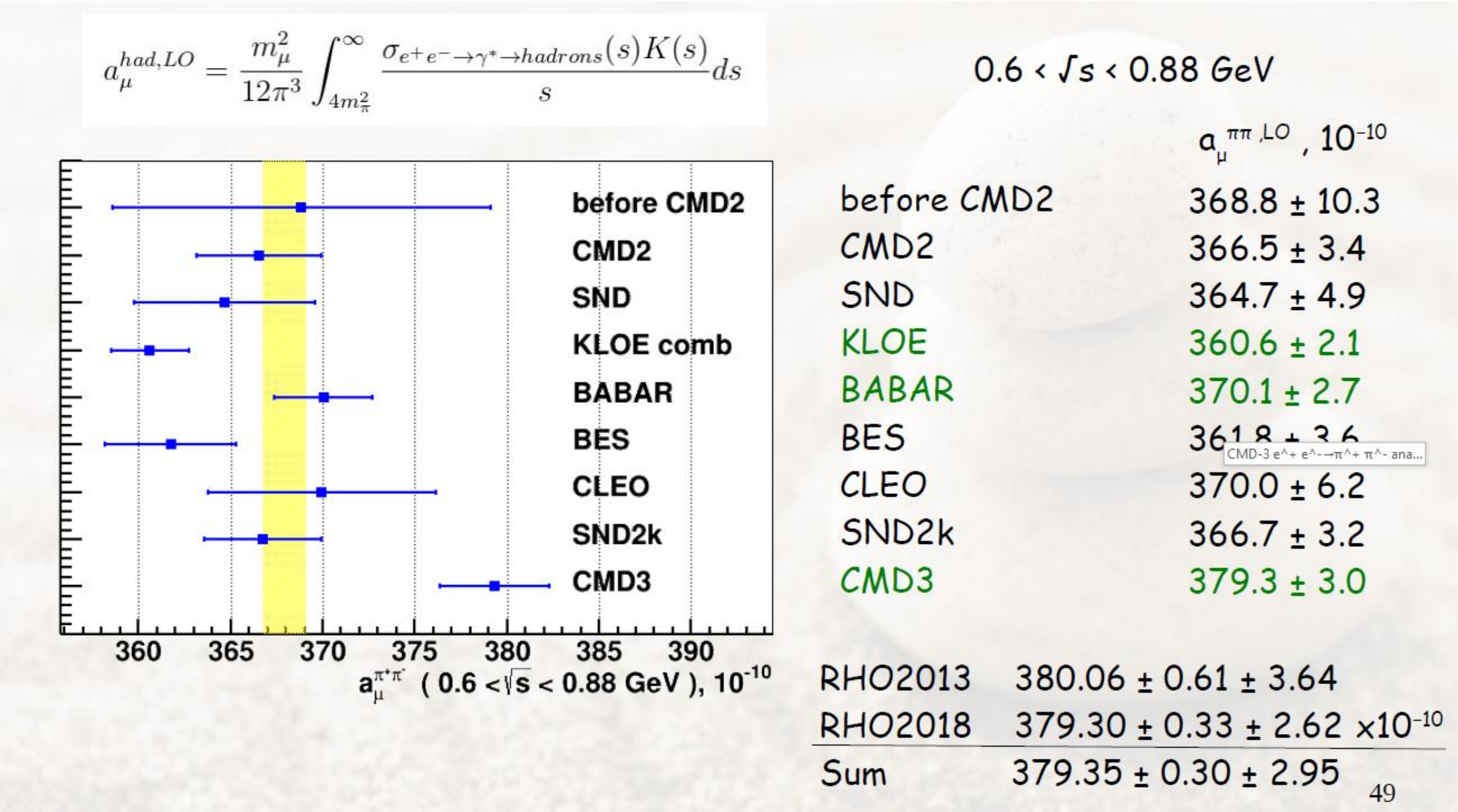


Comparison to other measurements

CMD-3 is systematically above previous measurements by ~2-5%



CMD-3 $e^+e^- \to \pi^+\pi^-$: contribution to g-2



$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty}\frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$

0.6 < √s < 0.88 GeV

	$a_\mu^{\pi\pi,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

CMD-3 e^+ e^-→π^+ π^- ana...

RHO2013	380.06 ± 0.61 ± 3.64
RHO2018	379.30 ± 0.33 ± 2.62 $\times 10^{-10}$
Sum	379.35 ± 0.30 ± 2.95

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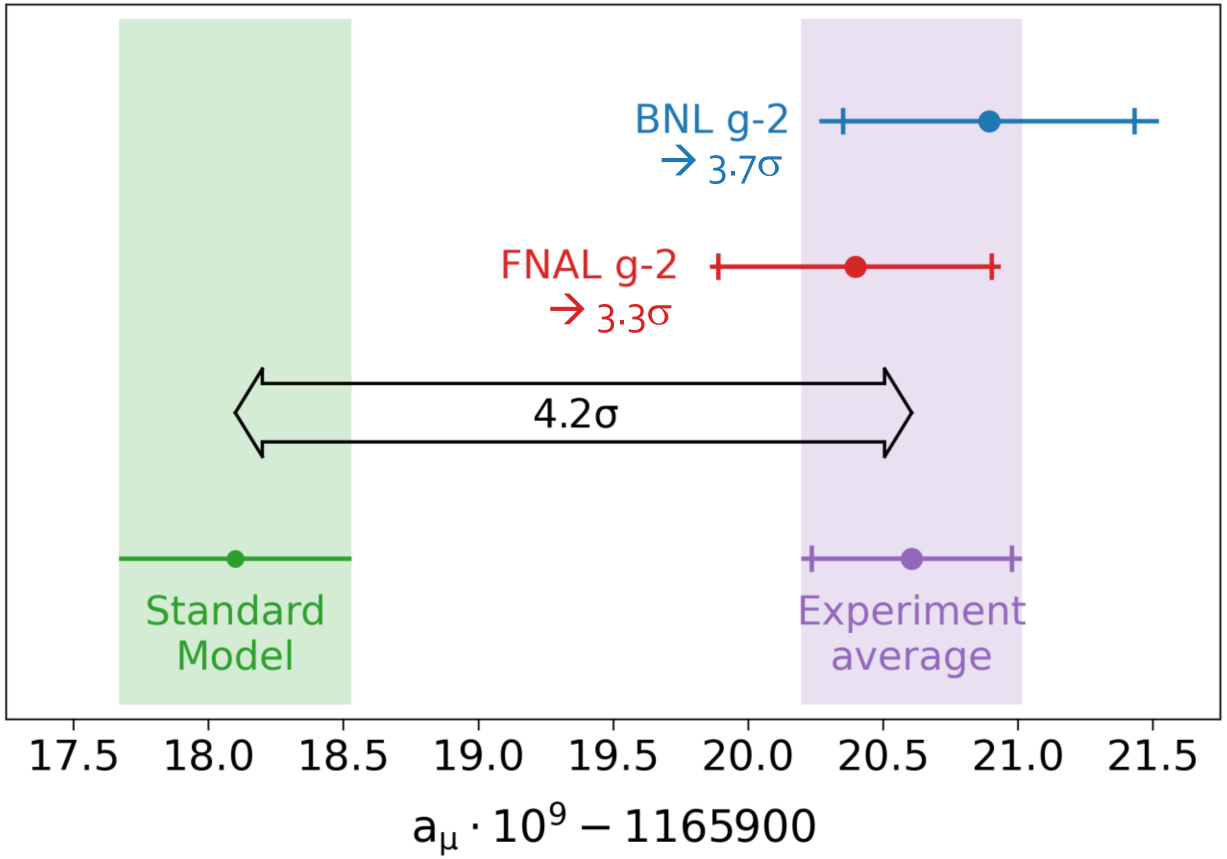
At the beginning of 2023…

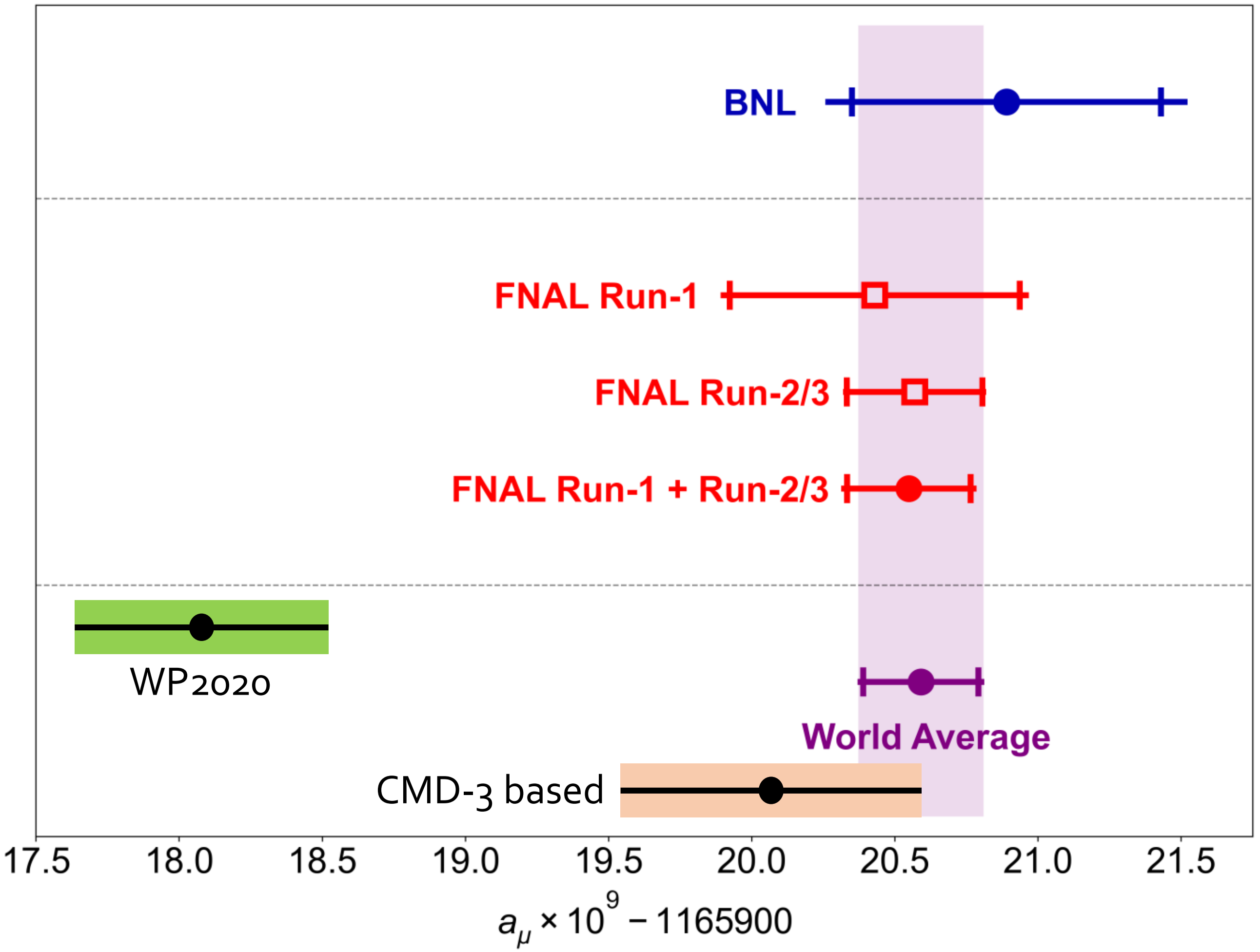
$a_\mu(SM) = 0.00116591810(43)$ → 368 ppb



$a_\mu(Exp) - a_\mu(SM) = 0.00000000251(59)$ → 4.2σ

Experiment vs SM prediction

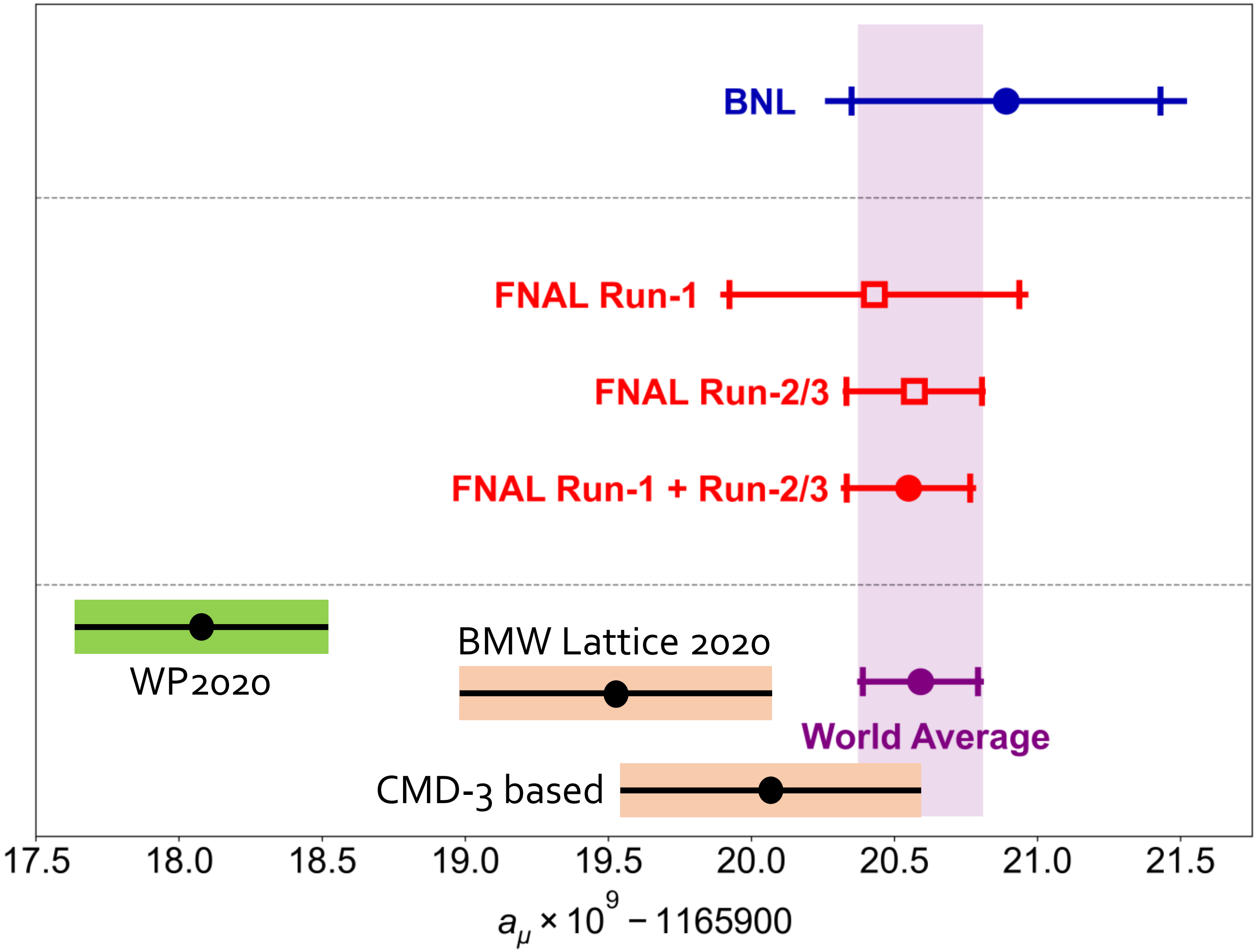
End of 2023



At the moment, the SM prediction for a_μ is unclear (due to hadronic contribution)

Experiment vs SM prediction

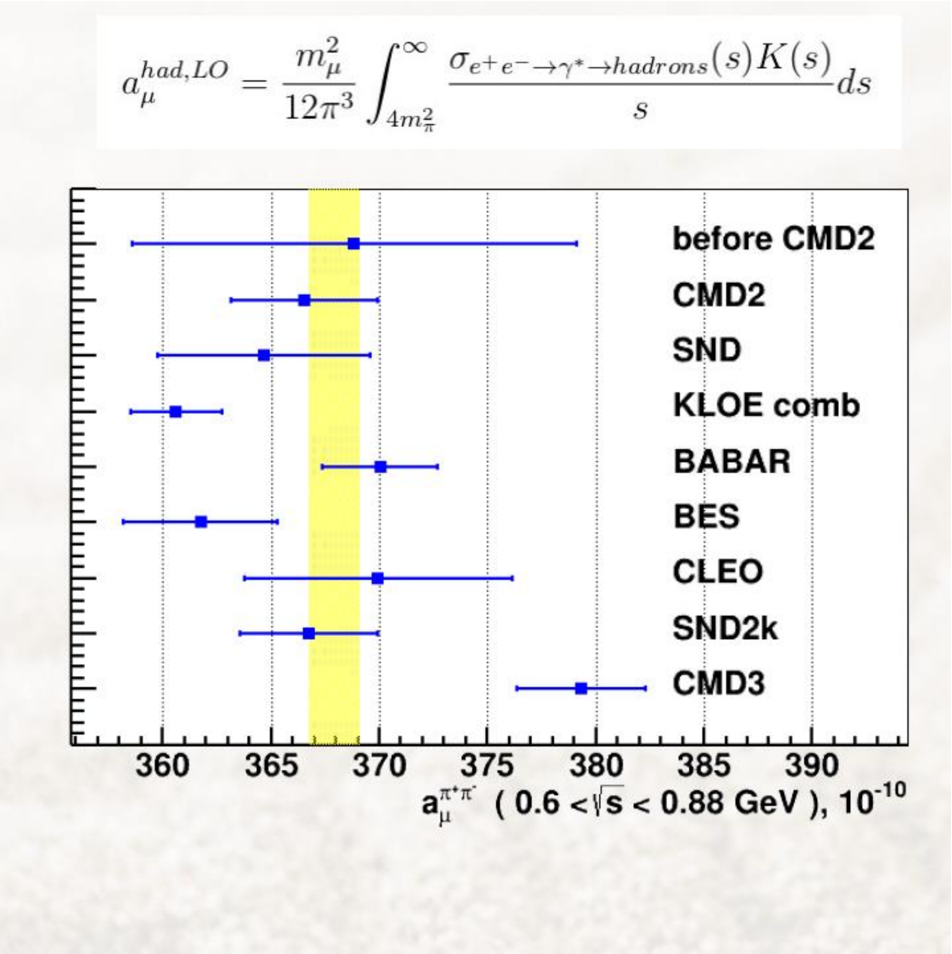
End of 2023



At the moment, the SM prediction for a_μ is unclear (due to hadronic contribution)

What’s next?

The status



$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty}\frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$

Discrepancies in data "blind" $a_\mu(SM)$

All (or all but one) existing measurements of $e^+e^- \to \pi^+\pi^-$ underestimated systematic uncertainty (at least at some energy range)

CMD-3 simply exaggerated the problem, but it was there already

CMD-3: what we could do wrong?

CMD-3 measurement has many internal cross-checks which doesn’t leave much space for unknowns.

- Is there problem with angle measurement (fiducial volume)?
  Unlikely: two systems are used; there is measurement of asymmetry; angle distribution agrees with simulation
- Is there problem with RC calculation?
  Unlikely as a source of discrepancy: CMD-2 and SND use the same code, and measurement of asymmetry agrees with RC MC generator. But there could be potential systematic shift in RC common for CMD-X/SND (e.g. for pions due to limitations of sQED).
- Is there problem with event separation?
  Unlikely: three methods agree (CMD-3 is the first measurement with several methods)
- Is there problem with trigger or detection efficiencies?
  Unlikely: should lead to shift of $\sigma(\mu\mu)$.
- Stupid mistake?
  Always possible, but we’ve done the whole analysis on MC data
- Unaccounted physical background which mimics $e^+e^- \to \pi^+\pi^-$?
  Possible, but we accounted for all known backgrounds from e^+e^- annihilation. Something else? Beam/residual gas interactions?

Prospects for SM prediction

Discrepancies in $e^+e^- \to H$ data make the SM prediction “blinded”

As of today, we don’t have established estimate of $a_\mu(SM)$

There are significant efforts to understand the discrepancies and to obtain additional new $e^+e^- \to H$ data:

- SND has the same amount of data collected as CMD-3, analysis is in progress
- BABAR is making reanalysis of old data using new approach (angular analysis)
- KLOE-2 started analysis of collected data, not analyzed before
- BELLE-II plans to do ISR measurement of $e^+e^- \to H$ cross sections

There is dedicated experiment, Muone, being prepared at CERN to measure hadronic contribution via $e\mu$ scattering

There is fast progress in lattice calculations

There are good chances to improve precision of SM prediction in coming years

Is there need for new measurements of hadronic cross sections?

Any value of $\Delta a_\mu(New\ Physics) = a_\mu(exp) - a_\mu(SM)$ is valuable!

FNAL expected precision of 140 ppb corresponds to $0.25\% \cdot a_\mu^{had,LO}$

HVP contribution: $a_\mu(had) = \int \sigma_{e^+e^-\to\text{адроны}}(s)\, K(s)\ ds$

In order to get HVP accuracy to match FNAL accuracy, cross sections need to be measured to ~0.2% (CMD-3: ~0.8%)

Channel	Contribution, $\cdot\ 10^{10}$ (KNT19)	Relative accuracy, need (now)
$\pi^+\pi^-$	504.23(1.90) (0.4%) ???	0.23% (0.8%)
$\pi^+\pi^-\pi^0$	46.63(94) (2.0%)	1.1% (1.5-3%)
$\pi^+\pi^-\pi^+\pi^-$	13.99(19) (1.4%)	0.8% (2-3%)
$\pi^+\pi^-\pi^0\pi^0$	18.15(74) (4.0%)	2.3% (5%)
K^+K^-	23.00(22) (1.0%)	0.6% (2%)
$K_S K_L$	13.04(19) (1.5%)	0.7% (2%)
$\boldsymbol{a_\mu(had;LO)}$	**692.8(2.4) (0.35%)**	**0.2%**

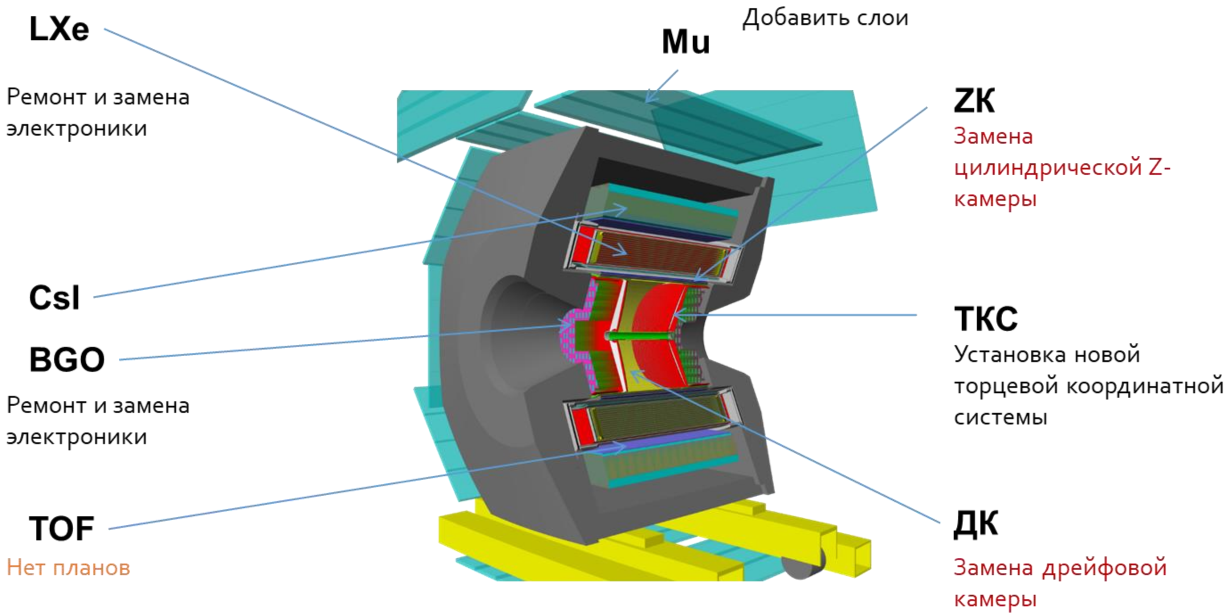
CMD-3 plans

The CMD-3 measurement is systematically limited – detector upgrade.

Detector upgrades under discussions: new drift chamber, new Z-chamber at inner and outer radii (probably, integrated with DC), *dedicated PID/TOF?,…*

The goal is to reach ~0.2-0.3% in $\sigma(e^+e^- \to \pi^+\pi^-)$

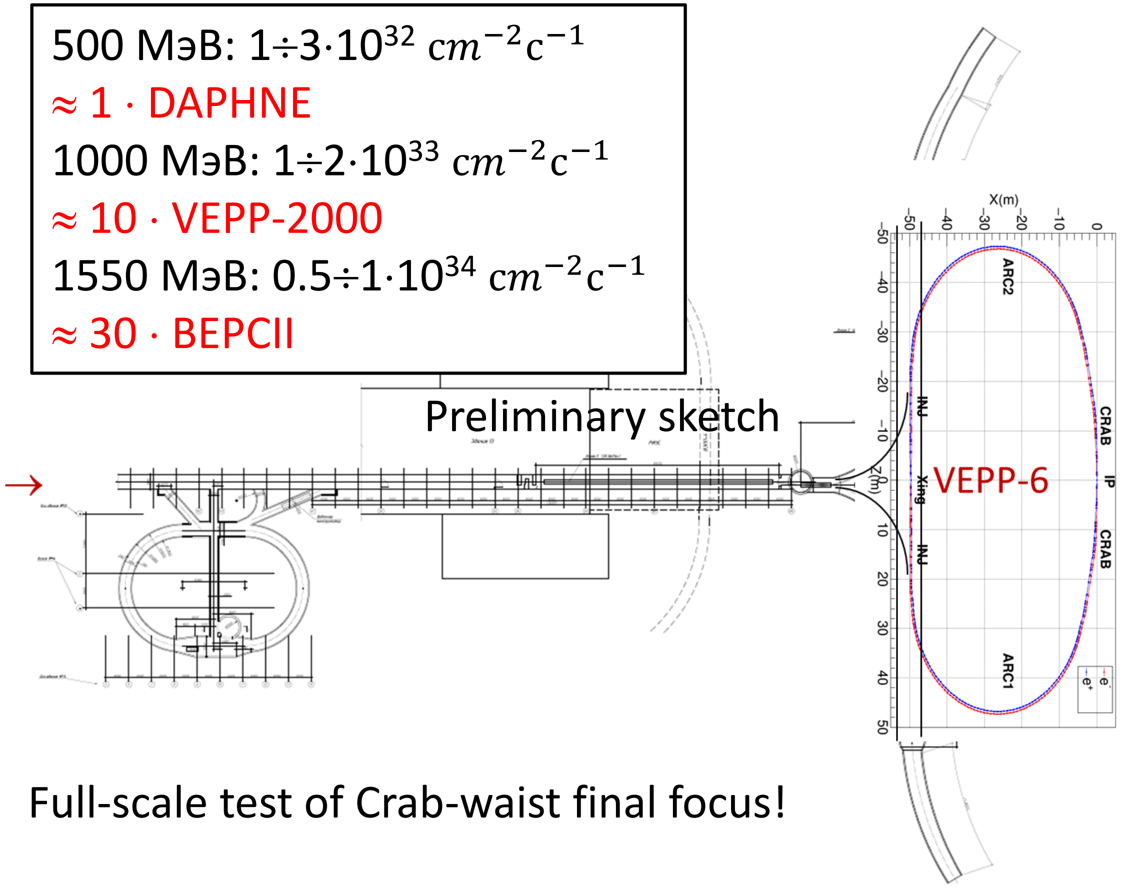
The precision critically depends on development on new generation of MC generators for radiative corrections



Under consideration: VEPP-6

- e^+e^- collider
  - Beam energy from <0.5 to 1.6 GeV (J/ψ) (2.0 GeV)
  - Luminosity $\mathcal{L} \approx 10^{\mathbf{34}}\ cm^{-2}c^{-1}$ @ 1.6 GeV
- General purpose detector
  - Tracking
  - Calorimetry
  - Particle ID
- Physics
  - J/ψ decays
  - Baryon thresholds
  - Measurement of R
  - … Complementary to Super charm-tau factory

500 МэВ: 1÷3·10^{32} $cm^{-2}c^{-1}$
≈ 1 · DAPHNE
1000 МэВ: 1÷2·10^{33} $cm^{-2}c^{-1}$
≈ 10 · VEPP-2000
1550 МэВ: 0.5÷1·10^{34} $cm^{-2}c^{-1}$
≈ 30 · BEPCII



Preliminary sketch

e+e- from → existing injector

Full-scale test of Crab-waist final focus!

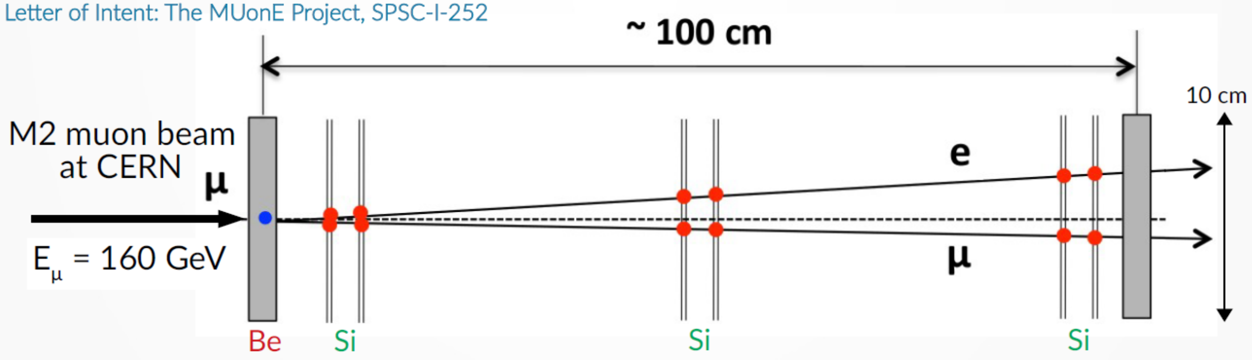
MUonE @CERN

Dedicated experiment to measure hadronic contribution in t-channel.

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

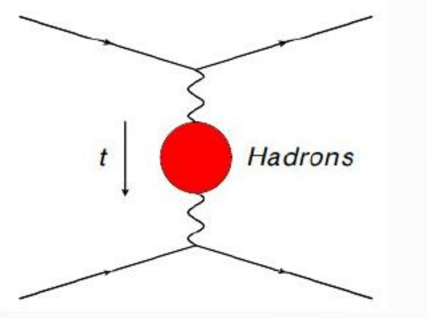
Lautrup, Peterman, De Rafael, Phys. Rep. C3 (1972), 193

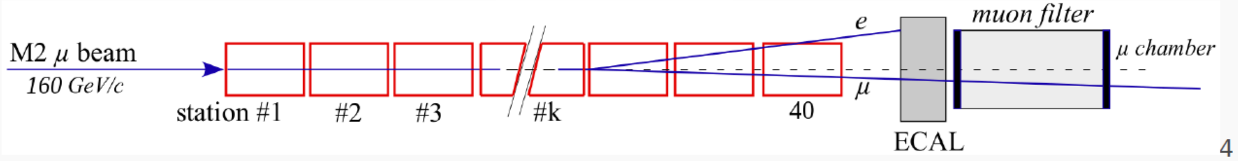
Letter of Intent: The MUonE Project, SPSC-I-252



Be (or C) target
1.5 cm thickness

Tracking system:
3 pairs of silicon strip detectors (CMS 2S modules)



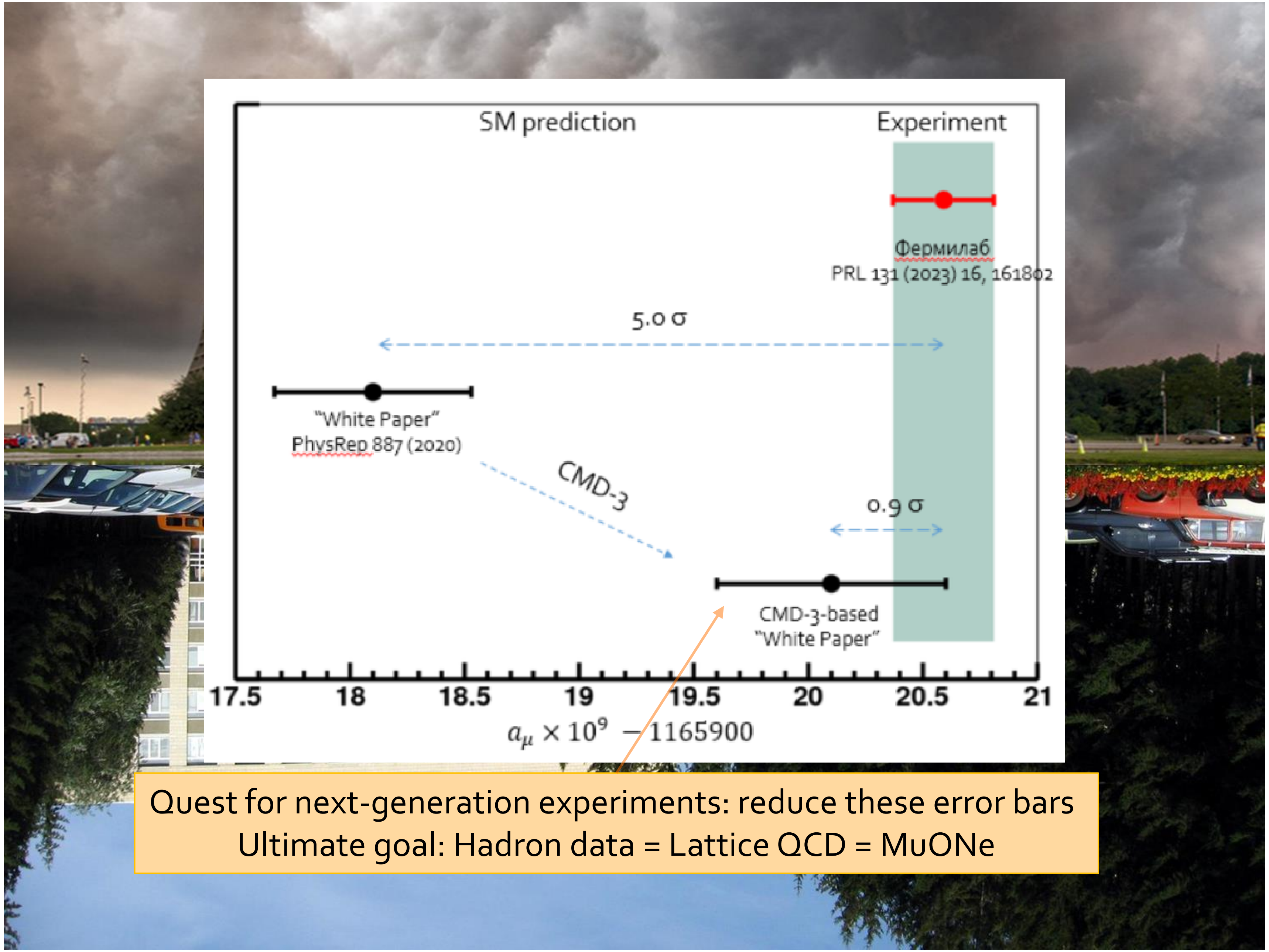


Measured: angular distribution of μe scattering; $4 \cdot 10^{12}$ events!

Now: proof-of-concept data taking; final result after LHC LS3 (2029-)

Conclusion

Conclusion



Quest for next-generation experiments: reduce these error bars
Ultimate goal: Hadron data = Lattice QCD = MuONe